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ROTATION EFFECT ON UNSTEADY MHD FLOW PAST AN IMPULSIVELY STARTED VERTICAL PLATE WITH VARIABLE TEMPERATURE IN POROUS MEDIUM

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ABSTRACT

This paper investigates the rotation effects on unsteady MHD flow past an impulsively started vertical plate with variable temperature in porous medium. Analytical solutions are found for the governing equations using the Laplace Transform method. A selected set of graphical results illustrating the effects of various parameters involved in the problem are presented and discussed. The numerical values of skin-friction have been tabulated.

Keywords: *Rotation effects, MHD, Temperature, Porous medium.*

1. INTRODUCTION

Magneto-hydrodynamic (MHD) boundary layer with heat transfer from vertical surfaces embedded in porous medium has been the subject of many engineering and geophysical applications such as geothermal reservoirs, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, power generation system and aero-dynamics, drying of porous solids, groundwater pollution, solar collector, heat exchanger, building heating and cooling underground energy transport etc. Magnetic field is also an important control parameter for heat transfer.

In view of their applications in industry and engineering, the study of uniform fluid flow on bodies of various geometries has been considered by many researchers using different analytical and numerical methods. For instance, Stewartson [1] worked on the impulsive motion of a flat plate in a viscous fluid. Radiation and mass transfer effects on two-dimensional flow past an impulsively started infinite vertical plate was studied by Prasad *et al.* [8]. Further Ibrahim and Makinde [10] considered chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate with suction. Again they [12] studied the radiation effect on chemically reacting magneto-hydrodynamics (MHD) boundary layer flow of heat and mass transfer through a porous vertical flat plate. Singh *et al.* [9] have analyzed the effect of thermally stratified ambient fluid on MHD convective flow along a moving non-isothermal vertical plate. Effects of mass transfer and free convection on the flow past an impulsively started vertical plate was studied by Soundalgekar [2]. Further Das *et al.* [13] considered the model, “An unsteady hydromagnetic flow of a heat absorbing dusty fluid past a permeable vertical Plate with ramped temperature”. Oscillating plate temperature effects on a flow past an infinite porous plate with constant suction and embedded in a porous medium was considered by Jaiswal and Soundalgekar [7]. Das *et al.* [4] analysed the effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Further the Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux was studied by Jha *et al.* [3]. Radiation and free convection flow past a moving plate was considered by Raptis and Perdikis [6]. Hossain and Takhar [5] have studied the radiation effects on mixed convection along a vertical plate with uniform surface temperature using finite difference method.

Further Rajput and Kumar [11] considered rotation and radiation effects on MHD flow past an impulsively started vertical plate with variable temperature. This paper investigates the rotation effects on unsteady MHD flow past an impulsively started vertical plate with variable temperature in porous medium. The results are shown with the help of graphs (Figure-1 to Figure-12) and table-1.

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2. MATHEMATICAL ANALYSIS

Consider an unsteady MHD flow of a viscous incompressible electrically conducting fluid past an impulsively started vertical plate with variable temperature. The fluid and the plate rotate as a rigid body with a uniform angular velocity $\bar{\Omega}$ about $\bar{z}$-axis in the presence of an applied uniform magnetic field B_o normal to the plate. The fluid motion is induced due to the impulsive movement of the plate as well as the free convection due to heating of the plate. Initially, at time $\bar{t} \leq 0$, the fluid and the plate are at rest and at a uniform temperature $\bar{T}_\infty$. At time $\bar{t} > 0$, the plate starts moving with a velocity u_o in its own plane and the temperature of the plate is raised to $\bar{T}_\omega$. As the plate occupying the plane $\bar{z}$ = 0 is of infinite extent, all the physical quantities depend only on $\bar{z}$ and $\bar{t}$. As the fluid is electrically conducting whose magnetic Reynolds number is very small and hence the induced magnetic field produced by the fluid motion is negligible in comparison to the applied one. Under the above assumptions, the governing equations with Boussinesq's approximation are as follows:

$$\frac{\partial \bar{u}}{\partial \bar{t}} - 2\bar{\Omega}\,\bar{v} = g\beta(\bar{T} - \bar{T}_\infty) + \nu\frac{\partial^2 \bar{u}}{\partial \bar{z}^2} - \frac{\sigma {B_o}^2}{\rho}\bar{u} - \frac{\mu}{K}\bar{u}, \tag{1}$$

$$\frac{\partial \bar{v}}{\partial \bar{t}} + 2\bar{\Omega}\,\bar{u} = \nu\frac{\partial^2 \bar{v}}{\partial \bar{z}^2} - \frac{\sigma {B_o}^2}{\rho}\,\bar{v} - \frac{\mu}{K}\,\bar{v}, \tag{2}$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} = \alpha\,\frac{\partial^2 \bar{T}}{\partial \bar{z}^2}. \tag{3}$$

