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NANO SEMI – GENERALIZED IRRESOLUTE MAPS IN NANO TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce and study new class of maps, namely nano semi-generalized irresolute maps, strongly nano semi-generalized continuous maps and perfectly nano semi-generalized continuous maps in nano topological spaces. We define and analyse some of the properties of these mappings in terms of nano sg-open sets, nano sg-closed sets.

Keywords: *Nano sg-closed sets, Nano sg-open sets, Nano sg-closure, Nano continuity, Nano sg-continuity, Nano – irresolute, Nano sg-irresolute, Strongly nano sg-continuity, Perfectly nano sg-continuity.*

1. INTRODUCTION

Bhattacharyya and Lahiri [1] introduced semi-generalized closed sets in topology and investigated some of their properties. Crossley and Hildebrand [5] introduced and studied irresolute functions in topological spaces. Sundaram *et al.*, [13] defined the semi – generalized continuous map and semi $T_{1/2}$ spaces. Di Maio and Noiri [7] studied the weak and strong forms of irresolute functions. Recently Sundaram [14] and Devi *et al.*, [6] have investigated gc – irresolute, αg – irresolute and gs – irresolute functions. The notion of nano topology was introduced by Lellis Thivagar [10] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He also analysed the nano forms of weakly open sets such as nano α -open sets, nano semi-open sets and nano pre-open sets. Bhuvaneswari and Ezhilarasi [3] studied nano semi – generalized continuous maps in nano topological spaces. The aim of this paper is to introduce the concepts of nano irresolute maps, nano semi – generalized irresolute maps in nano topological spaces and investigate some of their properties. We also establish various forms of continuities associated to nano semi-generalized closed sets, namely, strongly nano sg-continuity, perfectly nano sg-continuity.

2. PREMILINARIES

Definition: 2.1[1] A subset A of of a space (X, τ) is called a semi-generalized closed set if scl(A)$\subseteq$U whenever A $\subseteq$U and U is semi-open.

Definition: 2.2[8] The semi-generalized closure of a subset A of a space X is the intersection of all sg-closed sets containing A and is denoted by sgCl(A).

Definition:2.3[8] The semi-generalized interior of a subset A of a space X is the union of all sg-open sets contained in A and is denoted by sgInt(A).

Definition: 2.4 [**13**] A function f: X→Y is semi-generalized continuous (sg-continuous) if f^{-1}(V) is sg-closed set in X for every closed set V of Y or equivalently, a function f: X→Y is sg -continuous if and only if the inverse image of each open set is sg-open set.

Definition: 2.5 [**5**] Let f: (X,τ)→(Y,σ) be a map. Then *f* is said to be irresolute if $f^{-1}(V)$ is semi – open in (X,τ) for each semi – open set V of (Y,σ).

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Definition: 2.6 [**6**] Let f: (X,τ)→(Y,σ) be a map. Then *f* is said to be sg – irresolute if $f^{-1}(V)$ is sg – closed in (X,τ) for each sg – closed set V of (Y,σ).

Definition: 2.7 [**12**] A map f: (X,τ)→(Y,σ) is said to be strongly continuous if $f^{-1}(V)$ is both open and closed in (X,τ) for each subset V of (Y,σ).

Definition: 2.8 [9] A map f: (X,τ)→(Y,σ) is said to be strongly sg – continuous if $f^{-1}(V)$ is open in (X,τ) for each sg – open set V of (Y,σ).

Definition: 2.9 [9] A map f: (X,τ)→(Y,σ) is said to be perfectly sg - continuous if $f^{-1}(V)$ is clopen in (X,τ) for each sg – open set V of (Y,σ).

Definition: 2.10 [10] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X ⊆ U. Then,

(i) The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and is denoted by $L_R(X)$. $L_R(X) = U\{R(x): R(x) \subseteq X, x \in U\}$ where R(x) denotes the equivalence class determined by x ϵ U.

(ii) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$. $U_R(X) = U\{R(x): R(x) \cap X \neq \Phi, x \in U\}$.

(iii) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not–X with respect to R and it is denoted by $B_R(X)$. $B_R(X) = U_R(X) - L_R(X)$.

Property: 2.11[10] If (U, R) is an approximation space and X, Y⊆U, then

1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\Phi) = U_R(\Phi) = \Phi$
3. $L_R(U) = U_R(U) = U$
4. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
5. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
6. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
7. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
8. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
9. 9. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
10. $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
11. $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$.

