SPLITTING GRAPH ON EVEN SUM CORDIAL LABELING OF GRAPHS

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ABSTRACT

In this paper, we investigate the splitting graph of the family of bipartite graphs, paths and cycles are even sum cordial graphs and proved several classes of graphs such that \( P_m(n) + K_n \), \((K_n \cup P_m) + 2K_1\), \( <W_n(1), W_n(2)>\), \( B_n \), \( S(B_n, n) \), Helm graph \( H_n \) and Flower graph \( F_l n \) are even sum cordial graphs.

Keywords: Splitting graphs, Helm graphs, Flower graphs, Bistar.

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1. INTRODUCTION

The origin of graph labeling can be attributed to Rosa[7]. In [1], Cahit introduce the concept of cordial labeling of graph. For splitting graph we refer E.Sampath kumar and H.B.Waliker[8]. For different splitting graphs we refer R.Lawrence Razario raj[5, 6]. In [9, 10], Vaidya, et.al., introduced the concepts of Helm, Flower and Bistar graphs are divisor cordial labeling. In this paper, we investigate the splitting graph of the family of bipartite graphs, paths and cycles are even sum cordial graphs and proved several classes of graphs such that \( P_m(n) + K_n \), \((K_n \cup P_m) + 2K_1\), \( <W_n(1), W_n(2)>\), \( B_n \), \( S(B_n, n) \), Helm graph \( H_n \) and Flower graph \( F_l n \) are even sum cordial graphs.

2. PRELIMINARIES

Definition 2.1 [3] Let \( G = (V, E) \) be a simple graph and \( f: V \rightarrow \{1, 2, 3, \ldots |V|\} \) be a bijection. For each edge \( uv \), assign the label 1 if \( f(u) + f(v) \) is even and the label 0 otherwise. \( f \) is called an even sum cordial labeling if \( |e_f(0) - e_f(1)| \leq 1 \), where \( e_f(1) \) and \( e_f(0) \) denote the number of edges labeled with 1 and number of edges labeled with 0 respectively. A graph with an even sum cordial labeling is called an even sum cordial graph.

Proposition 2.1: [3] Any path is an even sum cordial graph.

Proposition 2.2: [3] Any cycle \( C_n \) is an even sum cordial graph except \( n = 6, 6 + 2d, \ldots \) when \( d = 4 \).

Definition 2.3: [6] A wheel graph \( W_n \) is a graph with \( n + 1 \) vertices formed by connecting a single vertex to all the vertices of \( n \) cycle.

Definition 2.3: [9] The Helm graph \( H_n \) is the graph obtained from a wheel \( W_n \) by attaching a pendent edge to each rim vertex. It consists three types of vertices:
- an apex of degree \( n \)
- \( n \) vertices of degree 4
- \( n \) pendent vertices.

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Definition 2.4: [9] The Flower graph $F_{ln}$ is the graph obtained from Helm $H_n$ by joining each pendent vertex to apex of the Helm $H_n$. It consists three types of vertices:
- an apex of degree $2n$
- $n$ vertices of degree $4$
- $n$ vertices of degree $2$.

Definition 2.5: [6] Consider two copies of graph $G$ namely $G_1$ and $G_2$. Then the graph $G' = \langle G_1, G_2 \rangle$ is a graph obtained by joining the apex vertices of $G_1$ and $G_2$ by a new vertex.

Definition 2.6: [10] The Bistar $B_{n,n}$ is a graph obtained by joining the two copies of $K_{1,n}$ by an edge is called bistar graph.

Definition 2.7: [5] The graph $P_m(+)K_n$ is a graph with the vertex set $V(G) = \{u_i, v_j, 1 \leq i \leq m, 1 \leq j \leq n\}$ and the edge set $E(G) = \{u_iu_{i+1}, u_1v_1, u_nv_n, 1 \leq i \leq (m-1), 1 \leq j \leq n\}$

Definition 2.8: [4] The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is another graph $G_3$, ie., $G_3 = G_1 \cup G_2$ whose vertex set $V_3 = V_1 \cup V_2$ and the edge set is $E_3 = E_1 \cup E_2$.