The boundary conditions taken are as under:

$$\left.\begin{array}{lll} \bar{t} \leq 0: & \bar{u} = 0,\ \bar{T} = \bar{T}_\infty & for\ all\ values\ of\ \bar{z}, \\ \bar{t} > 0: & \bar{u} = u_o,\ \bar{T} = \bar{T}_\infty + (\bar{T}_\omega - \bar{T}_\infty)\frac{{u_o}^2}{\nu}\bar{t} & for\ \bar{z} = 0, \\ & \bar{u} \to 0,\ \bar{T} \to \bar{T}_\infty & as\ \ \bar{z} \to \infty, \end{array}\right\} \tag{4}$$

where the symbols are: $\bar{T} -$ temperature of the fluid, $\bar{T}_\infty -$ temperature of the fluid far away from the plate, $\bar{T}_\omega -$ temperature at the wall, $B_o -$ exrernal magnetic field, $\bar{u}$ −primary velocity of the fluid, $\bar{v}$ −secondary velocity of the fluid, $u_o -$ velocity of the Plate, K −permeability parameter, $\bar{z} -$ spatial coordinate normal to the plate, $\bar{t}$ −time, β −volumetric coefficient of thermal expansion, α−thermal diffusivity, g− acceleration due to gravity, ρ− density, υ −kinematic viscosity, σ−Stefan-Boltzmann constant and $\bar{\Omega} -$ rotation parameter.

To obtain the equations in dimensionless form, the following non-dimensional quantities are introduced:

$$\left.\begin{array}{lll} u = \dfrac{\bar{u}}{u_o}, & v = \dfrac{\bar{v}}{u_o}, & t = \dfrac{{u_o}^2}{\upsilon}\bar{t}, \\ z = \dfrac{u_o}{\upsilon}\bar{z},\ \ G_r = \dfrac{g\beta\upsilon(\bar{T}_\omega - \bar{T}_\infty)}{{u_o}^3}, & & \Omega = \dfrac{\upsilon}{{u_o}^2}\,\bar{\Omega}, \\ \theta = \dfrac{(\bar{T} - \bar{T}_\infty)}{(\bar{T}_\omega - \bar{T}_\infty)}, & M = \dfrac{\sigma B_0^2\upsilon}{\rho {u_o}^2}, & \mathrm{P_r} = \dfrac{\upsilon}{\alpha}, \end{array}\right\} \tag{5}$$

where u is dimensionless primary velocity of the fluid, v − dimensionless secondary velocity of the fluid, z − dimensionless spatial coordinate normal to the plate, $\theta -$ dimensionless temperature, $\mathrm{P_r}$ − Prandtl number, G_r −thermal Grashof number t−dimensionless time, Ω − dimensionless rotation parameter and M−magnetic field parameter. The equations (1), (2), and (3) become:

$$\frac{\partial u}{\partial t} - 2\,\Omega v = G_r\theta + \frac{\partial^2 u}{\partial z^2} - \left(M + \frac{1}{K}\right)u\,, \tag{6}$$

$$\frac{\partial v}{\partial t} + 2\Omega u = \frac{\partial^2 v}{\partial z^2} - \left(M + \frac{1}{K}\right)v\ , \tag{7}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\mathrm{P_r}}\frac{\partial^2 \theta}{\partial z^2}. \tag{8}$$

The corresponding boundary conditions given in equation (4) become:

$$\left.\begin{array}{lll} t \leq 0: & u = 0,\ \theta = 0 & for\ all\ values\ of\ z, \\ t > 0: & u = 1,\ \theta = t & for\ z = 0, \\ & u \to 0,\ \theta \to 0 & as\ \ z \to \infty. \end{array}\right\} \tag{9}$$

To solve above system, take $q = u + iv$. Then using equations (6) and (7), we get,

$$\frac{\partial q}{\partial t} = G_r\theta + \frac{\partial^2 q}{\partial z^2} - mq \tag{10}$$

where $m = M + 2\Omega i + \frac{1}{K}$.

