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ON T-FUZZY BI-IDEALS AND (λ, μ)-FUZZY BI-IDEALS IN NEAR-RING WITH RESPECT TO t-NORM

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ABSRACT

In this paper we introduce the concept of T-fuzzy bi-ideals using t-norm and t-norm (λ,μ)-fuzzy bi-ideals in near-ring and investigate some of their properties.

1. INTRODUTION

The theory of fuzzy set was first inspired by Zadeh [5]. Triangular norms were introduced by Schweizer and Sklar[3,4] to model the distances in probabilistic metric spaces. P.Dheena, G.Mohanraj [2] and M.Akram [1] have studied several properties of T-fuzzy ideals of rings and T-fuzzy ideals of near-rings. In this paper we introduce the notion of T-fuzzy bi-ideals using t-norm and t-norm (λ,μ)-fuzzy bi-ideals in near-ring and investigate some related properties.

2. PRELIMINARIES

Definition 2.1: A fuzzy subset μ of a near-ring N is called T- fuzzy bi-ideal if

(1) $\mu(x-y) \geq T(\mu(x),\mu(y))$
(2) $\mu(xyz) \geq T(\mu(x),\mu(z))$ for all $x, y, z \in N$.

Definition 2.2: A mapping $f: N \to N'$ is called a near-ring homomorphism if $f(x+y)=f(x)+f(y)$ and $f(xy)=f(x)f(y)$ for all $x, y \in N$.

Definition 2.3: a quotient nearing is a near –ring that is the quotient of a near-ring and one of its bi-ideal I, denoted N/I. If I is a bi-ideal of a near-ring N and $a \in N$, then a coset of I is a set of the form $a+I = \{a+s/s \in I\}$.The set of all coset is denoted by N/I.

Theorem 2.4: If I is a bi-ideal of a near-ring N, the set N/I is a near-ring under the operations $(a+I)+(b+I) = (a+b)+I$ and $(a+I).(b+I)= (a.b)+I$.

Lemma 2.5:.If μ is a fuzzy bi-ideal of N, then $\mu(0) \geq \mu(x)$ for all $x \in N$.

Definition 2.6.[3]: A t-norm is a function $T:[0,1]x[0,1] \to [0,1]$ that satisfies the following conditions for all $x, y, z \in [0,1]$,

(1) $T(x,1) =x$,
(2) $T(x, y) =T(y, x)$,
(3) $T(x, T(y, z)) = T(T(x, y), z)$,
(4) $T(x, y) \leq T(x, z)$ whenever $y \leq z$.

A simple example of such defined t-norm is a function $T(x, y) = \min(x, y)$. In general case, $T(x, y) \leq \min(x, y)$ and $T(x, 0) = 0$ for all $x, y \in [0,1]$.

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Definition 2.7: A subgroup B of N is said to be bi-ideal if $BNB \subseteq B$.

Definition 2.8: A fuzzy subset μ of a near-ring N is called fuzzy bi-ideal if

(1) $\mu(x-y) \geq \min\{\mu(x),\mu(y)\}$
(2) $\mu(xyz) \geq \min\{\mu(x),\mu(z)\}$ for all x, y, z∈N.

Definition 2.9: A fuzzy bi-ideal μ of a near-ring N is said to be normal if μ(0)=1.

Definition 2.10:.Let N and N′ be two near-rings and 'f 'a function of N into N′.

(i) If λ is a fuzzy set in N′ , then the preimage of λ under 'f' is the fuzzy set in N defined by $\mu(x) = (\lambda o f)(x)=\lambda(f(x))$ for each x∈N,
(ii) If μ is a fuzzy set of N, then the image of μ under f is the fuzzy set in N′ defined by

$$f(\mu)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu(x) & \text{if } f^{-1}(y) \neq \phi, \\ 0 & \text{otherwise} \quad \text{for each } y\in N'. \end{cases}$$

Definition 2.11: Let μ and λ be a T-fuzzy bi-ideal of a Near-ring N. Then the direct product of T-fuzzy bi-ideal is defined by $(\mu\times\lambda)(x, y)=T(\mu(x), \lambda(y))$ for all x, y∈N.