Definition 2.9: [4] The joint $G_1 + G_2$ of $G_1$ and $G_2$ consists of $G_1 \cup G_2$ and all lines joining $V_1$ with $V_2$.

Definition 2.10: [8] (Splitting Graph) For each vertex $v$ of a graph $G$, take a new vertex $v'$, join $v'$ to all vertices of $G$ adjacent to $v$. The graph $S(G)$ thus obtained is called splitting graph of $G$.

3. MAIN RESULTS

Proposition 3.1: The graph $S(K_{1,m})$ is an even sum cordial graph.

Example 3.1: $S(K_{1,3})$

Proposition 3.2: The graph $S(K_{2,m})$ is an even sum cordial graph.

Example 3.2: $S(K_{2,3})$

Proposition 3.3: The graph $S(K_{1,n,n})$ is an even sum cordial graph.

Example 3.3: $S(K_{1,3,3})$
Proposition 3.4: The graph $S(P_n)$ is an even sum cordial graph except $n$ is multiple of 11.

Proof: Let $G$ be a graph $S(P_n)$. Let $v_1, v_2, \ldots, v_n$ be the vertices of $P_n$. By Proposition 2.1, we construct the even sum cordial path $P_n$. Then add the vertices $v_1, v_2, \ldots, v_n$ to the graph $P_n$. Assign the labeling for the vertices in the following way. Define $f: V(G) \rightarrow \{1, 2, 3, \ldots, 2n\}$ as follows $f(v_{n-i}) = n + i + 1, 0 \leq i \leq n-1$ and $|V(G)| = 2n, |E(G)| = 3(n-1)$. Now, we construct the splitting graph $S(P_n)$ by definition 2.10. Thus $|e_f(0) - e_f(1)| \leq 1$. Hence $G$ is an even sum cordial graph.

Example 3.4: $S(P_{12})$

Proposition 3.5: The graph $S(C_n)$ is an even sum cordial graph except $n = 6, 6 + d, 6 + 2d$, when $d = 4$.

Proof: Let $G$ be a graph $S(C_n)$. Let $v_1, v_2, \ldots, v_n$ be the vertices of $C_n$. By Proposition 2.2, we construct the even sum cordial cycle $C_n$. Then add the vertices $v_1, v_2, \ldots, v_n$ to the graph $C_n$. Assign the labeling for the vertices in the following way. Define $f: V(G) \rightarrow \{1, 2, 3, \ldots, 2n\}$ as follows $f(v_i) = n + i, 1 \leq i \leq n$ and $|V(G)| = 2n, |E(G)| = 3n$. Now, we draw the splitting graph $S(C_n)$ by definition 2.10. Thus $|e_f(0) - e_f(1)| \leq 1$. Hence $G$ is an even sum cordial graph.

Example 3.5: $S(C_{11})$

Proposition 3.6: The graph $H_n$ is an even sum cordial graph.

Proof: Let $G$ be a graph $H_n$. Let $x, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n$ be the vertices of $H_n$. Here $x$ is the apex vertex of degree $n$, let $v_1, v_2, \ldots, v_n$ are the vertices of degree 4 and $u_1, u_2, \ldots, u_n$ are the pendent vertices. Assign the labeling for the vertices in the following way. Define $f: V(G) \rightarrow \{1, 2, 3, \ldots, 2n+1\}$ as follows $f(x) = 1, f(v_i) = i + 1, 1 \leq i \leq n, f(u_k) = n + k + 1, 1 \leq k \leq n$ and $|V(G)| = 2n + 1, |E(G)| = 3n$. We draw the graph $H_n$ from the wheel graph $W_n$ by attaching a pendant edge to each rim vertex. Then we get $e_f(0) = e_f(1), \text{ when } n \text{ is even};$ $e_f(0) = e_f(1) + 1, \text{ when } n \text{ is odd}.$

Thus $|e_f(0) - e_f(1)| \leq 1$. Hence $G$ is an even sum cordial graph.
Example 3.6: \((H_5)\)