The boundary conditions (9) are reduced to:

$$\left.\begin{array}{lll} t \le 0: & q = 0,\ \theta = 0 & for\ all\ values\ of\ z, \\ t > 0: & q = 1,\ \theta = t & for\ z = 0, \\ & q \to 0,\ \theta \to 0 & as\ z \to \infty. \end{array}\right\} \tag{11}$$

The governing non-dimensional partial differential equations (8) and (10) subject to the above boundary conditions prescribed in equation (11) are solved using the Laplace Transform technique. The solution is as under:

$$q(z,t) = \frac{1}{2} e^{-\sqrt{m}\,z}\left\{1 + Erf\left(\frac{2t\sqrt{m} - z}{2\sqrt{t}}\right) + e^{2\sqrt{m}\,z}\, Erfc\left(\frac{2t\sqrt{m} + z}{2\sqrt{t}}\right)\right\}$$

$$+\frac{G_r}{4m^2}\left[\begin{array}{c} 2e^{-\sqrt{m}\,z}\, m\, t\, z \left(\begin{array}{c} -1 - e^{2\sqrt{m}\,z} - Erf\left(\frac{2t\sqrt{m} - z}{2\sqrt{t}}\right) \\ +e^{2\sqrt{m}\,z}\, Erf\left(\frac{2t\sqrt{m} + z}{2\sqrt{t}}\right) \end{array}\right) + e^{-\sqrt{m}\,z} z\sqrt{m} \left\{\begin{array}{c} 1 - e^{2\sqrt{m}\,z} + Erf\left(\frac{2t\sqrt{m} - z}{2\sqrt{t}}\right) \\ + \quad e^{2\sqrt{m}\,z}\, Erf\left(\frac{2t\sqrt{m} + z}{2\sqrt{t}}\right) \end{array}\right\} \\ +e^{-\sqrt{m}\,z} A_1 \left\{\left(\begin{array}{c} -1 - e^{2\sqrt{m}\,z} - Erf\left(\frac{2t\sqrt{m} - z}{2\sqrt{t}}\right) \\ +e^{2\sqrt{m}\,z}\, Erf\left(\frac{2t\sqrt{m} + z}{2\sqrt{t}}\right) \end{array}\right) + \right\} \\ -2A_1 e^{-z\sqrt{m + \frac{m}{A_1}} + \frac{mt}{A_1}} \left\{-1 - e^{2z\sqrt{m + \frac{m}{A_1}}} + Erf\left(\frac{z - 2t\sqrt{m + \frac{m}{A_1}}}{2\sqrt{t}}\right) + e^{2z\sqrt{m + \frac{m}{A_1}}}\, Erf\left(\frac{z + 2t\sqrt{m + \frac{m}{A_1}}}{2\sqrt{t}}\right)\right\} \end{array}\right]$$

$$-\frac{G_r\sqrt{P_r}}{2m^2\sqrt{\pi}}\left[\begin{array}{l} m\, z\left\{2e^{-\frac{z^2 P_r}{4t}}\sqrt{t} + \sqrt{\pi P_r}\, z\left(-1 + Erf\left(\frac{\sqrt{P_r}\, z}{2\sqrt{t}}\right)\right)\right\} + \frac{2mt\sqrt{\pi}}{\sqrt{P_r}}\left(-1 + Erf\left(\frac{\sqrt{P_r}\, z}{2\sqrt{t}}\right)\right) \\ -\frac{A_1\sqrt{\pi}}{\sqrt{P_r}}\, e^{-z\sqrt{\frac{mP_r}{A_1}} + \frac{mt}{A_1}}\left\{-1 - e^{2z\sqrt{\frac{mP_r}{A_1}}} - Erf\left(\frac{-z\sqrt{P_r} + 2t\sqrt{\frac{m}{A_1}}}{2\sqrt{t}}\right)\right. \\ \left. + e^{2z\sqrt{\frac{mP_r}{A_1}}}\, Erf\left(\frac{z\sqrt{P_r} + 2t\sqrt{\frac{m}{A_1}}}{2\sqrt{t}}\right)\right\} + \frac{2A_1\sqrt{\pi}}{\sqrt{P_r}}\left(-1 + Erf\left(\frac{\sqrt{P_r}\, z}{2\sqrt{t}}\right)\right) \end{array}\right],$$

