SOME NEW RESULTS ON 1-NEAR MEAN CORDIAL LABELING OF GRAPHS

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ABSTRACT

Let G = (V, E) be a simple graph. A surjective function \( f: V(G) \rightarrow \{0, 1, 2\} \) is said to be a 1-Near Mean Cordial Labeling if for each edge uv, the induced map

\[
\ell*(uv) = \begin{cases} 
0 & \text{if } \frac{f(u)+f(v)}{2} \text{ is an integer} \\
1 & \text{otherwise}
\end{cases}
\]

Satisfies the condition \( |e_f(0) - e_f(1)| \leq 1 \) where \( e_f(0) \) is the number of edges with 0 label and \( e_f(1) \) is the number of edges with 1 label.

G is said to be a 1-Near Mean Cordial Graph if it has a 1- Near Mean Cordial Labeling. In this paper, we proved that wheel, complete bipartite, helm, closed helm, flower, sunflower, and \( S(K_{1,n}) \) are 1- Near Mean Cordial Graphs.

Keywords: 1-Near Mean Cordial Labeling, 1-Near Mean Cordial Graph.

1. INTRODUCTION

All graphs considered here are finite, simple and undirected. Gallian [2] has given a dynamic survey of labeling. For graph theoretic terminologies and notations we follow Harary [3]. The concept of mean cordial labeling was introduced by Raja Ponraj, Muthirulan Sivakumar and Murugesan Sundaram in the year 2012 [1,4,5,7]. Let \( f \) be a function from \( V(G) \) to \{0, 1, 2\}. For each edge uv of G, assign the label \( \frac{f(u) + f(v)}{2} \). \( f \) is called a mean cordial labeling of G if \( |v_f(0) - v_f(1)| \leq 1 \) and \( |e_f(0) - e_f(1)| \leq 1 \), i, j \( \in \{0,1,2\} \) where \( v_f(x) \) and \( e_f(x) \) denote the number of vertices and edges labeled with \( x \) (\( x = 0, 1, 2 \)) respectively. A graph with a mean cordial labeling is called Mean Graph. K.Palani, J.Rejila Jeya Surya [6] introduced a new concept called 1-Near Mean Cordial Labeling and investigated some standard graphs.

2. PRELIMINARIES

We define the concept of 1-Near Mean Cordial Labeling as follows,

Let G = (V, E) be a simple graph. A surjective function \( f: V(G) \rightarrow \{0, 1, 2\} \) said to be a 1-Near Mean Cordial Labeling if for each edge uv, the induced map

\[
\ell*(uv) = \begin{cases} 
0 & \text{if } \frac{f(u)+f(v)}{2} \text{ is an integer} \\
1 & \text{otherwise}
\end{cases}
\]

Satisfies the condition \( |e_f(0) - e_f(1)| \leq 1 \) where \( e_f(0) \) is the number of edges with 0 label and \( e_f(1) \) is the number of edges with 1 label.

G is said to be a 1-Near Mean Cordial Graph if it has a 1- Near Mean Cordial Labeling. We proved that wheel, complete bipartite, helm, closed helm, flower, sunflower, and \( S(K_{1,n}) \) are 1- Near Mean Cordial Graphs.

Definition 2.1: A graph \( C_n+K_1 \) is called a wheel with n spokes and is denoted by \( W_n \).
Definition 2.2: A graph $G$ is called a complete bipartite graph $K_{m,n}$ with bipartition $V(G) = V_1 \cup V_2$ where $V_1 = \{x_1, x_2, ..., x_m\}$ and $V_2 = \{y_1, y_2, ..., y_n\}$ and all vertices in $V_1$ are adjacent to all vertices in $V_2$ but no vertices in $V_1$ and $V_2$.

Definition 2.3: The helm $H_n$, is the graph obtained from a wheel by attaching a pendant edge at each vertex of the $n$-cycle.

Definition 2.4: A closed helm $CH_n$, is a graph obtained from a helm by joining each pendant vertex to the central vertex of the helm.

Definition 2.5: A flower $Fl_n$, is the graph obtained from a helm graph by joining each pendant vertex to the central vertex of the helm graph.

Definition 2.6: The sunflower graph $v[n, s, t]$ is the resultant graph obtained from the flower graph of wheels $W_n$ by adding $n$-1 pendant edges to the central vertex.

Definition 2.7: For each vertex $v$ of a graph $G$ take a new vertex $v_0$. Join $v_0$ to all the vertices of $G$ adjacent to $v$. The graph $S(G)$ thus obtained is called splitting graph of $G$.

3. MAIN RESULTS

Theorem 3.1: The wheel $W_n$ is a 1-Near Mean Cordial Graph.

Proof: Let $G = (V, E)$ be a simple graph.

Let $G$ be $W_n$.