3. THE QUOTIENT NEAR-RINGS VIA T-FUZZY BI-IDEALS

Theorem 3.1: Let I be a bi-ideal of a near-ring N. If μ is a T-fuzzy bi-ideal of N, then the fuzzy set $\overline{\mu}$ of N/I defined by $\overline{\mu}(a+I) = \sup\limits_{x \in I}\mu(a+x)$ is a T-fuzzy bi-ideal of the quotient near-ring N/I with respect to I.

Proof: Let N be a near-ring and μ be a T-fuzzy bi-ideal of N.

Let a, b∈N such that a+I=b+I. Then b= a+y for some y∈I.

Thus $\overline{\mu}(b+I) = \sup\limits_{x \in I}\mu(b+x) = \sup\limits_{x \in I}\mu(a+y+x) = \sup\limits_{z = x + y \in I}\mu(a+z)=\overline{\mu}(a+I)$. Hence $\overline{\mu}$ is well defined.

Let x+I, y+I∈ N/I,

$$\begin{aligned} \overline{\mu}((x+I)-(y+I)) &= \overline{\mu}((x-y)+I) \\ &= \sup_{z \in I}\mu((x-y)+z) \\ &= \sup_{z = u - v \in I}\mu((x-y)+(u-v)) \\ &= \sup_{z = u - v \in I}\mu((x+u)-(y+v)) \\ &\geq \sup_{u, v \in I}T(\mu(x+u) , \mu(y+v)) \\ &= T\{\sup_{u \in I}\mu(x+u), \sup_{v \in I}\mu(y+v)\} \\ &= T(\overline{\mu}(x+I),\overline{\mu}(y+I)) \end{aligned}$$

Therefore $\overline{\mu}((x+I)-(y+I)) \geq T(\overline{\mu}(x+I), \overline{\mu}(y+I))$.

For x+I, y+I, z+I∈N/I,

$$\begin{aligned} \overline{\mu}((x+I)(y+I)(z+I)) &= \overline{\mu}((xyz)+I) \\ &= \sup_{u \in I}\mu(xyz+u) \\ &= \sup_{u \in I}\mu((x+u)(y+u)(z+u)) \\ &\geq \sup_{u \in I}T(\mu(x+u) ,\mu(z+u)) \\ &= T\{\sup_{u \in I}\mu(x+u), \sup_{u \in I}\mu(z+u)\} \\ &= T(\overline{\mu}(x+I), \overline{\mu}(z+I)). \end{aligned}$$

Therefore $\overline{\mu}((x+I)(y+I)(z+I)) \geq T(\overline{\mu}(x+I), \overline{\mu}(z+I))$.

Thus $\overline{\mu}$ is a T-fuzzy bi-ideal of N/I.

Theorem 3.2: Let I be a bi-ideal of near-ring N. Then there is one-to-one correspondence between the set of T-fuzzy bi-ideals μ of N such that μ(0) =μ(s) for all s∈I and the set of all T-fuzzy bi-ideals $\overline{\mu}$ of N/I.

Proof: Let I be a bi-ideal of a near-ring N. Let μ be T-fuzzy bi-ideal of a near-ring N. Using theorem 3.1, we prove that $\overline{\mu}$ defined by $\overline{\mu}$(a+I) = $\sup_{x \in I}$μ(a+x) is a T-fuzzy bi-ideal of N/I. Since μ(0) =μ(s) for all s∈I, μ(a+s)≥ T(μ(a), μ(s)) = μ(a). Again, μ(a)=μ(a+s-s) ≥ T(μ(a+s), μ(s))=μ(a+s).Thus μ(a+s) =μ(a) for all s ∈I, that is, $\overline{\mu}$(a+I) = μ(a). Hence the correspondence μ ↦ $\overline{\mu}$ is one-to-one. Let $\overline{\mu}$ be a T-fuzzy bi-ideal of N/I and defined fuzzy set μ in N by μ(a) =$\overline{\mu}$(a + I) for all a∈I.

For any x, y∈N,

$$
\begin{aligned}
\mu(x-y) = \overline{\mu}((x-y) + I) &= \overline{\mu}((x+I) - (y+I)) \\
&\geq T(\overline{\mu}(x + I), \overline{\mu}(y + I)) \\
&= T(\mu(x), \mu(y))
\end{aligned}
$$

Therefore μ(x-y) ≥ T(μ(x), μ(y)).

