ON OUT AND IN BINARY NEIGHBORHOOD GRAPHS

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ABSTRACT

The out binary neighborhood graph \( B^{-}_{md}(H) \) of a directed graph \( H = (V,E) \) is the undirected graph with vertex set \( V \cup S^{-} \) where \( S^{-} \) is the set of all non-empty out open neighborhood sets of \( H \) in which two vertex \( u,v \) are adjacent if \( u,v \in S^{-} \) and \( u \cap v \neq \phi \) or \( u \in V \) and \( v \) is the out open neighborhood set of \( u \). Similarly we can define the in open binary neighborhood graph. In this paper some properties of these new graphs are discussed.

Keywords: Neighborhood set, In and out binary neighborhood graph, connected graph, Eulerian.

1. INTRODUCTION

We are considering only finite, simple, directed or undirected graphs. Consider a graph \( G = (V,E) \) and a vertex \( v \) in \( V \). Then the open neighborhood set of \( v \) is the set \( N(v) = \{ u \in V : uv \in E \} \). The idea of neighborhood graph was introduced by Kulli in [1] and according to him for a graph \( G = (V,E) \) the neighborhood graph of \( G \) is the graph with vertex set \( V \cup S \) where \( S \) is the set of all open neighborhood sets of vertices of \( G \) in which two vertex \( u,v \) are adjacent if \( u,v \in S \) and \( u \cap v \neq \phi \) or \( u \in V \) and \( v \) is an open neighborhood set containing \( u \).

Let \( H = (V,E) \) be a directed graph and \( v \) be a vertex in \( V \). Then the open neighborhood set of \( v \) is the set \( N^{-}(v) = \{ u \in V : vu \in E \} \) and the in open neighborhood set of \( v \) is \( N^{+}(v) = \{ u \in V : uv \in E \} \). We devote this paper to introduce a new type of graph using these definitions.

2. OUT AND IN BINARY NEIGHBORHOOD GRAPHS

Definition 2.1 Out binary neighborhood graph: The out binary neighborhood graph \( B^{-}_{md}(H) \) of a directed graph \( H = (V,E) \) is the undirected graph with vertex set \( V \cup S^{-} \) where \( S^{-} \) is the set of all non-empty out open neighborhood sets of vertices of \( H \), in which two vertex \( u,v \) are adjacent if \( u,v \in S^{-} \) and \( u \cap v \neq \phi \) or \( u \in V \) and \( v \) is the out open neighborhood set of \( u \).

Definition 2.2 In binary neighborhood graph: The in binary neighborhood graph \( B^{+}_{md}(H) \) of a directed graph \( H = (V,E) \) is the undirected graph with vertex set \( V \cup S^{+} \), where \( S^{+} \) is the set of all non-empty in open neighborhood sets of vertices of \( H \), in which two vertex \( u,v \) are adjacent if \( u,v \in S^{+} \) and \( u \cap v \neq \phi \) or \( u \in V \) and the in open neighborhood set of \( u \).

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Example 2.3: In figure 1, 2 and 3 a directed graph $H$ and its $B_{nd}^-(H)$ and $B_{nd}^+(H)$ are given. For $H$ in figure 1 the out open neighborhood sets of $H$ are $N^-(A) = \{C\}, N^-(B) = \{C, D\}, N^-(C) = \{D\}, N^-(D) = \{\varphi\}$ and the in open neighborhood sets of $H$ are $N^+(A) = \{B\}, N^+(B) = \varphi, N^+(C) = \{A, B\}, N^+(D) = \{B, C\}$.

Result 2.4: For any directed graph $H$, if a vertex $v$ in $H$ isolated then it remains isolated in both $B_{nd}^+(H)$ and $B_{nd}^-(H)$.

Definition 2.5 Source, sink vertex [2]: A vertex $v$ of a directed graph $H$ is said to be a sink vertex if $N^-(v) = \varphi$ and is said to be a source vertex if $N^+(v) = \varphi$.

Theorem 2.6:

a. A vertex $v$ in $H$ is isolated in $B_{nd}^-(H)$ iff it is sink vertex in $H$

b. A vertex $v$ in $H$ is isolated in $B_{nd}^+(H)$ iff it is source in $H$

Proof:

a. A vertex $v$ in $H$ is isolated in $B_{nd}^-(H)$ iff $N^-(v) = \varphi$ iff $v$ is sink vertex in $H$.

b. A vertex $v$ in $H$ is isolated in $B_{nd}^+(H)$ iff $N^+(v) = \varphi$ iff it is a source in $H$.