$$\theta(z,t) = t - tErf\left(\frac{\sqrt{P_r}\, z}{2\sqrt{t}}\right) - \frac{z^2}{2}\left\{-1 + Erf\left(\frac{\sqrt{P_r}\, z}{2\sqrt{t}}\right)\right\} P_r - \frac{e^{-\frac{z^2 P_r}{4t}}\, z\sqrt{P_r t}}{\sqrt{\pi}},$$

where $A_1 = P_r - 1$.

3. SKIN FRICTION

The skin-friction components τ_x and τ_y are obtained as:

$$\tau_x + i\,\tau_y = -\left(\frac{\partial q}{\partial z}\right)_{z=0}$$

$$= \frac{1}{2}\sqrt{m}\left\{1 + \mathrm{Erf}\left(\sqrt{mt}\right) + \mathrm{Erfc}\left(\sqrt{mt}\right)\right\} - \frac{1}{2}\left\{-\frac{2e^{-mt}}{\sqrt{\pi t}} + 2\sqrt{m}\,\mathrm{Erfc}\left(\sqrt{mt}\right)\right\}$$

$$-\frac{G_r}{4m^2}\left[\begin{array}{c} 4m^{3/2}t + 2\sqrt{m}\,\mathrm{Erf}\left(\sqrt{mt}\right) + 2mt\left\{-2\sqrt{m} + \frac{2e^{-mt}}{\sqrt{\pi t}} + 2\sqrt{m}\,\mathrm{Erf}\left(\sqrt{mt}\right)\right\} \\ +4\sqrt{m}\,\mathrm{A}_1 + 2\mathrm{A}_1\left\{-2\sqrt{m} + \frac{2e^{-mt}}{\sqrt{\pi t}} + 2\sqrt{m}\,\mathrm{Erf}\left(\sqrt{mt}\right)\right\} \\ -2\mathrm{A}_1 e^{\frac{mt}{A_1}}\left\{\frac{2e^{-t\left(m + \frac{m}{A_1}\right)}}{\sqrt{\pi t}} - 2\sqrt{m + \frac{m}{A_1}} + 2\mathrm{Erf}\left[\sqrt{t}\sqrt{m + \frac{m}{A_1}}\right]\sqrt{m + \frac{m}{A_1}}\right\} \\ -4e^{\frac{mt}{A_1}}\sqrt{m + \frac{m}{A_1}}\, A_1 \end{array}\right]$$

$$+\frac{G_r}{2m^2\sqrt{\pi}}\left[\begin{array}{c} 4m\sqrt{P_r t} + \sqrt{P_r}\frac{2A_1}{\sqrt{t}} - 2\sqrt{\pi \mathrm{P}_r}\, e^{\frac{mt}{A_1}}\sqrt{\frac{m}{A_1}}\, A_1 \\ -e^{\frac{mt}{A_1}}\sqrt{\pi}A_1\left(\frac{2e^{-\frac{mt}{A_1}}\sqrt{P_r}}{\sqrt{\pi}\sqrt{t}} - 2\sqrt{\frac{m}{A_1}}\sqrt{P_r} + 2\mathrm{Erf}\left[\sqrt{t}\sqrt{\frac{m}{A_1}}\right]\sqrt{\frac{m}{A_1}}\sqrt{P_r}\right) \end{array}\right]$$

4. RESULT AND DISCUSSION

In order to get a physical insight of the problem, a representative set of numerical results is shown graphically in Figures 1–12.Primary velocity profiles are shown in figures - 1 to 5. From figure -1, it is clear that the primary velocity increases when G_r is increased (keeping other parameters M = 2.0, P_r = 0.71, K = 0.5, Ω = 0.5, t = 0.2 constant). Primary velocity profile for different values of K is shown in figure - 2 and it shows that primary velocity increases with increase in K. Also primary velocity decreases with increase in M (figure - 3) and Ω (figure - 4). But it increases with t (figure - 5).