Definition:2.12 [10] Let U be the universe , R be an equivalence relation on U and the Nano topology $\tau_R(X) = \{U, \Phi, L_R(X), U_R(X), B_R(X)\}$ where X ⊆ U. Then $\tau_R(X)$ satisfies the following axioms:

(i) U and Φ ϵ $\tau_R(X)$.

(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. (U, $\tau_R(X)$) is called the Nano topological space. Elements of the Nano topology are known as nano open sets in U. Elements of $[\tau_R(X)]^c$ are called nano closed sets with $[\tau_R(X)]^c$ being called Dual Nano topology of $\tau_R(X)$. If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set B = { U, $L_R(X)$, $B_R(X)$} is the basis for $\tau_R(X)$.

Definition: 2.13 [10] If $(U, \tau_R(X))$ is a Nano topological space with respect to X where X ⊆ U and if A ⊆ U, then

(i) The nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by NInt(A). NInt(A) is the largest nano open subset of A.

(ii) The nano closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by NCl(A). NCl(A) is the smallest nano closed set containing A.

Remark: 2.14 [11] Throughout this paper, U and V are non-empty, finite universes; X ⊆ U and Y ⊆ V ; U/R and V/R' denote the families of equivalence classes by equivalence relations R and R' respectively on U and V. $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ are the Nano topological spaces with respect to X and Y respectively.

Definition: 2.15[11] A subset A of a Nano topological space $(U, \tau_R(X))$ is said to be nano dense if NCl(A)=U**.**

Definition: 2.16[2] If $(U,\tau_R(X))$ is a Nano topological space with respect to X where X ⊆U and if A ⊆ U, then

(i) The nano semi-closure of A is defined as the intersection of all nano semi-closed sets containing A and is denoted by NsCl(A). NsCl(A) is the smallest nano semi-closed set containing A and NsCl(A) ⊆A.

(ii) The nano semi-interior of A is defined as the union of all nano semi-open subsets of A and is denoted by NsInt(A). NsInt(A) is the largest nano semi open subset of A and NsInt(A) ⊆A.

Definition: 2.17[2] A subset A of $(U,\tau_R(X))$ is called nano semi-generalized closed set (Nsg-closed) if NsCl(A)$\subseteq$V and A$\subseteq$V and V is nano semi-open in $(U,\tau_R(X))$. The subset A is called nano sg-open in $(U,\tau_R(X))$ if A^c is nano sg-closed.

Definition: 2.18[11] Let $(U,\tau_R(X))$ and $(V,\tau_{R'}(Y))$ be two Nano topological spaces. Then a mapping $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano continuous on U if the inverse image of every nano open set in V is nano open in U.

Definition: 2.19[2] If $(U,\tau_R(X))$ is a Nano topological space with respect to X where X ⊆ U and if A ⊆ U, then

(i) The nano semi- generalized closure of A is defined as the intersection of all nano semi-generalized closed sets containing A and is denoted by NsgCl(A). NsgCl(A) is the smallest nano semi-generalized closed set containing A and if A is a nano sg-closed set, then NsgCl(A) = A.

(ii) The nano semi-generalized interior of A is defined as the union of all nano semi-generalized open subsets of A and is denoted by NsgInt(A). NsgInt(A) is the largest nano semi-generalized open subset of A. If A is nano sg-open set, then NsgInt(A) = A.

Definition: 2.20[3] Let $(U,\tau_R(X))$ and $(V,\tau_{R'}(Y))$ be two Nano topological spaces. Then a map $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano sg-continuous on U if the inverse image of every nano open set in V is nano sg-open in U.

3. Nsg – IRRESOLUTE MAPS

In this section, we introduce the concepts of nano irresolute maps, Nsg – irresolute maps in Nano topological spaces and investigate some of their properties.

Definition: 3.1 A map $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano irresolute if the inverse image $f^{-1}(A)$ of every nano semi – open set A in V is nano semi – open in U.

Definition:3.2 A map $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano sg – irresolute (Nsg - irresolute) if the inverse image $f^{-1}(A)$ of every Nsg - closed set A in V is Nsg – closed in U.

