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SOME RESULTS ON SOFT RELATIONS

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ABSTRACT

In this paper, we established the Commutative law, Associative law, Distributive law and De Morgan's laws on soft relations.

Keywords: *Cartesian products, Inverse of a soft set relation, Complement of a soft set relation, Union of soft set relation, Intersection of soft set relation.*

1. INTRODUCTION

Soft theory was initiated by the Russian researcher Molodtsov in 1999. Molodtsov proposed the soft set as a completely generic mathematical tool for modeling uncertainties. There is no limited condition to the description of objects; so researchers can choose the form of parameters they need. There are many mathematical tools available for modeling complex systems such as probability theory, fuzzy set theory, interval mathematics etc. But there are inherent difficulties associated with each of these techniques. Soft set theory has a rich potential for application in many directions, some of which are reported by Molodtsov [1] in his work. He successfully 1 applied soft set theory in areas such as the smoothness of function, game theory, operation research, Riemann integration and so on. Presently, work on the soft set theory is making progress rapidly.

This paper organized as follows: Section 2 summarizes basic notions about soft sets. Section 3 focuses results on soft set relations.

2. PRELIMINARIES

Definition 2.1. [3] Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the set of all subsets of U. Let $A \subseteq E$, and F be a mapping $F : A \rightarrow P(U)$, then the pair (F, A) is called a **soft set** over U, In other words, a soft set over U is a parameterized family of subsets of the universe U.

Example 2.2. Let us consider a soft set (F, E) which describes the "attractiveness of houses" that Mr. X is considering for purchase.

Suppose that there are six houses in the universe
$U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ under consideration, and that
$E = \{e_1, e_2, e_3, e_4, e_5\}$ is a set of decision parameters. The e_i ($i = 1, 2, 3, 4, 5$) stands for the parameters "expensive", "beautiful", "wooden", "cheap", and "in green surroundings" respectively.

Consider the mapping F given by "houses (.)", where (.) is to be filled in by one of the parameters $e_i \in E$. For instance, $F(e_1)$ means "houses (expensive)", and its functional value is the set $\{h \in U : h$ is an expensive house$\}$. Suppose that $F(e_1) = \{h_2, h_4\}$, $F(e_2) = \{h_1, h_3\}$, $F(e_3) = \{h_1\}$, $F(e_4) =\{h_3, h_5\}$ and $F(e_5) = \{h_1, h_5\}$. Then we can view the soft set (F, E) as consisting of the following collection of approximations:
(F, E) = {(expensive houses, $\{h_2, h_4\}$); (beautiful houses, $\{h_1, h_3\}$); (wooden houses, $\{h_1\}$); (cheap houses, $\{h_3, h_5\}$); (in the green surrounding, $\{h_1, h_5\}$)}Each approximation has two parts: a predicate and an approximate value set.

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Definition 2.3: [5] A soft set (F, A) over a universe U is said to be **null soft set** denoted by $\tilde{\phi}$, if $\forall\ e \in A$, $F(e) = \phi$.

Definition 2.4: [5] A soft set (F, A) over a universe U is called **absolute soft set** denoted by $(F, \tilde{A})$, if $\forall\ e \in A$, $F(e) = U$. clearly $\tilde{A}^c = \tilde{\phi}$ and $\tilde{\phi}^c = \tilde{A}$.

Definition 2.5: [7] Let $E = \{e_1, e_2, e_3, ..., e_n\}$ be a set of parameters. The **NOT set** of E denoted $\neg E$ is defined as $\neg E = \{\neg e_1, \neg e_2, \neg e_3, ..., \neg e_n\}$.

Definition 2.6: [5] For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a **soft subset** of (G, B) if

1) $A \subset B$, and
2) $\forall\ \varepsilon \in A$, $F(\varepsilon)$ and $G(\varepsilon)$ are identical approximations.

We write $(F, A) \tilde{\subseteq} (G, B)$

(F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(F, A) \tilde{\supseteq} (G, B)$.

Definition 2.7: [5] Two soft sets (F, A) and (G, B) over a common universe U are said to be **soft equal** if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.8: [5] The **complement of a soft set** (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \neg A)$ where $F^c : \neg A \to P(U)$ is a mapping given by $F^c = U - F(\neg\alpha)$, $\forall\ \alpha \in \neg A$.