For any x, y, z∈N, we have

$$
\begin{aligned}
\mu(xyz) &= \overline{\mu}((xyz) + I) = \overline{\mu}((x + I)(y + I)(z+I)) \\
&\geq T(\overline{\mu}(x + I), \overline{\mu}(z + I)) \\
&= T(\mu(x), \mu(z)).
\end{aligned}
$$

Therefore μ(xyz) ≥ T(μ(x), μ(z)).

Thus μ is a T-fuzzy bi-ideal of N. Note that μ(z) = $\overline{\mu}$(z + I) = $\overline{\mu}$(I) for all z∈I, which shows that μ(z) =μ(0) for all z∈I.

Theorem 3.3: Let T be t-norm and I be a bi-ideal of a near-ring N. Then for all α∈[0,1], there exists a T-fuzzy bi-ideal μ of N such that μ(0) = α and U(μ ; α) = I.

Proof: Let μ : N→[0,1] be a fuzzy subset of N defined by μ(x) =$\begin{cases} \alpha \ if \ x \epsilon I \\ 0 \ oherwise \end{cases}$ Where α is a fixed number in [0,1].clearly U(μ ; α)= I and μ(0)=α. Let x, y∈N, then a routine calculation shows that μ is a T-fuzzy bi-ideal of N.

Theorem 3.4: Let μ be a T-fuzzy bi-ideal of a near-ring N and let μ(0)=α. Then the fuzzy subset μ^* of the quotient near-ring N / $U(\mu;\alpha)$ defined by μ^*(x+ $U(\mu;\alpha)$) = μ(x) for all x∈N is a T-fuzzy bi-ideal of N/ $U(\mu;\alpha)$.

Proof: The proof is straightforward.

Theorem 3.5: Let I be a bi-ideal of a near-ring N and ϕ be a T-fuzzy bi-ideal of N/I such that ϕ(x+I)=ϕ(x) only if x∈I. Then there exists a T-fuzzy bi-ideal of N such that$U(\mu\ ;\alpha)$=I, $\alpha = \mu(0)$,and ϕ=μ^*.

Proof: Let ϕ be a T-fuzzy bi-ideal of N/I. Define a T-fuzzy bi-ideal μ of N by μ(x)= ϕ(x+I) for all x∈N.

For x, y∈N, μ(x-y)=ϕ((x-y)+I) ≥ T(ϕ(x+I), ϕ(y+I)) =T(μ(x), μ(y))

For x, y, z∈N, μ(xyz)=ϕ(xyz+I) ≥T(ϕ(x+I), ϕ(z+I))=T(μ(x), μ(z)). Therefore μ is a T-fuzzy bi-ideal of N.

Next, we prove that
$U(\mu;\alpha)$ =I.Let x∈ U(μ; α)⇔μ(x)≥$\alpha = \mu(0)$⇔μ(x)≥$\mu(0)$⇔μ(x)= $\mu(0)$⇔ϕ(x+I)= ϕ(0+I))⇔ϕ(x+I) = ϕ(I)⇔ x∈I.

Hence U(μ ; α)=I. Finally, we prove that μ^* = ϕ, Now, μ^*(x+I) =μ^*(x+ $U(\mu;\alpha)$) = μ(x)= ϕ(x+I). Therefore μ^*= ϕ.

Theorem 3.6: An onto homomorphic image of a T-fuzzy bi-ideal with sup property is a T-fuzzy bi-ideal.

Proof: Let N and N′ be two near-rings. Let f: N→N ′be an epimorphism and μ be a T-fuzzy bi-ideal of N with sup property and λ be the image of μ under f.

Let f(x), f(y), f(z)∈f(N) and let $x_0 \in f^{-1}(f(x))$, $y_0 \in f^{-1}(f(y))$, $z_0 \in f^{-1}(f(z))$ such that $\mu(x_0) = \sup_{t \in f^{-1}(f(x))} \mu(t)$,
$\mu(y_0) = \sup_{t \in f^{-1}(f(y))} \mu(t)$, $\mu(z_0) = \sup_{t \in f^{-1}(f(z))} \mu(t)$.