Example: 3.3 Let U = {a, b, c, d} with U/R ={{a},{c},{b, d}}.Let X={a, b}⊆ U. Then $\tau_R(X)$ = {U, ϕ, {a}, {a, b, d},{b, d}}. Nano sg-open sets are {U, ϕ, {a, c, d}, {a, b, d}, {a, b, c},{b, c, d}, {b, d}, {a, d}, {a, b}, {a, c}, {a}, {b}, {d}}. Nano sg-closed sets are {U, ϕ, {a, c, d}, {a, b, c},{b, c, d}, {b, d}, {c, d},{b, c},{a, c}, {a}, {b}, {c}, {d}}. Let V = {x, y, z, w} with V/R' = {{x},{y, z},{w}}.Let Y={x, z}⊆ V. Then $\tau_{R'}(Y)$ = {V, ϕ, {x}, {y, z}, {x, y, z}}. Nano sg-open sets are {V, ϕ, {y, z, w}, {x, z, w}, {x, y, w},{x, y, z}, {y, z}, {x, w}, {x, y}, {x, z}, {x}, {y}, {z}}. Nano sg-closed sets are {V, ϕ, {x}, {y},{z}, {w}, {x, w},{y, z},{z, w}, {y, w}, {y, z, w}, {x, z, w}, {x, y, w}}. Define $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ as f(a) = x, f(b) = y, f(c) = w, f(d) = z. Then f^{-1}(V) = U, $f^{-1}(\phi) = \phi$, f^{-1}({x, y, w}) = {a, b, c}, f^{-1}({x, z, w}) = {a, c, d}, f^{-1}({y, z, w}) = {b, d, c}, f^{-1}({x}) = {a}, f^{-1}({w}) = {c}, f^{-1}({y}) = {b}, f^{-1}({z}) = {d}, f^{-1}({y, z}) = {b, d}, f^{-1}({z, w}) = {c, d}, f^{-1}({x, w}) = {a, c}, f^{-1}({y, w}) = {b, c}. Thus the inverse image of every Nsg – closed set in V is Nsg – closed in U. Hence $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is Nsg – irresolute.

Theorem: 3.4 A function $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is Nsg – irresolute if and only if for every Nsg – open set A in $(V,\tau_{R'}(Y))$, the inverse image $f^{-1}(A)$ is Nsg - open in $(U,\tau_R(X))$.

Proof: The proof follows from the fact that the complement of a Nsg – closed set is Nsg – open set. If $f:(U,\tau_R(X))\to (V,\tau_{R'}(Y))$ be Nsg – irresolute, then for every Nsg–closed set B of V, $f^{-1}(B)$ is Nsg–closed in $(U,\tau_R(X))$.If A is Nsg – open subset of V, then A^c is Nsg – closed subset. Hence $f^{-1}(A^c)$ is Nsg – closed in U. But $f^{-1}(A^c) = (f^{-1}(A))^c$ and hence $(f^{-1}(A))^c$ is Nsg – closed so that $f^{-1}(A)$ is Nsg – open.

Conversely, if for all Nsg – open subsets A of V, $f^{-1}(A)$ is Nsg – open in U. If B is any Nsg – closed subset of V, then B^c is Nsg – open in V. Hence by the given hypothesis, $f^{-1}(B^c)$ is Nsg – open in U. But $f^{-1}(B^c) = (f^{-1}(B))^c$ which is Nsg – open in V and hence $f^{-1}(B)$ is Nsg – closed in V. This implies that $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg – irresolute.

Theorem: 3.5 If a map $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg – irresolute, then it is Nsg-continuous but not conversely.

Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be Nsg – irresolute map. Then the inverse image $f^{-1}(A)$ of every Nsg – closed set A in V is Nsg – closed in U. Since every nano closed set is nano sg - closed, the inverse image of every nano closed set in V is Nsg – closed in U whenever the inverse image of every Nsg – closed set is Nsg – closed. Hence Nsg – irresolute function is Nsg-continuous.

The converse of the above theorem need not be true as seen from the following example.