Let $V(G) = \{u, v_i: 1 \leq i \leq n\}$ and $E(G) = \{(uv_i): 1 \leq i \leq n\} \cup \{(v_i, v_{i+1}): 1 \leq i \leq n - 1\} \cup \{v_nv_1\}$

Define $f: V(G) \rightarrow \{0, 1, 2\}$ by

$f(u) = 1$

$f(v_i) = \begin{cases} 
0 & i \equiv 1 \mod 2 \\
2 & i \equiv 0 \mod 2
\end{cases} \quad 1 \leq i \leq n$

The induced edge labeling are

$f^*(uv_i) = 1, \quad 1 \leq i \leq n$

$f^*(v_iv_{i+1}) = 0, \quad 1 \leq i \leq n - 1$

$f^*(v_nv_1) = 0$

Here, $e_1(0) = e_1(1) = n$

Hence the graph satisfies the condition $|e_1(0) - e_1(1)| \leq 1$

Therefore, the wheel $W_n$ is a 1-near mean cordial graph.

Illustration 1: The 1- near mean cordial graph of $W_5$ is shown in the figure 1
Theorem 3.2: The complete bipartite graph, \( K_{m,n} \) is a 1-Near Mean Cordial Graph.

Proof: Let \( G = (V, E) \) be a simple graph.

Let \( G \) be \( K_{m,n} \)

Let \( V(G) = \{u_i: 1 \leq i \leq m, v_j: 1 \leq j \leq n\} \) and \( E(G) = \{(u_i v_j : 1 \leq i \leq m, 1 \leq j \leq n}\} \)

Define \( f: V(G) \rightarrow \{0, 1, 2\} \) by

\[
\begin{cases}
0 & i \equiv 1 \mod 4 \\
1 & i \equiv 0,2 \mod 4, 1 \leq i \leq m,
1 \equiv 3 \mod 4 \\
2 & i \equiv 1 \mod 2, 1 \leq j \leq n,
\end{cases}
\]

The induced edge labeling are

Case-(i): when \( m \) is even and \( n \) is even or odd

\[
f^*(u_{2i-1}v_j) = \begin{cases}
0 & j \equiv 0 \mod 2 \\
1 & j \equiv 1 \mod 2, 1 \leq j \leq n
\end{cases}
\]

\[
f^*(u_{2i}v_j) = \begin{cases}
0 & j \equiv 1 \mod 2 \\
1 & j \equiv 0 \mod 2, 1 \leq j \leq n
\end{cases}
\]

Here, \( e_f(0) = e_f(1) = mn \)

Case-(ii): when \( m \) is odd and \( n \) is even or odd

\[
f^*(u_{2i-1}v_j) = \begin{cases}
0 & j \equiv 0 \mod 2 \\
1 & j \equiv 1 \mod 2, \frac{m+1}{2} \leq j \leq n
\end{cases}
\]

\[
f^*(u_{2i}v_j) = \begin{cases}
0 & j \equiv 1 \mod 2 \\
1 & j \equiv 0 \mod 2, \frac{m+1}{2} \leq j \leq n
\end{cases}
\]

Here, \( e_f(0) = \frac{mn}{2} \) \( n \) is odd \( e_f(1) = \frac{mn}{2} \) \( n \) is even

Hence the graph satisfies the condition \( |e_f(0) - e_f(1)| \leq 1 \)

Therefore, the complete bipartite graph, \( K_{m,n} \) is a 1-near mean cordial graph.

Illustration 2: The 1-near mean cordial graph of \( K_{4,3} \) and \( K_{3,3} \) are shown in the figure 2(a) and figure 2(b)
**Theorem 3.3:** The helm $H_n$ is a 1-Near Mean Cordial Graph.

**Proof:** Let $G = (V, E)$ be a simple graph.

Let $G$ be $H_n$.

Let $V(G) = \{u, v_i: 1 \leq i \leq n, w_i: 1 \leq i \leq n\}$ and

$E(G) = \{(uv_i), (wv_i): 1 \leq i \leq n\} \cup \{(v_iv_{i+1}): 1 \leq i \leq n - 1\} \cup \{v_nv_1\}$

Define $f: V(G) \rightarrow \{0, 1, 2\}$ by

$f(u) = 0$

$f(v_i) = \begin{cases} 0 & i \equiv 1 \mod 2 \\ 2 & i \equiv 0 \mod 2 \end{cases} \quad 1 \leq i \leq n$

$f(w_i) = \begin{cases} 1 & i \equiv 0 \mod 2 \\ 2 & i \equiv 1 \mod 2 \end{cases} \quad 1 \leq i \leq n$

The induced edge labeling are

$f^*(uv_i) = 1, \quad 1 \leq i \leq n$

$f^*(v_iv_{i+1}) = 0, \quad 1 \leq i \leq n - 1$

$f^*(v_nv_1) = 0$

$f^*(wv_i) = \begin{cases} 0 & i \equiv 1 \mod 2 \\ 1 & i \equiv 0 \mod 2 \end{cases} \quad 1 \leq i \leq n$

Here, $e_f(0) = \begin{cases} \frac{3n+1}{2} & n \text{ is odd} \\ \frac{3n}{2} & n \text{ is even} \end{cases}$

$e_f(1) = \begin{cases} \frac{3n-1}{2} & n \text{ is odd} \\ \frac{3n}{2} & n \text{ is even} \end{cases}$

Hence the graph satisfies the condition $|e_f(0) - e_f(1)| \leq 1$

Therefore, the helm $H_n$ is a 1-near mean cordial graph.