Now,

(i) $\lambda(f(x)-f(y)) = \sup_{t \in f^{-1}(f(x)-f(y))} \mu(t)$

$\geq \mu(x_0-y_0)$

$\geq T\{\mu(x_0), \mu(y_0)\}$

$= T\{\sup_{t \in f^{-1}(f(x))} \mu(t), \sup_{t \in f^{-1}(f(y))} \mu(t)\}$

$= T\{\lambda(f(x)), \lambda(f(y))\}$

Therefore $\lambda(f(x)-f(y)) \geq T\{\lambda(f(x)), \lambda(f(y))\}$.

(ii) $\lambda(f(x)f(y)f(z)) = \lambda(f(xyz))$

$= \sup_{t \in f^{-1}(f(xyz))} \mu(t)$

$\geq \mu(x_0y_0z_0)$

$\geq T\{\mu(x_0), \mu(z_0)\}$

$= T\{\sup_{t \in f^{-1}(f(x))} \mu(t), \sup_{t \in f^{-1}(f(z))} \mu(t)\}$

$= T\{\lambda(f(x)), \lambda(f(z))\}$

Therefore $\lambda(f(x)f(y)f(z)) \geq T\{\lambda(f(x)), \lambda(f(z))\}$. Hence λ is a T-fuzzy bi-ideal of N′.

Theorem 3.7: An onto homomorphic pre-image of a T-fuzzy bi-ideal of near-ring is a T-fuzzy bi-ideal.

Proof: Let N and N′ be two near-rings. Let f: N→N′be an epimorphism.

Let x, y∈N,

(i) $\mu(x-y) = (\lambda o f)(x-y)$

$= \lambda(f(x-y))$

$= \lambda((f(x)-f(y))$

$\geq T\{\lambda(f(x)), \lambda(f(y))\}$

$= T\{(\lambda o f)(x), (\lambda o f)(y)\}$

$= T\{\mu(x), \mu(y)\}$

Therefore $\mu(x-y) \geq T(\mu(x), \mu(y))$.

Similarly we can prove that $\mu(xyz) \geq T(\mu(x), \mu(z))$ for all x, y, z ∈N. Hence μ is a T-fuzzy bi-ideal of N.

Theorem 3.8: Let N and N′ be two near-rings. If μ and λ are T-fuzzy bi-ideals of N and N′ respectively, then μxλ is a T-fuzzy bi-ideal of the direct product NxN′.

Proof: Let $(x_1, y_1), (x_2, y_2) \in N \times N'$.

Now, $(\mu \times \lambda)((x_1,y_1)-(x_2,y_2)) = (\mu \times \lambda)(x_1-x_2, y_1-y_2)$

$= T[\mu(x_1-x_2), \lambda(y_1-y_2)]$

$\geq T[T(\mu(x_1), \mu(x_2)), T(\lambda(y_1), \lambda(y_2))]$

$= T(T[T(\mu(x_1), \mu(x_2)), \lambda(y_1)], \lambda(y_2))$

$= T[T(\lambda(y_1), T(\mu(x_1), \mu(x_2))), \lambda(y_2)]$

$= T(T[T(\lambda(y_1), (\mu(x_1)), \mu(x_2)], \lambda(y_2))$

$= T[T(\mu(x_1), (\lambda(y_1)), T(\mu(x_2), \lambda(y_2))]$

$= T[(\mu \times \lambda)(x_1, y_1), (\mu \times \lambda)(x_2,y_2)]$

Therefore $(\mu \times \lambda)((x_1, y_1) - (x_2, y_2)) \geq T((\mu \times \lambda)(x_1, y_1), (\mu \times \lambda)(x_2, y_2))$.