Example:3.6 Let U ={a,b,c,d} with U/R ={{a},{c},{b,d}}.Let X={a,b}⊆U. Then $\tau_R(X)$ ={U, ϕ,{a},{a,b,d},{b,d}}.
Nano sg - closed sets are {U, ϕ, {a,c,d}, {a,b,c},{b,c,d}, {b,d}, {c,d},{b,c},{a,c}, {a}, {b}, {c}, {d}}.
Let V = {x,y,z,w} with V/R' = {{x},{y,z},{w}}. Let Y= {x,z} ⊆ V. Then $\tau_{R'}(Y)$ = {V, ϕ,{x},{y,z},{x,y,z}}.
Nano sg-closed sets are {V, ϕ, {x}, {y},{z}, {w}, {x,w},{y,z},{z,w}, {y,w}, {y,z,w}, {x,z,w}, {x,y,w}}.
Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ as f(a) = y, f(b) = x, f(c) = w, f(d) = z. Then f is Nsg – continuous since the inverse image of every nano closed set in V is Nsg – closed in U. But f is not Nsg – irresolute since f^{-1}({y,z}) = {a,d} is not Nsg – closed in U even though {y,z} is Nsg – closed in V. Hence a Nsg-continuous function is not Nsg – irresolute.

Theorem: 3.7 Let $(U,\tau_R(X))$, $(V,\tau_{R'}(Y))$ and $(W,\tau_{R''}(Z))$ be Nano topological spaces. If the functions $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ and $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ are both Nsg – irresolute, then $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is Nsg – irresolute.

Proof: As the map $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ is Nsg – irresolute, the inverse image $g^{-1}(A)$ of every Nsg – open set A in W is Nsg – open in V. Hence $g^{-1}(A)$ is a Nsg – open set in V and $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ being Nsg – irresolute implies that $f^{-1}[g^{-1}(A)]$ is Nsg – open in U. Thus $(g\circ f)^{-1}(A)=f^{-1}[g^{-1}(A)]$ is Nsg – open in U for every Nsg – open set $g^{-1}(A)$ in V. Hence $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is Nsg – irresolute.

Theorem: 3.8 Let $(U,\tau_R(X))$, $(V,\tau_{R'}(Y))$ and $(W,\tau_{R''}(Z))$ be Nano topological spaces. For any Nsg – irresolute map $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ and any Nsg – continuous map $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$, the composition $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is Nsg – continuous.

Proof: Let A be a nano closed set in W. Since the map $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ is Nsg – continuous, the inverse image $g^{-1}(A)$ is Nsg – closed in V. Since the map $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg – irresolute, the inverse image $f^{-1}[g^{-1}(A)]$ of Nsg – closed set $g^{-1}(A)$ in V is Nsg – closed in U. Thus the inverse image $(g\circ f)^{-1}(A)$ is Nsg – closed in U for every Nsg – closed set A in W. Hence the composition $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is Nsg – continuous.

Theorem: 3.9 If a map $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg – irresolute , then for every subset A of $(U,\tau_R(X))$, $f(NsgCl(A))\subseteq NsCl(f(A))$.

Proof: If A is the subset of U, then consider $NsCl(f(A))$ which is Nsg – closed in V. By definition 3.2, $f^{-1}(NsCl(f(A)))$ is Nsg – closed in V. Furthermore, $A\subset f^{-1}(f(A))\subset f^{-1}(NsCl(f(A)))$.

By the destination of nano semi - generalized closure, $NsgCl(A)\subseteq f^{-1}(NsCl(f(A)))$ and consequently, $f(NsgCl(A))\subseteq f(f^{-1}(NsCl(f(A))))\subseteq Nscl(f(A))$

Example3.10 In Example 3.3, let A={b,d}$\subseteq$ U. Now f(A) =f({b,d}) ={y,z}. $NsCl(f(A))=NsCl(\{y,z\})=\{y,z\}$ is Nsg – closed in V. Now $f^{-1}(NsCl(f(A)))$ = $f^{-1}(\{y,z\})=\{b,d\}$ is Nsg–closed in U.
Also $f^{-1}(f(A))=f^{-1}(\{y,z\})=\{b,d\}$. Hence $A\subset f^{-1}(f(A))\subset f^{-1}(NsCl(f(A)))$.
Also, $NsgCl(A)=NsgCl(\{b,d\})=\{b,d\}$ .Hence $NsgCl(A)\subseteq f^{-1}(NsCl(f(A)))$ .Now $f(NsgCl(A))=f(\{b,d\})=\{y,z\}$ Now, we have $f(f^{-1}(NsCl(f(A))))=f(\{b,d\})=\{y,z\}$. Hence it is obvious that $f(NsgCl(A))\subseteq f(f^{-1}(NsCl(f(A))))\subseteq Nscl(f(A))$.