Let us call F^c to be the soft complement function of F.

Definition 2.9: [2] If (F, A) and (G, B) be two soft sets then "(F, A) **AND** (G, B)" is denoted by $(F, A) \wedge (G, B)$ and is defined by $(F, A) \wedge (G, B) = (H, A \times B)$ where

$$H(\alpha, \beta) = F(\alpha) \cap G(\beta),\ \forall\ (\alpha, \beta) \in A \times B$$

Definition 2.10: [2] If (F, A) and (G, B) be two soft sets then "(F, A) *OR* (G, B)" is denoted by $(F, A) \vee (G, B)$ and is defined by $(F, A) \vee (G, B) = (O, A \times B)$ where

$$O(\alpha, \beta) = F(\alpha) \cup G(\beta),\ \forall\ (\alpha, \beta) \in A \times B$$

Definition 2.11: [3] The **intersection of two soft sets** (F, A) and (G, B) over U is the soft set (H, C). where $C = A \cap B$ and $\forall\ \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon)$ or $G(\varepsilon)$ (as both are same set). This is denoted by $(F, A) \tilde{\cap} (G, B) = (H, C)$.

Definition 2.12: [3] The **union of two soft sets** of (F, A) and (G, B) over the common universe U is the soft set (H, C) where $C = A \cup B$ $\forall\ \varepsilon \in C$.

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & if \quad \varepsilon \in A - B \\ G(\varepsilon) & if \quad \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon) & if \quad \varepsilon \in A \cap B \end{cases}$$

This relationship is denoted by $(F, A) \tilde{\cup} (G, B) = (H, C)$.

3. SOFT RELATIONS

Definition 3.1: [5] Let (F, A) and (G, B) be two soft sets over U, then the **Cartesian product** of (F, A) and (G, B) is define as $(F, A) \times (G, B) = (H, A \times B)$, where $A \times B \to P(U \times U)$ and $H(a,b) = F(a) \times G(b)$ where $(a, b) \in A \times B$.

That is, $H(a, b) = \{(h_i, h_j) : \text{where } h_i \in F(a) \text{ and } h_j \in G(b)\}$

The Cartesian product of three or more nonempty soft sets can be defined by generalizing the definition of the Cartesian product of two soft sets. The Cartesian $(F_1,A) \times (F_2,A) \times ... \times (F_n,A)$ of the nonempty soft sets $(F_1,A), (F_2,A), ..., (F_n,A)$ is the soft sets of all ordered n−tuples $(h_1, h_2, ..., h_n)$ where $h_i \in F_i(a)$.

Definition 3.2: [7] Let (F, A) and (G, B) be two soft sets over U, then a relation from (F, A) to (G, B) is a soft subset of $(F, A) \times (G, B)$. A **relation** from (F, A) to (G, B) is of the form (H_1, S) where $S \subset A \times B$ and $H_1(a, b) = H(a, b)$ $\forall\ a, b \in S$. Any subset of $(F, A) \times (F, A)$ is called a relation on (F, A). In an equivalent way, we can define the relation R on the soft set (F, A) in the parameterized form as follows.

If $(F, A) = \{F(a), F(b), ...\}$, then $F(a) R\, F(b)$ if and only if $F(a) \times F(b) \in R$.

Definition 3.3: [5] Let R be a soft set relation from (F, A) to (G, B) , then the **domain of R** denoted as dom R, is defined as the soft set (D, A_1) where

$A_1 = \{a \in A : H(a, b) \in R$ for some $b \in B\}$ and $D(a_1) = F(a_1)$, $\forall\ a_1 \in A$.

The **range of R** denoted by ran R, is defined as the soft set (T, B_1) , where $B_1 \subset B$ and $B_1 = \{b \in B : H(a, b) \in R$ for some $a \in A\}$ and $T(b_1) = G(b_1)$, $\forall\ b_1 \in B$, where ran R = T.

Example 3.4: [5] Let us consider an example to illustrate a relation on soft set. Let U denotes set of people in a social gathering.

$U = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}\}$.

Let A denotes different job categories.

Take A = {charteredaccountant, doctors, engineers, teachers}

That is $A = \{c, d, e, t\}$.

Let B denote the qualification of people.