Let $(x_1, y_1), (x_2, y_2), (x_3, y_3) \in N \times N'$,

Now, $(\mu \times \lambda)((x_1, y_1)(x_2, y_2)(x_3, y_3)) = (\mu \times \lambda)(x_1x_2x_3, y_1y_2y_3)$

$= T(\mu(x_1x_2x_3), \lambda(y_1y_2y_3))$

$\geq T[T(\mu(x_1), \mu(x_3)), T(\lambda(y_1), \lambda(y_3))]$

$= T(T[T(\mu(x_1), \mu(x_3)), \lambda(y_1)], \lambda(y_3))$

$= T[T(\lambda(y_1), T(\mu(x_1), \mu(x_3))), \lambda(y_3)]$

$= T(T[T(\lambda(y_1), (\mu(x_1)), \mu(x_3)], \lambda(y_3))$

$= T[T(\mu(x_1), (\lambda(y_1)), T(\mu(x_3), \lambda(y_3))]$

$= T[(\mu \times \lambda)(x_1, y_1), (\mu \times \lambda)(x_3, y_3)]$

Therefore $(\mu \times \lambda)((x_1, y_1)(x_2, y_2)(x_3, y_3)) \geq T((\mu \times \lambda)(x_1, y_1), (\mu \times \lambda)(x_3, y_3))$.

Hence $\mu \times \lambda$ is a T-fuzzy bi-ideal of the direct product $N \times N'$.

Theorem 3.9: Let μ be a T-fuzzy bi-ideal of N. If $\mu(x+y) = \mu(0)$, then $\mu(x) = \mu(y)$.

Proof: Assume that $\mu(x+y) = \mu(0)$ for all $x, y \in N$.

Then
$$\begin{aligned}\mu(x) &= \mu((x+y)-y)\\ &\geq T(\mu(x+y), \mu(y))\\ &= T(\mu(0), \mu(y))\\ &= \mu(y)\end{aligned}$$
Therefore $\mu(x) \geq \mu(y)$.

Now,
$$\begin{aligned}\mu(y) &= \mu((x+y)-x)\\ &\geq T(\mu(x+y), \mu(x))\\ &= T(\mu(0), \mu(x))\\ &= \mu(x)\end{aligned}$$
Therefore $\mu(y) \geq \mu(x)$. Hence $\mu(x)=\mu(y)$.

Theorem 3.10:. Let N be a near-ring and μ be a T-fuzzy bi-ideal of N. Then the set $N_\mu = \{x \in N / \mu(0) = \mu(x)\}$ is a bi-ideal of N.

Proof: Let μ be a T-fuzzy bi-ideal of N. Let $x, y \in N_\mu \Rightarrow \mu(x) = \mu(y) = \mu(0)$. Consider $\mu(x-y) \geq T(\mu(x), \mu(y)) \geq T(\mu(0), \mu(0)) \geq \mu(0)$ and so $\mu(x-y) = \mu(0) \Rightarrow x-y \in N_\mu$ and hence N_μ is a subgroup of N. Let $x, z \in N_\mu$ and $y \in N$. Then $\mu(x) = \mu(z) = \mu(0)$. Now $\mu(xyz) \geq T(\mu(x), \mu(z)) \geq T(\mu(0), \mu(0)) = \mu(0)$ and so $\mu(xyz) = \mu(0) \Rightarrow xyz \in N_\mu$. Hence N_μ is a bi-ideal of N.

Theorem 3.11: If $\{\mu_i / i \in \wedge\}$ is a family of T-fuzzy bi-ideals of a near-ring N, then $\bigvee_{i\in\wedge}\mu_i$ is also a T-fuzzy bi-ideal of N where $\bigvee_{i\in\wedge}\mu_i$ is defined by $(\bigvee_{i\in\wedge}\mu_i)(x) = \sup\{\mu_i(x) / i \in \wedge\}$ for all $x \in N$.

Proof: Let $\{\mu_i / i \in \wedge\}$ be a family of T-fuzzy bi-ideals of N and $x, y, z \in N$. Then we have that
$$\begin{aligned}(\bigvee_{i\in\wedge}\mu_i)(x-y) &= \sup\{\mu_i(x-y)/i \in \wedge\}\\ &\geq \sup\{T(\mu_i(x), \mu_i(y)) / i \in \wedge\}\\ &= T(\{\sup\{\mu_i(x)/i \in \wedge\}, \sup\{\mu_i(y) / i \in \wedge\})\\ &= T((\bigvee_{i\in\wedge}\mu_i)(x), (\bigvee_{i\in\wedge}\mu_i)(y))\end{aligned}$$

Therefore $(\bigvee_{i\in\wedge}\mu_i)(x-y) \geq T((\bigvee_{i\in\wedge}\mu_i)(x), (\bigvee_{i\in\wedge}\mu_i)(y))$.