Theorem:3.11 Let $(U,\tau_R(X))$, $(V,\tau_{R'}(Y))$ and $(W,\tau_{R''}(Z))$ be Nano topological spaces. If f : $(U,\tau_R(X))$ $\to$ $(V,\tau_{R'}(Y))$ is Nsg – continuous and g : $(V,\tau_{R'}(Y))$ $\to$ $(W,\tau_{R''}(Z))$ is nano continuous , then the composition $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is Nsg – continuous.

Proof: Let A be a nano closed set in W. Since the map g : $(V,\tau_{R'}(Y))$ $\to$ $(W,\tau_{R''}(Z))$ is nano continuous, the inverse image $g^{-1}(\mathrm{A})$ is nano closed in V. Since the map f : $(U,\tau_R(X))$ $\to$ $(V,\tau_{R'}(Y))$ is Nsg – continuous, for any nano closed set $g^{-1}(\mathrm{A})$ in V, $f^{-1}[g^{-1}(A)]$ is Nsg – closed in U. Now $(g\circ f)^{-1}(A)=f^{-1}[g^{-1}(A)]$ is Nsg – closed in U . Hence the inverse image of a nano closed set A in W is Nsg – closed set in U. Hence the composition $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is Nsg – continuous.

Definition:3.12 A map $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is strongly nano continuous if $f^{-1}(\mathrm{A})$ is both nano open and nano closed in $(U,\tau_R(X))$ for each subset A of $(V,\tau_{R'}(Y))$.

Definition:3.13 A map $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is strongly nano sg –continuous (strongly Nsg -continuous) if $f^{-1}(\mathrm{A})$ is nano open $(U,\tau_R(X))$ for every Nsg – open set in $(V,\tau_{R'}(Y))$.

Theorem:3.14 If the map $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is strongly Nsg –continuous, then it is nano continuous but not conversely.

Proof: Let A be a nano open set in $(V,\tau_{R'}(Y))$. As every nano open set is Nsg – open , A is Nsg- open in V.
By given hypothesis , $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is strongly Nsg –continuous and hence $f^{-1}(\mathrm{A})$ is nano open in U for every Nsg – open set A in V. Hence $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano continuous.

The converse of the above theorem need not be true as seen from the following example.

Example: 3.15 Let U = {a,b,c,d} with $\tau_R(X)$ ={U, ϕ ,{a},{a,b,d},{b,d}}. Let V ={x,y,z,w} with $\tau_{R'}(Y)$ ={ V, ϕ, {x}, {y,z}, {x,y,z} } . Define a map f : $(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ as f(a) = x, f(b) = y, f(c) = w, f(d) = z. Then f is nano continuous since the inverse image of every nano open set in V is nano open in U. But f is not strongly Nsg – continuous since f^{-1}({x,z,w}) = {a,c,d} is not nano open in U even though {x,z,w} is Nsg – open in V. Hence a nano continuous function is not strongly Nsg – continuous.

Theorem: 3.16 If a function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is strongly nano continuous, then it is strongly Nsg – continuous.

Proof: Let A be a Nsg – open subset of $(V,\tau_{R'}(Y))$ .The map $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is strongly nano continuous and hence the inverse image of every subset of V is both nano open and nano closed in $(U,\tau_R(X))$. Hence $f^{-1}(\mathrm{A})$ is nano open in $(U,\tau_R(X))$ for every Nsg–open subset A of $(V,\tau_{R'}(Y))$. Hence $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is strongly Nsg – continuous.

Theorem: 3.17 A map $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is strongly Nsg – continuous if and only if the inverse image of every Nsg- closed set in $(V,\tau_{R'}(Y))$ is nano closed in $(U,\tau_R(X))$.

Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a strongly Nsg –continuous map, F be a Nsg – closed set in $(V,\tau_{R'}(Y))$ and hence V – F is Nsg – open set in V. Since f is strongly Nsg –continuous, the inverse image of every Nsg – open set in $(V,\tau_{R'}(Y))$ is nano open in $(U,\tau_R(X))$. Hence $f^{-1}(\mathrm{V-F})$ is nano open in U.Also $f^{-1}(\mathrm{V-F}) = f^{-1}(\mathrm{V}) - f^{-1}(\mathrm{F})$ = U – $f^{-1}(\mathrm{F})$ is nano open in U. Thus $f^{-1}(\mathrm{F})$ is nano closed in U.Hence the inverse image of every Nsg – closed set in $(V,\tau_{R'}(Y))$ is nano closed in U if f is strongly Nsg –continuous.