**Illustration 3:** The 1-near mean cordial graph of $H_5$ is shown in the figure 3.
**Theorem 3.4:** A closed helm $CH_n$ is a 1-Near Mean Cordial Graph.

**Proof:** Let $G = (V, E)$ be a simple graph.

Let $G$ be $CH_n$.

Let $V(G) = \{u, v_i : 1 \leq i \leq n, w_i : 1 \leq i \leq n\}$ and

$E(G) = \{(uv_i), (w_iw_i) : 1 \leq i \leq n\} \cup \{(v_iv_{i+1}), (w_iw_{i+1}) : 1 \leq i \leq n - 1\} \cup \{(v_nv_1), (w_rw_1)\}$

Define $f : V(G) \to \{0, 1, 2\}$ by

$$
f(u) = 1,
\begin{align*}
f(v_i) &= 0, & i &\equiv 1 \mod 2 \\
f(v_i) &= 2, & i &\equiv 0 \mod 2
\end{align*}
$$ \hspace{1cm} 1 \leq i \leq n

$f(w_i) = 1$, \hspace{1cm} 1 \leq i \leq n

The induced edge labeling are

$f^*(uv_i) = 1$, \hspace{1cm} 1 \leq i \leq n
\begin{align*}
f^*(v_iv_{i+1}) &= 0, & 1 \leq i \leq n - 1 \\
f^*(w_iw_{i+1}) &= 0, & 1 \leq i \leq n - 1
\end{align*}

Here, $e_t(0) = e_t(1) = 2n$

Hence the graph satisfies the condition $|e_t(0) - e_t(1)| \leq 1$

Therefore, the closed helm $CH_n$ is a 1-near mean cordial graph.

**Illustration 4:** The 1-near mean cordial graph of $CH_4$ is shown in the figure 4,

![Figure 4: CH4](image)

**Theorem 3.5:** A flower graph $Fl_n$ is a 1-Near Mean Cordial Graph.

**Proof:** Let $G = (V, E)$ be a simple graph.

Let $G$ be $Fl_n$.

Let $V(G) = \{u, v_i : 1 \leq i \leq n, w_i : 1 \leq i \leq n\}$ and

$E(G) = \{(uv_i), (u_iw_i), (w_iw_i) : 1 \leq i \leq n\} \cup \{(v_iv_{i+1}) : 1 \leq i \leq n - 1\} \cup \{(v_nv_1)\}$
Define $f : V(G) \rightarrow \{0, 1, 2\}$ by

$$f(u) = 1$$

$$f(v_i) = \begin{cases} 0 & i \equiv 1 \mod 2 \\ 2 & i \equiv 0 \mod 2 \end{cases} \quad 1 \leq i \leq n$$

$$f(w_i) = \begin{cases} 1 & i \equiv 0 \mod 2 \\ 2 & i \equiv 1 \mod 2 \end{cases} \quad 1 \leq i \leq n$$

The edge induced labeling are,

$$f^*(uvi) = 1, \quad 1 \leq i \leq n$$

$$f^*(vivi+1) = 0, \quad 1 \leq i \leq n - 1$$

$$f^*(vnv1) = 0,$$  

$$f^*(uwi) = \begin{cases} 0 & i \equiv 0 \mod 2 \\ 1 & i \equiv 1 \mod 2 \end{cases} \quad 1 \leq i \leq n$$

$$f^*(wivi) = \begin{cases} 1 & i \equiv 0 \mod 2 \\ 0 & i \equiv 1 \mod 2 \end{cases} \quad 1 \leq i \leq n$$

Here, $e_1(0) = e_1(1) = 2n$

Hence the graph satisfies the condition $|e_1(0) - e_1(1)| \leq 1$

Therefore, a flower graph $F_l_n$ is a 1-near mean cordial graph.

**Illustration 5:** The 1-near mean cordial graph of $F_3$ is shown in the figure 5.

![Figure 5: $F_3$](image)

**Theorem 3.6:** The sunflower graph $S_n$ is a 1-Near Mean Cordial Graph.

**Proof:** Let $G = (V, E)$ be a simple graph.

Let $G$ be $S_n$.