Now,
$$\begin{aligned}(\bigvee_{i\in\wedge}\mu_i)(xyz) &= \sup\{\mu_i(xyz) / i \in \wedge\}\\ &\geq \sup\{T(\mu_i(x), \mu_i(z)) / i \in \wedge\}\\ &= T(\sup\{\mu_i(x)/i \in \wedge\}, \sup\{\mu_i(z) / i \in \wedge\})\\ &= T((\bigvee_{i\in\wedge}\mu_i)(x), (\bigvee_{i\in\wedge}\mu_i)(z))\end{aligned}$$

Therefore $(\bigvee_{i\in\wedge}\mu_i)(xyz) \geq T((\bigvee_{i\in\wedge}\mu_i)(x), (\bigvee_{i\in\wedge}\mu_i)(z))$. Hence $\bigvee_{i\in\wedge}\mu_i$ is a T-fuzzy bi-ideal of N.

Theorem 3.12: Let μ be a T-fuzzy bi-ideal of a near-ring N and μ be a fuzzy subset in N defined by $\hat{\mu}(x) = \frac{1}{\mu(0)}\mu(x)$ for all $x \in N$. Then $\hat{\mu}$ is a normal T-fuzzy bi-ideal of N.

Proof: Let μ be a T-fuzzy bi-ideal of a near-ring N. Clearly, $\hat{\mu}(0) = \frac{1}{\mu(0)}\mu(0) = 1$, $\hat{\mu}$ is normal.

For any $x, y \in N$,

Then
$$\begin{aligned}\hat{\mu}(x-y) &= \frac{1}{\mu(0)}\mu(x-y)\\ &\geq \frac{1}{\mu(0)}\min\{\mu(x), \mu(y)\}\\ &= \min\{\frac{\mu(x)}{\mu(0)}, \frac{\mu(y)}{\mu(0)}\}\\ &= \min\{\hat{\mu}(x), \hat{\mu}(y)\}\end{aligned}$$

Therefore $\hat{\mu}(x-y) \geq \min\{\hat{\mu}(x), \hat{\mu}(y)\}$. Similarly $\hat{\mu}(xyz) \geq \min\{\hat{\mu}(x), \hat{\mu}(z)\}$ for all x, y, z∈N. Hence $\hat{\mu}$ is a normal T-fuzzy bi-ideal of N and obviously $\mu \subseteq \hat{\mu}$.

Corollary 3.13: If μ is a T-fuzzy bi-ideal of a near-ring N satisfying $\hat{\mu}$ (x) = 0 for some x∈N, then μ(x)= 0 also.

4. t-NORM (λ, μ)-FUZZY bi-IDEALS

Definition 4.1: Let A be a fuzzy subset of N. Then A is called a t-norm (λ μ)-fuzzy bi-ideal of N if for all x, y, z∈N,

(i) $A(x-y) \vee \lambda \geq t(t(A(x), A(y)), \mu)$

(ii) $A(xyz) \vee \lambda \geq t(t(A(x), A(z)), \mu)$.

Theorem 4.2: Let A be a fuzzy subset of N. Then A is a t-norm (λ,μ)-fuzzy bi-ideal of N iff A_α is a bi-ideal of N for all $\alpha \in (\lambda, \mu)$.

Proof: Let A be a t-norm (λ, μ)-fuzzy bi-ideal of N. Let x, y ∈ $A_\alpha \Rightarrow A(x) \geq \alpha, A(y) \geq \alpha$. Consider $A(x-y) \vee \lambda \geq t(t(A(x), A(y)), \mu) \geq t(t(\alpha, \alpha), \mu) = \alpha$ (since α>λ). That is $A(x-y) \vee \lambda \geq \alpha \Rightarrow A(x-y) \geq \alpha \Rightarrow x-y \in A_\alpha$. Hence A_α is a subgroup of N. Let x, z∈$A_\alpha \Rightarrow A(x) \geq \alpha, A(z) \geq \alpha$ and y∈N. Then $A(x-y) \vee \lambda \geq t(t(A(x), A(z)), \mu) \geq t(t(\alpha, \alpha), \mu) = \alpha$ (since α>λ). That is $A(xyz) \vee \lambda \geq \alpha \Rightarrow A(xyz) \geq \alpha \Rightarrow xyz \in A_\alpha$. Hence A_α is a bi-ideal of N.