Conversely, let the inverse image of every Nsg – closed set in V be nano closed in U. Let H be a Nsg – open set in $(V,\tau_{R'}(Y))$, then V – H is Nsg – closed in V. By given hypothesis, $f^{-1}(\mathrm{V-H})$ is nano closed in $(U,\tau_R(X))$ and hence $f^{-1}(\mathrm{V-H}) = f^{-1}(\mathrm{V}) - f^{-1}(\mathrm{H})$ = U – $f^{-1}(\mathrm{H})$ is nano closed in $(U,\tau_R(X))$. Hence $f^{-1}(\mathrm{H})$ is nano open in U for every Nsg – open set H in V. Thus the inverse image of every Nsg – open set in $(V,\tau_{R'}(Y))$ is nano open in $(U,\tau_R(X))$ .Hence $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is strongly Nsg – continuous.

Corollary: 3.18 If $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ and $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ are strongly Nsg – continuous , then their composition $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is also strongly Nsg – continuous.

Theorem: 3.19 Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ and $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ be any two maps. Then their composition $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is

(i) Strongly Nsg – continuous if g is strongly Nsg – continuous and f is nano continuous.
(ii) Nsg – irresolute if g is strongly Nsg – continuous and f is Nsg – continuous (or f is Nsg – irresolute.)
(iii) Strongly Nsg – continuous if g is strongly nano continuous and f is nano irresolute.
(iv) Nano continuous if g is Nsg – continuous and f is strongly Nsg – continuous.

Proof:

(i) Let A be a Nsg – open set in $(W,\tau_{R''}(Z))$. Since $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ is strongly Nsg – continuous, $g^{-1}(\mathrm{A})$ is nano open in V. Since $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano continuous, $f^{-1}[g^{-1}(A)]$ is nano open in $(U,\tau_R(X))$. Now $(g\circ f)^{-1}(A)=f^{-1}[g^{-1}(A)]$ is nano open in $(U,\tau_R(X))$ for every Nsg – open set A in $(W,\tau_{R''}(Z))$. Hence $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is strongly Nsg – continuous.

(ii) Let A be a Nsg – open set in $(W,\tau_{R''}(Z))$. Since $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ is strongly Nsg – continuous, $g^{-1}(\mathrm{A})$ is nano open in V. Since $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg -continuous, $f^{-1}[g^{-1}(A)]$ is Nsg – open in $(U,\tau_R(X))$. Now $(g\circ f)^{-1}(A)=f^{-1}[g^{-1}(A)]$ is Nsg – open in $(U,\tau_R(X))$ for every Nsg – open set A in $(W,\tau_{R''}(Z))$. Hence $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is Nsg – irresolute.

(iii) Let A be a Nsg–open set in $(W,\tau_{R''}(Z))$ Since $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ is strongly nano continuous, $g^{-1}(\mathrm{A})$ is both nano closed and nano open in V. As every nano open set is nano semi – open , $g^{-1}(\mathrm{A})$ is nano semi – open in V. Since $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano irresolute, $f^{-1}[g^{-1}(A)]$ is nano semi – open in $(U,\tau_R(X))$. Now $(g\circ f)^{-1}(A) = f^{-1}[g^{-1}(A)]$ is nano semi – open in $(U,\tau_R(X))$ as g is strongly nano continuous. Hence the map $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is strongly Nsg – continuous.

(iv) Let A be a nano open set in $(W,\tau_{R''}(Z))$ Since $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ is Nsg – continuous, $g^{-1}(A)$ is Nsg – open in $(V,\tau_{R'}(Y))$. Since $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is strongly Nsg – continuous, $f^{-1}[g^{-1}(A)]$ is nano open in $(U,\tau_R(X))$. Now $(g\circ f)^{-1}(A) = f^{-1}[g^{-1}(A)]$ is nano open in $(U,\tau_R(X))$. Hence the map $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is nano continuous for every nano open set A in $(W,\tau_{R''}(Z))$.

Definition: 3.20 A map $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is perfectly nano sg –continuous (perfectly Nsg - continuous) if the inverse image of every Nsg – open set in $(V,\tau_{R'}(Y))$ is both nano open and nano closed in $(U,\tau_R(X))$.