Take B = {B.Sc.,B.T ech.,MBBS,M.Sc.}

That is $B = \{b_1, b_2, m_1, m_2\}$.

Then the soft set (F, A) is given by$\{F(c) = \{p_1, p_2\};\ F(d) = \{p_4, p_5\}; F(e) = \{p_7, p_9\}; F(t) = \{p_3, p_4, p_7\}\}$ and it describes people having different jobs and the soft set (G, B) is given by $\{G(b_1) = \{p_1, p_6, p_8, p_{10}\}; G(b_2) = \{p_3, p_6, p_7, p_9\}; G(m_1) = \{p_3, p_4, p_5, p_8\}, G(m_2) = \{p_3, p_8\}\}$ and it represents the people qualified in various courses.

Define a relation R from (F, A) to (G, B) as follows:

$F(a)RG(b)$ iff $F(a) \subseteq G(b)$.

Then $R = \{F(d) \times G(m_1), F(e) \times G(b_2)$.

Then dom$R = (D, A_1)$ where A_1 = {doctors, engineers} and $D(a) = F(a)$ for every $a \in A_1$.

Similarly ran$R = \{T, B_1\}$ where B_1 = {B.T ech.,MBBS} and $T(b) = G(b)$ for every $b \in B_1$.

Definition 3.5: [6] Let I be a soft set relation on (F, A) . If $\forall\ a, b\ \in A$ and a$\neq$ b , $F(a) \times F(a)\ \in I$, but $F(a) \times F(b) \notin I$, then I is called the **identity soft set relation**.

Example 3.6: Let $(F, A) = \{F(a), F(b), F(c)\}$ with a relation defined as

$I = \{F(a) \times F(a), F(b) \times F(b), F(c) \times F(c)\}$

Definition 3.7: [6] The **inverse of a soft set relation** R denoted by R^{-1} is defined by

$R^{-1} = \{F(b) \times F(a) : F(\text{a})RF(b)\}$

Example 3.8: Let $(F, A) = \{F(a), F(b), F(c)\}$ with a relation R defined as $R = \{F(a) \times F(a), F(b) \times F(c), F(c) \times F(a)\}$.

$R^{-1} = \{F(a) \times F(a), F(c) \times F(b), F(a) \times F(c)\}$.

Definition 3.9: [6] The **complement of a soft set relation** R on (F, A) denoted as R^c is defined by $R^c = \{F(a) \times F(b) \mid F(a) \times F(b) \notin R,\ \forall$ a, b $\in$ A}.

Definition 3.10: [6] The **union of two soft set relation** R and Q on (F, A) denoted as $R \cup Q$ is defined by $R \cup Q = \{F(a) \times F(b) \mid F(a) \times F(b) \in \text{R}$ or $F(a) \times F(b) \in Q\}$.

Definition 3.11: [6] The **intersection of two soft set relation** R and Q on (F, A) denoted as $R \cap Q$ is defined by $R \cap Q = \{F(a) \times F(b) \mid F(a) \times F(b) \in \text{R}$ and $F(a) \times F(b)\ \in Q\}$.

Example 3.12: Consider a soft set (F, A) over U where $U = \{h_1, h_2, h_3, h_4\}$, $A = \{m_1, m_2\}$ and $F(m_1) = \{h_1, h_2\}$, $F(m_2) = \{h_2, h_3, h_4\}$. A soft set relation R on (F, A) is defined as

$R = \{F(m_1) \times F(m_1), F(m_2) \times F(m_1)\}$. Then

$R^c = \{F(m_1) \times F(m_2), F(m_2) \times F(m_2)\}$.

Another soft set relation Q on (F, A) is defined as $Q = \{F(m_1) \times F(m_1), F(m_2) \times F(m_2)\}$. Then

$R \cup Q = \{F(m_1) \times F(m_1), F(m_2) \times F(m_1), F(m_2) \times F(m_2)\}$.

$R \cap Q = \{F(m_1) \times F(m_1)\}$.

Definition 3.13: [6] Let R, Q be two soft set relation on (F, A). $\forall\, a, b \in A$, if $F(a) \times F(b) \in R \Rightarrow F(a) \times F(b) \in Q$, then we call $R \subset Q$ (or $R \leq Q$).