**Proposition 3.7:** The graph \(F_{l_n}\) is an even sum cordial graph.

**Proof:** Let \(G\) be a graph \(F_{l_n}\). Let \(x, v_1, v_2,\ldots, v_n\) be the vertices of \(F_{l_n}\). Here \(x\) is the apex vertex of degree \(2n\), \(v_1, v_2,\ldots, v_n\) are the vertices of degree 4 and \(u_1, u_2,\ldots, u_n\) are the vertices of degree 2. Assign the labeling for the vertices in the following way. Define \(f: V(G) \rightarrow \{1, 2, 3, \ldots, 2n + 1\}\) as follows:

\[
f(x) = 1, \quad f(v_i) = i + 1, 1 \leq i \leq n, \quad f(u_k) = n + k + 1, 1 \leq k \leq n\]

and \(|V(G)| = 2n + 1, |E(G)| = 4n\). We draw the graph \(F_{l_n}\) from the graph \(H_n\) by joining each pendant vertex of \(H_n\) to the apex vertex of \(H_n\) by an edge. Then we get \(e_f(0) = e_f(1)\).

Thus \(|e_f(0) - e_f(1)| \leq 1\).

Hence \(G\) is an even sum cordial graph.

Example 3.7: \((F_{l_5})\)

**Proposition 3.8:** The graph \(P_m (+) \overline{K}_n\) is an even sum cordial graph.

**Example 3.8:**

\[
m=7,n=4 \quad m=8,n=4
\]
**Proposition 3.9:** The graph \((\overline{K_n} \cup P_n) + 2K_1\) is an even sum cordial graph.

**Example 3.9:** \((\overline{K_5} \cup P_6) + 2K_1\)

![Graph](image)

**Proposition 3.10:** The graph \((W^1_n, W^2_n)\) is an even sum cordial graph.

**Proof:** Let \(G\) be a graph \((W^1_n, W^2_n)\). Let \(u, v, u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\) be the vertices of \(G\). Let \(u_1, u_2, \ldots, u_n\) be the rim vertices of \(W^1_n\) and \(v_1, v_2, \ldots, v_n\) be the rim vertices of \(W^2_n\).

Let \(u\) and \(v\) be the apex vertices of \(W^1_n\) and \(W^2_n\) respectively and \(x\) be a common vertex of \(W^1_n, W^2_n\). Then \(|V(G)| = 2n + 3, |E(G)| = 4n + 2\).

Define \(f: V(G) \to \{1, 2, 3, \ldots, 2n + 3\}\) as follows; Construction of \(W^1_n\). Wheel graph \(W^1_n\) is a graph with \(n + 1\) vertices formed by connecting single vertex \(u\) to all the vertices \(u_1, u_2, \ldots, u_n\) of \(n\) cycle. Assign the label of \(W^1_n\) by \(f(u_i) = 2i, 1 \leq i \leq n\) and \(f(u) = 1\).

Construction of \(W^2_n\). Wheel graph \(W^2_n\) is a graph with \(n + 1\) vertices formed by connecting single vertex \(v\) to all the vertices \(v_1, v_2, \ldots, v_n\) of \(n\) cycle. Assign the label of \(W^2_n\) by \(f(v_i) = 2i + 1, 1 \leq i \leq n\) and \(f(v) = 2n + 2\) and \(f(x) = 2n + 3\). Then connect the vertices \(u\) and \(v\) to the vertex \(x\) by an edge. Then we get \(e_f(0) = e_f(1)\). Thus \(|e_f(0) - e_f(1)| \leq 1\). Hence \(G\) is an even sum cordial graph.

**Example 3.10:** \((W^1_8, W^2_8)\)

![Graph](image)

**Proposition 3.11:** The graph \(B_{n,n}\) is an even sum cordial graph.

**Example 3.11:**

![Graph](image)

**Proposition 3.12:** The graph \(S(B_{n,n})\) is an even sum cordial graph.
Example 3.12:

4. CONCLUSIONS

In this paper, we established the splitting graph of the family of bipartite graphs, paths and cycles are even sum cordial graphs and proved several classes of graphs such that \( P_m(+)\overline{K_n}(\overline{K_n} \cup P_m) + 2K_1, (W_n^{(1)}, W_n^{(2)}), B_{n,n} \otimes (B_{n,n}), \) Helm graph \( H_n \) and Flower graph \( F_{l,n} \) are even sum cordial graphs.

REFERENCES


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