Secondary velocity profiles are shown in figures-6 to 10. Figure - 6 shows that secondary velocity decreases when G_r is increased. Figure -7 shows that it decreases when K is increased. From figure – 8, it is observed that, the secondary velocity increases when M increases. Figure – 9 and 10 respectively show that it decreases with Ω and t.

Temperature profiles are illustrated in figure – 11 and 12 for different values of P_r and time. In figure 11, it can be seen that the temperature of the fluid is inversely proportional to the value of Prandtl number P_r. Thus, the increase in P_r reduces the temperature in the system. This is due to the fact that there would be a decrease of thermal boundary layer thickness with the increase of Prandtl number P_r. Also thermal boundary layer increases with time (figure - 12).

The effects of various parameters on the skin-friction are shown in table - 1. It is found from table - 1, that when the values of Ω, M and P_r are increased (keeping other parameters fixed), the value of τ_x is increased but if values of t, G_r and K are increased, τ_x gets decreased. Also it is observed that τ_y decreases with M & P_r and it is increased when t, G_r, Ω and K are increased.

5. CONCLUSION

Conclusions of this study are as follows:

- Primary velocity (u) increases with the increase in G_r , K and t and decreases with increase in M and Ω.
- Secondary velocity (v) increases with the increase in M and decreases with increase in Ω, G_r , K and t.
- Skin friction-
    - τ_x increases when M, Ω and P_r are increased but it decreases with G_r, K, and t.
    - τ_y increases when G_r, K, Ω and t are increased but it decreases if M and P_r are increased.

