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A STUDY ON MULTI FUZZY RW-CLOSED, MULTI FUZZY RW-OPEN SETS IN MULTI FUZZY TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we have studied some of the properties of multi fuzzy rw-closed and multi fuzzy rw-open sets in multi fuzzy topological spaces and have proved some results on these.

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Key Words: *fuzzy subset, multi fuzzy subset, multi fuzzy topological spaces, multi fuzzy rw-closed, multi fuzzy rw-open.*

INTRODUCTION

The concept of a fuzzy subset was introduced and studied by L.A.Zadeh [15] in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers have motivated us to work on this paper. C.L.Chang [4] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Many researchers like R.H.Warren [14], K.K.Azad [1], G.Balasubramanian and P.Sundaram [2, 3], S.R.Malghan and S.S.Benchalli [10, 11] and many others have contributed to the development of fuzzy topological spaces. We introduce the concept of multi fuzzy rw-closed and multi fuzzy rw-open sets in multi fuzzy topological spaces and have established some results.

1. PRELIMINARIES

1.1 Definition[15]: Let X be a non-empty set. A **fuzzy subset A** of X is a function A: X → [0, 1].

1.2 Definition: A **multi fuzzy subset** A of a set X is defined as an object of the form $A = \{\langle x, A_1(x), A_2(x), A_3(x), \ldots, A_n(x) \rangle / x\in X\}$, where $A_i : X\to[0, 1]$ for all i. It is denoted as $A = \langle A_1, A_2, A_3, \ldots, A_n\rangle$.

1.3 Definition: Let A and B be any two multi fuzzy subsets of a set X. We define the following relations and operations:

(i) $A \subseteq B$ if and only if $A_i(x) \leq B_i(x)$ for all i and for all x in X.
(ii) $A = B$ if and only if $A_i(x) = B_i(x)$ for all i and for all x in X.
(iii) $A^c = 1-A = \langle 1-A_1, 1-A_2, 1-A_3, \ldots, 1-A_n \rangle$.
(iv) $A\cap B= \{\langle x, \min\{A_1(x), B_1(x)\}, \min\{A_2(x), B_2(x)\}, \ldots, \min\{A_n(x), B_n(x)\}\rangle / x\in X\}$.
(v) $A\cup B=\{\langle x, \max \{A_1(x), B_1(x) \}, \max\{A_2(x), B_2(x)\}, \ldots, \max\{A_n(x), B_n(x)\}\rangle / x\in X\}$.

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1.4 Definition: Let X be a set and $\Im$ be a family of multi fuzzy subsets of X. The family $\Im$ is called a multi fuzzy topology on X if and only if $\Im$ satisfies the following axioms

(i) $0, \bar{1} \in \Im$,

(ii) If $\{A_i ; i \in I\} \subseteq \Im$, then $\bigcup_{i \in I} A_i \in \Im$,

(iii) If $A_1, A_2, A_3, \ldots A_n \in \Im$, then $\bigcap_{i=1}^{i=n} A_i \in \Im$.

The pair $(X, \Im)$ is called a multi fuzzy topological space. The members of $\Im$ are called multi fuzzy open sets in X. A multi fuzzy set A in X is said to be multi fuzzy closed set in X if and only if A^c is a multi fuzzy open set in X.

1.5 Definition: Let $(X, \Im)$ be a multi fuzzy topological space and A be a multi fuzzy set in X. Then $\cap\{B: B^c \in \Im$ and $B \supseteq A\}$ is called multi fuzzy closure of A and is denoted by mfcl(A).

1.6 Theorem: Let A and B be two multi fuzzy sets in multi fuzzy topological space $(X, \Im)$. Then the following results are true,

(i) mfcl(A) is a multi fuzzy closed set in X.
(ii) mfcl(A) is the least multi fuzzy closed set containing A.
(iii) A is a multi fuzzy closed if and only if A = mfcl(A).
(iv) mfcl($\bar{0}$) = $\bar{0}$, $\bar{0}$ is the empty multi fuzzy set
(v) mfcl(mfcl(A)) = mfcl(A).
(vi) mfcl($A \cup B$) = mfcl(A) $\cup$ mfcl(B).
(vii) mfcl(A) $\cap$ mfcl(B) $\supseteq$ mfcl($A \cap B$).

1.7 Definition: Let $(X, \Im)$ be a multi fuzzy topological space and A be a multi fuzzy set in X. Then $\cup\{B: B \in \Im$ and $B \subseteq A\}$ is called multi fuzzy interior of A and is denoted by mfint(A).

1.8 Theorem: Let $(X, \Im)$ be a multi fuzzy topological space, A and B be two multi fuzzy sets in X. The following results hold good,

(i) mfint (A) is a multi fuzzy open set in X.
(ii) mfint(A) is the largest multi fuzzy open set in X which is less than or equal to A.
(iii) A is a multi fuzzy open set if and only if A = mfint (A) .
(iv) $A \subseteq B$ implies mfint (A) $\subseteq$ mfint(B).
(v) mfint (mfint (A)) = A.
(vi) mfint($A \cap B$) = mfint(A) $\cap$ mfint(B).
(vii) mfint(A) $\cup$ mfint(B) $\subseteq$ mfint($A \cup B$).
(viii) mfint($\bar{1}$ –A) = $\bar{1}$ – mfcl(A).
(ix) mfcl($\bar{1}$ – A) = $\bar{1}$ – mfint(A).