Theorem: 3.21 A map $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is perfectly Nsg –continuous , then it is strongly Nsg – continuous.

Proof: Since $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is perfectly Nsg – continuous, $f^{-1}(A)$ is both nano open and nano closed in $(U,\tau_R(X))$ for every Nsg – open set A in $(U,\tau_R(X))$. By definition, $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is strongly Nsg – continuous.

Theorem: 3.22 If the map $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is strongly Nsg – continuous , then it is perfectly Nsg – continuous.

Proof: Let A be a Nsg –open set in $(V,\tau_{R'}(Y))$. Since $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is strongly nano continuous, $f^{-1}(A)$ is both nano open and nano closed in $(U,\tau_R(X))$. Hence $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is perfectly Nsg –continuous.

Theorem: 3.23 Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a map. Then the following are equivalent.

(i) f is strongly nano continuous.
(ii) f is strongly Nsg – continuous
(iii) f is perfectly Nsg – continuous

Proof: The proof follows from the theorems 3.16, 3.21 and 3.22

Theorem: 3.24 Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ and $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ be any two maps. Then their composition $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is

(i) Nsg – continuous if g is strongly nano continuous and f is Nsg – continuous.
(ii) Nsg – irresolute if g is perfectly Nsg – continuous and f is Nsg – continuous (or f is Nsg – irresolute.)
(iii) Strongly Nsg – continuous if g is perfectly Nsg – continuous and f is nano continuous.
(iv) Perfectly Nsg – continuous if g is strongly nano continuous and f is perfectly Nsg – continuous.

Proof:

(i) Let A be a nano open set in $(W,\tau_{R''}(Z))$. Since $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ is strongly nano continuous, $g^{-1}(A)$ is both nano open and nano closed in $(V,\tau_{R'}(Y))$. Since $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg – continuous, $f^{-1}[g^{-1}(A)]$ is Nsg – open in $(U,\tau_R(X))$. Now $(g\circ f)^{-1}(A) = f^{-1}[g^{-1}(A)]$ is Nsg – open in $(U,\tau_R(X))$ for every nano – open set A in $(W,\tau_{R''}(Z))$. Hence $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is Nsg – continuous.

(ii) Let A be a Nsg – open set in $(W,\tau_{R''}(Z))$. Since $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ is perfectly Nsg – continuous, $g^{-1}(A)$ is both nano open and nano closed in V. Since $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is

Nsg –continuous, $f^{-1}[g^{-1}(A)]$ is Nsg – open in $(U,\tau_R(X))$. Now $(g\circ f)^{-1}(A)=f^{-1}[g^{-1}(A)]$ is Nsg – open in $(U,\tau_R(X))$ for every Nsg – open set A in $(W,\tau_{R''}(Z))$. Hence $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is Nsg – irresolute.

(iii) Let A be a Nsg – open set in $(W,\tau_{R''}(Z))$. Since $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ is perfectly Nsg – continuous, $g^{-1}(\mathrm{A})$ is both nano closed and nano open in V. Since $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano continuous, $f^{-1}[g^{-1}(A)]$ is nano open in $(U,\tau_R(X))$. Now $(g\circ f)^{-1}(A)=f^{-1}[g^{-1}(A)]$ is nano open in $(U,\tau_R(X))$. Hence the inverse image of every Nsg – open set in $(W,\tau_{R''}(Z))$ is nano open in $(U,\tau_R(X))$. Thus the map $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is strongly Nsg – continuous.

(iv) Let A be a Nsg–open set in $(W,\tau_{R''}(Z))$ Since $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ is strongly nano continuous, $g^{-1}(\mathrm{A})$ is both nano open and nano closed in $(V,\tau_{R'}(Y))$. As every nano open set is Nsg – open, $g^{-1}(\mathrm{A})$ is both Nsg – open and Nsg – closed Since $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is perfectly Nsg – continuous, $f^{-1}[g^{-1}(A)]$ is both nano open and nano closed in $(U,\tau_R(X))$. Now $(g\circ f)^{-1}(A)=f^{-1}[g^{-1}(A)]$ is both nano open and nano closed in $(U,\tau_R(X))$ for every Nsg–open set A in $(W,\tau_{R''}(Z))$. Hence the map $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is perfectly Nsg – continuous.

Corollary: 3.25 If $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ and $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ are perfectly Nsg – continuous , then their composition $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is also perfectly Nsg – continuous.

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