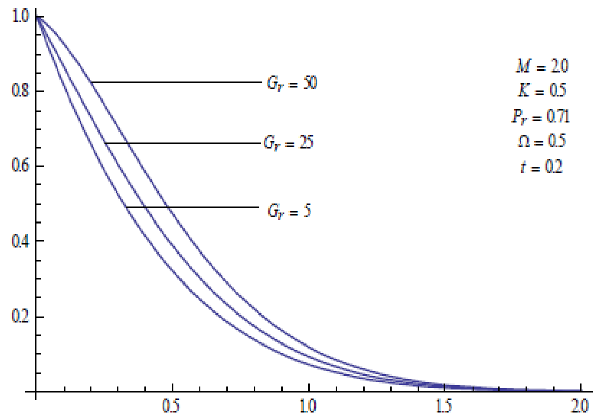

Figure 1: u v/s z

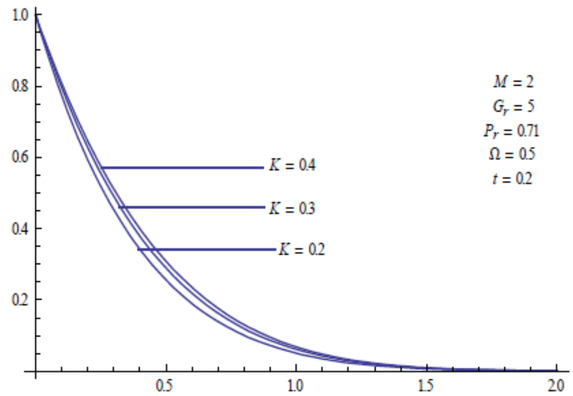

Figure 2: u v/s z

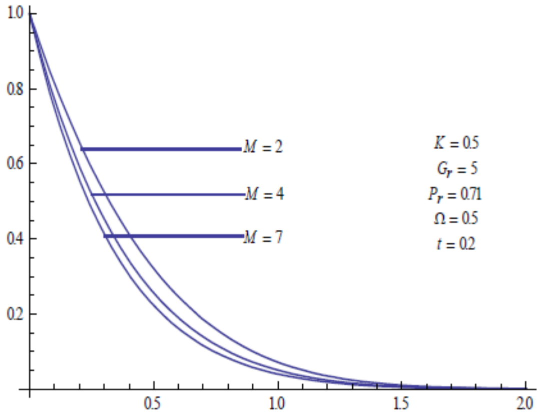

Figure 3: u v/s z

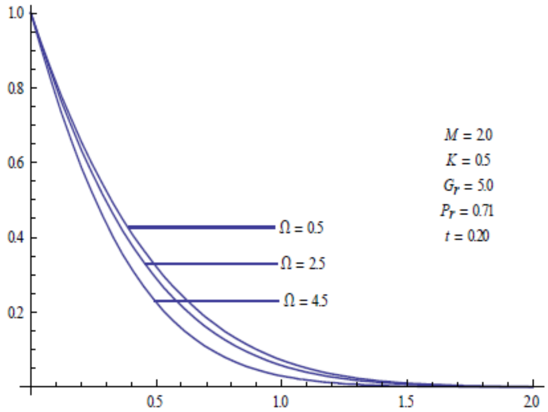

Figure 4: u v/s z

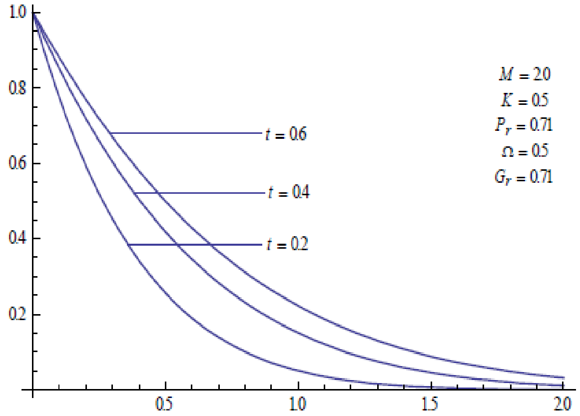

Figure 5: u v/s z

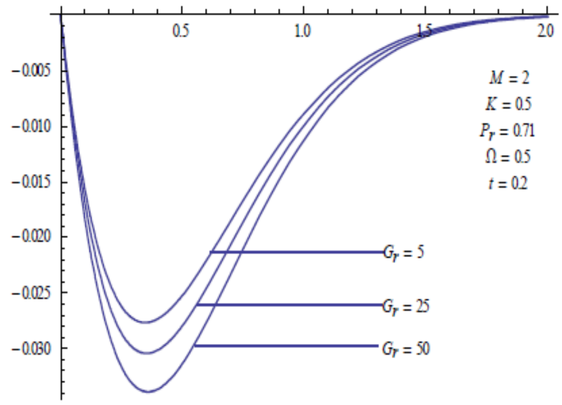

Figure 6: v v/s z

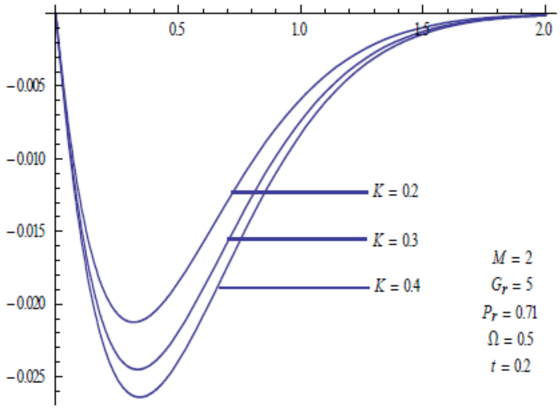

Figure 7: v v/s z

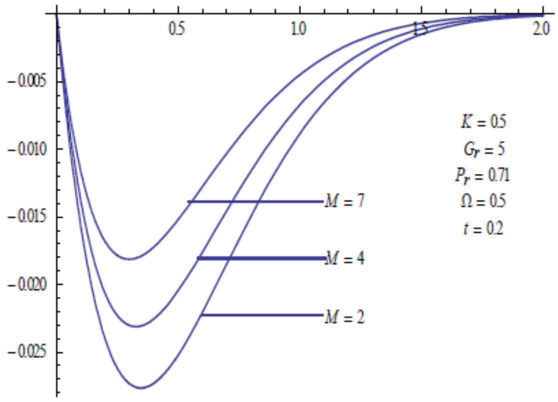

Figure 8: v v/s z

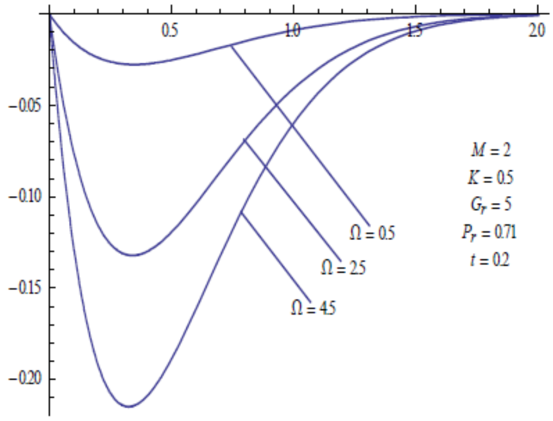

Figure 9: v v/s z

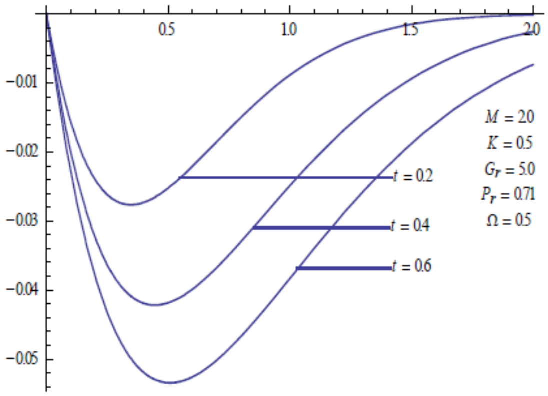

Figure 10: v v/s z

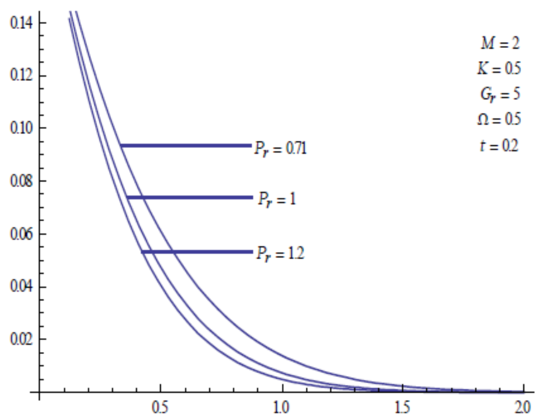

Figure 11: θ v/s z

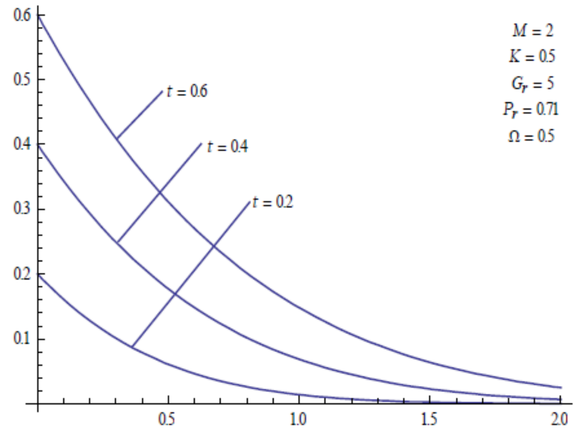

Figure 12: θ v/s z