Commutative law, associative law, distributive law, De Morgan's law on soft set relation are establish below

Theorem 3.14: (Commutative law) Let P and Q be two soft set relation on (F, A). Then
i) $P \cup Q = Q \cup P$,
ii) $P \cap Q = Q \cap P$.

Proof: Let $F(a) \times F(b) \in P \cup Q$.
Then $F(a) \times F(b) \in P$ *or* $F(a) \times F(b) \in Q$.
$\Rightarrow F(a) \times F(b) \in Q$ *or* $F(a) \times F(b) \in P$.
$\Rightarrow F(a) \times F(b) \in Q \cup P$.
Hence $P \cup Q \subseteq Q \cup P$. (1)

Now, let $F(a) \times F(b) \in P \cup Q$.
Then $F(a) \times F(b) \in Q$ *or* $F(a) \times F(b) \in P$.
$\Rightarrow F(a) \times F(b) \in P$ *or* $F(a) \times F(b) \in Q$.
$\Rightarrow F(a) \times F(b) \in P \cup Q$.
Hence $Q \cup P \subseteq P \cup Q$. (2)

From (1) and (2)
$P \cup Q = Q \cup P$.

The second part is prove similarly.

Theorem 3.15: (Associative law) Let P, Q and R be three soft set relation on (F, A). Then
i) $(P \cup Q) \cup R = P \cup (Q \cup R)$,
ii) $(P \cap Q) \cap R = P \cap (Q \cap R)$.

Proof: Let $F(a) \times F(b) \in (P \cup Q) \cup R$.
Then $F(a) \times F(b) \in (P \cup Q)$ *or* $F(a) \times F(b) \in R$
$\Rightarrow (F(a) \times F(b) \in P$ *or* $F(a) \times F(b) \in Q)$ *or* $F(a) \times F(b) \in R$.
$\Rightarrow F(a) \times F(b) \in P$ *or* $(F(a) \times F(b) \in Q$ *or* $F(a) \times F(b) \in R)$.
$\Rightarrow F(a) \times F(b) \in P$ *or* $(F(a) \times F(b) \in Q \cup R)$.
$\Rightarrow F(a) \times F(b) \in P \cup (Q \cup R)$.
Hence $(P \cup Q) \cup R \subseteq P \cup (Q \cup R)$. (1)

Now, let $F(a) \times F(b) \in P \cup (Q \cup R)$.
$\Rightarrow F(a) \times F(b) \in P$ *or* $(F(a) \times F(b) \in Q \cup R)$.
$\Rightarrow F(a) \times F(b) \in P$ *or* $(F(a) \times F(b) \in Q$ *or* $F(a) \times F(b) \in R)$.
$\Rightarrow (F(a) \times F(b) \in P$ *or* $F(a) \times F(b) \in Q)$ *or* $F(a) \times F(b) \in R$
$\Rightarrow F(a) \times F(b) \in (P \cup Q)$ *or* $F(a) \times F(b) \in R$
$\Rightarrow F(a) \times F(b) \in (P \cup Q) \cup R$.
Hence $P \cup (Q \cup R) \subseteq (P \cup Q) \cup R$. (2)

From (1) and (2)
$(P \cup Q) \cup R = P \cup (Q \cup R)$.
The second part is proving similarly.

Theorem 3.16: (Distributive law) Let P, Q and R be three soft set relation on (F, A). Then
i) $(P \cup Q) \cap R = (P \cap R) \cup (Q \cap R)$,
ii) $(P \cap Q) \cup R = (P \cup R) \cap (Q \cup R)$.

Proof: Let $F(a) \times F(b) \in (P \cup Q) \cap R$.
Then $F(a) \times F(b) \in (P \cup Q)$ *and* $F(a) \times F(b) \in R$
$\Rightarrow (F(a) \times F(b) \in P$ *or* $F(a) \times F(b) \in Q)$ *and* $F(a) \times F(b) \in R$.
$\Rightarrow (F(a) \times F(b) \in P$ *and* $(F(a) \times F(b) \in R)$ *or* $(F(a) \times F(b) \in Q$ *and* $F(a) \times F(b) \in R)$.
$\Rightarrow (F(a) \times F(b) \in P \cap R)$ *or* $(F(a) \times F(b) \in Q \cap R)$.
$\Rightarrow F(a) \times F(b) \in (P \cap R) \cup (Q \cap R)$.
Hence $(P \cup Q) \cap R \subseteq (P \cap R) \cup (Q \cap R)$. (1)