Result 2.7

a. If $v$ is a vertex of the directed graph $H$, then the degree of $v$ in $B_{nd}^-(H)$ is equal to 0 if $v$ is a sink in $H$ and equal to 1 if $v$ is not a sink in $H$.

b. If $v$ is a vertex of the directed graph $H$, then the degree of $v$ in $B_{nd}^+(H)$ is equal to 0 if $v$ is a source in $H$ and equal to 1 if $v$ is not a source in $H$.

Remark 2.8: Result 2.7 is the reason behind the name “in / out binary neighborhood graph”.

Definition 2.9 I-sequence collection: Let $\mathcal{S} = \{S_1, S_2, \ldots, S_n\}$ be a collection of non-empty sets. Then we call $\mathcal{S}$ an I-sequence collection if there exist a sequential arrangement $S_{k_1}, S_{k_2}, \ldots, S_{k_n}$ of elements of $\mathcal{S}$ such that it contains all elements of $\mathcal{S}$ at least once and for all $t, 1 \leq t \leq n$, there exist at least one $d < t$ with $S_{k_d} \cap S_{k_t} \neq \varphi$.

Lemma 2.10: For any directed graph $H = (V, E)$, the out binary neighborhood graph $B_{nd}^-(H) \{or\ B_{nd}^+(H)\}$ is connected then degree each out open neighborhood \{ respectively in open neighborhood\} of vertices of $H$ has degree at least 2 in $B_{nd}^-(H)$ \{or $B_{nd}^+(H)$\}.

Proof: follows from the connectedness of the graph and by result 2.7.
Theorem 2.11:

a. For any directed graph $H = (V, E)$, if the out binary neighborhood graph $B^-_n(H)$ is connected then $S^-$, the set of all non-empty out open neighborhood sets of vertices of $H$, is an I-sequence collection and $N^-(v) \neq \emptyset$ for all $v$ in $V$.

b. For any directed graph $H = (V, E)$, if the in binary neighborhood graph $B^+_n(H)$ is connected then $S^+$, the set of all non-empty in open neighborhood sets of vertices of $H$, is an I-sequence collection and $N^+(v) \neq \emptyset$ for all $v$ in $V$.

Proof:

a. Let $B^-_n(H)$ is connected. Then clearly $N^-(v) \neq \emptyset$ for all $v$ in $V$. We need only to prove $S^-$ is an I-sequence collection. For let $v_1, v_2, ..., v_n$ be the vertices of $H$ and $N^-(v_i)$ be the out open neighborhood set of some vertex $v_i$ of $V$. Then by lemma 2.10, there must exist another out open neighborhood set $N^-(v_j)$ of some vertex $v_j \neq v_i$ of $V$, such that $N^-(v_i)$ and $N^-(v_j)$ are adjacent in $B^-_n(H)$. Then by definition $N^-(v_i) \cap N^-(v_j) \neq \emptyset$.

Now suppose we have an I-sequence collection $\{N^-(v_{k1}), N^-(v_{k2}), ..., N^-(v_{kt})\}$ for $1 \leq t < n$. Our aim is find a $N^-(v_{k,t+1})$ such that $\{N^-(v_{k1}), N^-(v_{k2}), ..., N^-(v_{kt}), N^-(v_{k,t+1})\}$ is I-sequence collection. If possible no such $N^-(v_{k,t+1})$ exist. But that means none of the remaining out open neighborhood sets is adjacent to any of the sets in $\{N^-(v_{k1}), N^-(v_{k2}), ..., N^-(v_{kt})\}$. But then by connectedness of $B^-_n(H)$ at least one vertex in $V$ has degree greater than 1. This is a contradiction. Hence by mathematical induction, the result follows.

b. Similar proof follows

The following theorem is useful,

Theorem 2.12: A connected graph $G$ is Eulerian\cite{3} if and only if every vertex of $G$ has even degree.

Theorem 2.13: For any directed graph $H = (V, E)$, the out binary neighborhood graph $B^-_n(H)$ or the in binary neighborhood graph $B^+_n(H)$ are never Eulerian

Proof: follows from theorem 2.12 and result 2.7.

REFERENCES


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