Let $V(G) = \{u, v_i : 1 \leq i \leq n, w_i : 1 \leq i \leq n, x_i : 1 \leq i \leq n\}$ and

$$E(G) = \{(uv_i), (uw_i), (ux_i), (wivi) : 1 \leq i \leq n\} \cup \{(vivi+1) : 1 \leq i \leq n - 1\} \cup (vavn1)$$

Define $f : V(G) \rightarrow \{0, 1, 2\}$ by
\[ f(u) = 1 \]
\[ f(v) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2 \\ 2 & \text{if } i \equiv 0 \mod 2 \end{cases} \quad 1 \leq i \leq n \]
\[ f(w) = \begin{cases} 1 & \text{if } i \equiv 0 \mod 2 \\ 2 & \text{if } i \equiv 1 \mod 2 \end{cases} \quad 1 \leq i \leq n \]
\[ f(x) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2 \\ 1 & \text{if } i \equiv 0 \mod 2 \end{cases} \quad 1 \leq i \leq n \]

The edge induced labeling are,
\[ f^*(uvi) = 1, \quad 1 \leq i \leq n \]
\[ f^*(vivi+1) = 0, \quad 1 \leq i \leq n-1 \]
\[ f^*(v_nv1) = 0, \]
\[ f^*(uw) = \begin{cases} 0 & \text{if } i \equiv 0 \mod 2 \\ 1 & \text{if } i \equiv 1 \mod 2 \end{cases} \quad 1 \leq i \leq n \]
\[ f^*(wv) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2 \\ 1 & \text{if } i \equiv 0 \mod 2 \end{cases} \quad 1 \leq i \leq n \]
\[ f^*(ux) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2 \\ 1 & \text{if } i \equiv 0 \mod 2 \end{cases} \quad 1 \leq i \leq n \]

Here, \( e_f(0) = \begin{cases} \frac{5n+1}{2} & \text{if } n \text{ is odd} \\ \frac{5n}{2} & \text{if } n \text{ is even} \end{cases} \)
\[ e_f(1) = \begin{cases} \frac{5n-1}{2} & \text{if } n \text{ is odd} \\ \frac{5n}{2} & \text{if } n \text{ is even} \end{cases} \]

Hence the graph satisfies the condition \( |e_f(0) - e_f(1)| \leq 1 \)

Therefore, the sunflower graph \( S_n \) is a 1-near mean cordial graph.

**Illustration 6:** The 1-near mean cordial graph of \( S_4 \) is shown in the figure 6
Theorem 3.7: The splitting graph $S(K_{1,n})$ is a 1-Near Mean Cordial Graph.

Proof: Let $G = (V, E)$ be a simple graph.

Let $G$ be $S(K_{1,n})$.

Let $V(G) = \{u, v, u_i : 1 \leq i \leq n, v_i : 1 \leq i \leq n\}$ and $E(G) = \{(uu_i), (uv_i), (vu_i) : 1 \leq i \leq n\}$

Define $f: V(G) \to \{0, 1, 2\}$ by

$f(u) = 1$

$f(v) = 0$

$f(u_i) = \begin{cases} 0 & i \equiv 0 \mod 3 \\ 1 & i \equiv 1 \mod 3 \\ 2 & i \equiv 2 \mod 3 \end{cases} \quad 1 \leq i \leq n$

$f(v_i) = \begin{cases} 0 & i \equiv 0 \mod 2 \\ 1 & i \equiv 1 \mod 2 \end{cases} \quad 1 \leq i \leq n$

The edge induced labeling are,

$f^*(uu_i) = \begin{cases} 0 & i \equiv 1 \mod 3 \\ 1 & i \equiv 0, 2 \mod 3 \end{cases} \quad 1 \leq i \leq n$

$f^*(vu_i) = \begin{cases} 0 & i \equiv 0, 2 \mod 3 \\ 1 & i \equiv 1 \mod 3 \end{cases} \quad 1 \leq i \leq n$

$f^*(uv_i) = \begin{cases} 0 & i \equiv 1 \mod 2 \\ 1 & i \equiv 0 \mod 2 \end{cases} \quad 1 \leq i \leq n$

Here, $e_t(0) = \begin{cases} \frac{3n+1}{2} & n \text{ is odd} \\ \frac{3n}{2} & n \text{ is even} \end{cases}$

$e_t(1) = \begin{cases} \frac{3n-1}{2} & n \text{ is odd} \\ \frac{3n}{2} & n \text{ is even} \end{cases}$

Hence the graph satisfies the condition $|e_t(0) - e_t(1)| \leq 1$

Therefore, the splitting graph $S(K_{1,n})$ is a 1-near mean cordial graph.

Illustration 7: The 1-near mean cordial graph of $S(K_{1,4})$ is shown in the figure 7,
REFERENCES


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