Table 1: Skin friction for different parameters

G_r	K	M	Ω	t	P_r	τ_x	τ_y
05	0.5	2	0.5	0.2	0.71	1.992	0.201
25	0.5	2	0.5	0.2	0.71	1.319	0.214
50	0.5	2	0.5	0.2	0.71	0.477	0.230
05	0.2	2	0.5	0.2	0.71	2.552	0.174
05	0.3	2	0.5	0.2	0.71	2.251	0.188
05	0.4	2	0.5	0.2	0.71	2.091	0.196
05	0.5	2	0.5	0.2	0.71	1.992	0.201
05	0.5	4	0.5	0.2	0.71	2.374	0.182
05	0.5	7	0.5	0.2	0.71	2.884	0.159
05	0.5	2	0.5	0.2	0.71	1.992	0.201
05	0.5	2	2.5	0.2	0.71	2.118	0.985
05	0.5	2	4.5	0.2	0.71	2.384	1.984
05	0.5	2	0.5	0.2	0.71	1.992	0.201
05	0.5	2	0.5	0.4	0.71	1.599	0.245
05	0.5	2	0.5	0.6	0.71	1.257	0.275
05	0.5	2	0.5	0.2	0.71	1.992	0.201
05	0.5	2	0.5	0.2	3.00	2.044	0.200
05	0.5	2	0.5	0.2	7.00	2.072	0.199

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