Now, let $F(a) \times F(b) \in (P \cap R) \cup (Q \cap R)$.
$\Rightarrow (F(a) \times F(b) \in P \cap R)$ or $(F(a) \times F(b) \in Q \cap R)$.
$\Rightarrow (F(a) \times F(b) \in P$ and $F(a) \times F(b) \in R)$ or $(F(a) \times F(b) \in Q$ and $F(a) \times F(b) \in R)$.
$\Rightarrow (F(a) \times F(b) \in P$ or $F(a) \times F(b) \in Q)$ and $F(a) \times F(b) \in R$
$\Rightarrow F(a) \times F(b) \in (P \cup Q)$ and $F(a) \times F(b) \in R$
$\Rightarrow F(a) \times F(b) \in (P \cup Q) \cap R$.
Hence $(P \cap R) \cup (Q \cap R) \subseteq (P \cup Q) \cap R$. (2)

From (1) and (2)
$(P \cup Q) \cap R = (P \cap R) \cup (Q \cap R)$.

The second part is proving similarly.

Theorem 3.17: De Morgan's law
i) $(P \cup Q)^c = P^c \cap Q^c$,
ii) $(P \cap Q)^c = P^c \cup Q^c$.

Proof: Let $\forall\, a, b \in A, F(a) \times F(b) \in (P \cup Q)^c$.
Then $F(a) \times F(b) \notin (P \cup Q)$.
$\Rightarrow F(a) \times F(b) \notin P$ and $F(a) \times F(b) \notin Q$.
$\Rightarrow F(a) \times F(b) \in P^c$ and $F(a) \times F(b) \in Q^c$.
$\Rightarrow F(a) \times F(b) \in P^c \cap Q^c$.
Hence $(P \cup Q)^c \subseteq P^c \cap Q^c$. (1)

Now, let $F(a) \times F(b) \in P^c \cap Q^c$.
Then $F(a) \times F(b) \in P^c$ and $F(a) \times F(b) \in Q^c$.
$\Rightarrow F(a) \times F(b) \notin P$ or $F(a) \times F(b) \notin Q$
$\Rightarrow F(a) \times F(b) \notin (P \cup Q)$.
$\Rightarrow F(a) \times F(b) \in (P \cup Q)^c$.
Hence $P^c \cap Q^c \subseteq (P \cup Q)^c$. (2)

From (1) and (2)
$(P \cup Q)^c = P^c \cap Q^c$.

The second part is proving similarly.

Theorem 3.18: Let P and Q be two soft set relations on (F, A). Then
i) $(P \cup Q)^{-1} = P^{-1} \cup Q^{-1}$,
ii) $(P \cap Q)^{-1} = P^{-1} \cap Q^{-1}$.

Proof: *Let* $\forall\, a, b \in A, F(a) \times F(b) \in (P \cup Q)^{-1}$.
Then $F(b) \times F(a) \in (P \cup Q)$.
$\Rightarrow F(b) \times F(a) \in P$ or $F(b) \times F(a) \in Q$.
$\Rightarrow F(a) \times F(b) \in P^{-1}$ or $F(a) \times F(b) \in Q^{-1}$.
$\Rightarrow F(a) \times F(b) \in P^{-1} \cup Q^{-1}$.
Hence $(P \cup Q)^{-1} \subseteq P^{-1} \cup Q^{-1}$. (1)

Now, let $F(a) \times F(b) \in P^{-1} \cup Q^{-1}$.
Then $F(a) \times F(b) \in P^{-1}$ or $F(a) \times F(b) \in Q^{-1}$.
$\Rightarrow F(b) \times F(a) \in P$ or $F(b) \times F(a) \in Q$
$\Rightarrow F(b) \times F(a) \in (P \cup Q)$.
$\Rightarrow F(a) \times F(b) \in (P \cup Q)^{-1}$.
Hence $P^{-1} \cup Q^{-1} \subseteq (P \cup Q)^{-1}$. (2)

From (1) and (2)
$(P \cup Q)^{-1} = P^{-1} \cap Q^{-1}$.

The second part is proving similarly.

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