Conversely suppose that A_α is a bi-ideal of N for all α∈(λ, μ). Suppose $A(x-y) \vee \lambda < t(t(A(x), A(y)), \mu) = \alpha$. Then $A(x-y) \vee \lambda < \alpha \Rightarrow A(x-y) < \alpha$ (since α>λ) $\Rightarrow x-y \notin A_\alpha$, a contradiction to A_α is a bi-ideal of N. Hence $A(x-y) \vee \lambda \geq t(t(A(x), A(y)), \mu)$. Similarly we can prove that $A(xyz) \vee \lambda \geq t(t(A(x), A(z)), \mu)$. Hence A is a t-norm (λ, μ)-fuzzy bi-ideal of N.

Theorem 4.3: Let f: $N_1 \rightarrow N_2$ be a onto homomorphism of a near-ring N and let A be a t-norm (λ, μ)-fuzzy bi-ideal of N_1. Then f(A) is a t-norm (λ, μ)-fuzzy bi-ideal of N_2 where f(A)(y) = sup{A(x) / f(x) = y} for all y∈N_2.

Proof: Let f: $N_1 \rightarrow N_2$ be a onto homomorphism of a near-ring N and let A be a t-norm (λ, μ)-fuzzy bi-ideal of N_1.

For any $y_1, y_2 \in N_2$, we have

(i)
$$\begin{aligned} f(A)(y_1-y_2)\vee\lambda &= \sup\{A(x_1-x_2) / f(x_1-x_2) = y_1-y_2\}\vee\lambda \\ &= \sup\{A(x_1-x_2) \vee\lambda / f(x_1-x_2) = y_1-y_2\} \\ &\geq \sup(t(t(A(x_1), A(x_2)), \mu) / f(x_1) = y_1, f(x_2) = y_2) \\ &= t(t(\sup\{A(x_1) / f(x_1) = y_1\}, \sup\{A(x_2) / f(x_2) = y_2)\}), \mu) \\ &= t(t(f(A)(y_1), f(A)(y_2)), \mu) \end{aligned}$$

Therefore $f(A)(y_1-y_2)\vee\lambda \geq t(t(f(A)(y_1), f(A)(y_2)), \mu)$.

Similarly we can prove that $f(A)(y_1y_2y_3)\vee\lambda \geq t(t(f(A)(y_1), f(A)(y_3)), \mu)$. Hence f(A) is a t-norm (λ,μ)-fuzzy bi-ideal of N_2.

Theorem 4.4: Let f: $N_1 \rightarrow N_2$ be a onto homomorphism of a near-ring N and let B be a t-norm (λ,μ)-fuzzy bi-ideal of N_2. Then $f^{-1}(B)$ is a t-norm (λ,μ)-fuzzy bi-ideal of N_1, where $f^{-1}(B)(x) = B(f(x))$ for all x∈N_1.

Proof: For any $x_1, x_2 \in N_1$,

$$\begin{aligned} f^{-1}(B)(x_1- x_2)\vee\lambda &= B(f(x_1 - x_2))\vee\lambda \\ &= B(f(x_1) - f(x_2))\vee\lambda \\ &\geq t(t(B(f(x_1), B(f(x_2)), \mu) \\ &= t(t(f^{-1}(B)(x_1), f^{-1}(B)(x_2)), \mu) \end{aligned}$$

Therefore $f^{-1}(B)(x_1- x_2)\vee\lambda \geq t(t(f^{-1}(B)(x_1), f^{-1}(B)(x_2)), \mu)$. Similarly we can prove that $f^{-1}(B)(x_1x_2x_3)\vee\lambda \geq t(t(f^{-1}(B)(x_1), f^{-1}(B)(x_3)), \mu)$. Hence $f^{-1}(B)$ is a t-norm (λ, μ)-fuzzy bi-ideal of N_1.

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