1.9 Definition: Let $(X, \Im)$ be a multi fuzzy topological space and A be multi fuzzy set in X. Then A is said to be

(i) multi fuzzy semiopen if and only if there exists a multi fuzzy open set V in X such that $V \subseteq A \subseteq$ mfcl(V).
(ii) multi fuzzy semiclosed if and only if there exists a multi fuzzy closed set V in X such that mfint(V) $\subseteq A \subseteq V$.
(iii) multi fuzzy regular open set of X if mfint(mfcl(A)) = A.
(iv) multi fuzzy regular closed set of X if mfcl(mfint(A)) = A.
(v) multi fuzzy regular semiopen set of X if there exists a multi fuzzy regular open set V in X such that $V \subseteq A \subseteq$ mfcl(V). We denote the class of multi fuzzy regular semiopen sets in multi fuzzy topological space X by MFRSO(X).
(vi) multi fuzzy generalized closed (mfg-closed) if mfcl(A) $\subseteq$ V whenever $A \subseteq V$ and V is multi fuzzy open set and A is multi fuzzy generalized open if $\bar{1}$ – A is multi fuzzy generalized closed.

1.10 Theorem: The following are equivalent:

(i) A is a multi fuzzy semiclosed set.
(ii) A^C is a multi fuzzy semiopen set.
(iii) mfint(mfcl(A)) $\subseteq$ A.
(iv) mfcl(mfint(A^C)) $\supseteq$ A.

1.11 Theorem: Any union of multi fuzzy semiopen sets is a multi fuzzy semiopen set and any intersection of multi fuzzy semiclosed sets is a multi fuzzy semiclosed.

1.12 Remark: l\
(i) Every multi fuzzy open set is a multi fuzzy semiopen but not conversely.
(ii) Every multi fuzzy closed set is a multi fuzzy semi-closed set but not conversely.
(iii) The closure of a multi fuzzy open set is multi fuzzy semiopen set.
(iv) The interior of a multi fuzzy closed set is multi fuzzy semi-closed set.

1.13 Theorem: A multi fuzzy set A of a multi fuzzy topological space X is a multi fuzzy regular open if and only if A^C is multi fuzzy regular closed set.

1.14 Remark: (i) Every multi fuzzy regular open set is a multi fuzzy open set but not conversely. (ii) Every multi fuzzy regular closed set is a multi fuzzy closed set but not conversely.

1.15 Theorem: (i) The closure of a multi fuzzy open set is a multi fuzzy regular closed.
(ii) The interior of a multi fuzzy closed set is a multi fuzzy regular open set.

1.16 Theorem: (i) Every multi fuzzy regular semiopen set is a multi fuzzy semiopen set but not conversely. (ii) Every multi fuzzy regular closed set is a multi fuzzy regular semiopen set but not conversely. (iii) Every multi fuzzy regular open set is a multi fuzzy regular semiopen set but not conversely.

1.17 Theorem: Let $(X, \Im)$ be a multi fuzzy topological space and A be multi fuzzy set in X. Then the following conditions are equivalent:
(i) A is multi fuzzy regular semiopen.
(ii) A is both multi fuzzy semiopen and multi fuzzy semi-closed.
(iii) A^c is multi fuzzy regular semiopen in X.

1.18 Definition: An multi fuzzy set A of a multi fuzzy topological space $(X, \Im)$ is called:
(i) multi fuzzy g-closed if mfcl(A) $\subseteq$ V whenever A $\subseteq$ V and V is multi fuzzy open set in X.
(ii) multi fuzzy g-open if its complement A^c is multi fuzzy g-closed set in X.
(iii) multi fuzzy rg-closed if mfcl(A) $\subseteq$ V whenever A $\subseteq$ V and V is multi fuzzy regular open set in X.
(iv) multi fuzzy rg-open if its complement A^c is multi fuzzy rg-closed set in X.
(v) multi fuzzy w-closed if mfcl(A) $\subseteq$ V whenever A $\subseteq$ V and V is multi fuzzy semi open set in X.
(vi) multi fuzzy w-open if its complement A^c is multi fuzzy w-closed set in X.
(vii) multi fuzzy gpr-closed if pcl(A) $\subseteq$ V whenever A $\subseteq$ V and V is multi fuzzy regular open set in X.
(viii) multi fuzzy gpr-open if its complement A^c is multi fuzzy gpr-closed set in X.

1.19 Definition: Let $(X, \Im)$ be a multi fuzzy topological space. A multi fuzzy set A of X is called multi fuzzy regular w-clsoed (briefly, multi fuzzy rw-closed) if mfcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is multi fuzzy regular semiopen in multi fuzzy topological space X.

Note: We denote the family of all multi fuzzy regular w-closed sets in multi fuzzy topological space X by MFRWC(X).

1.20 Definition: A multi fuzzy set A of a multi fuzzy topological space X is called a multi fuzzy regular w-open (briefly, multi fuzzy rw-open) set if its complement A^C is a multi fuzzy rw-closed set in multi fuzzy topological space X.

Note: We denote the family of all multi fuzzy rw-open sets in multi fuzzy topological space X by MFRWO(X).

2. SOME PROPERTIES

2.1 Theorem: Every multi fuzzy closed set is an multi fuzzy rw-closed set in a multi fuzzy topological space X.

Proof: Let $A = \langle A_1, A_2, A_3, \ldots, A_i \rangle$ be a multi fuzzy closed set in a multi fuzzy topological space X. Let $B = \langle B_1, B_2, B_3, \ldots, B_i \rangle$ be a multi fuzzy regular semiopen set in X such that $A \subseteq B$. Since A is multi fuzzy closed, mfcl(A) = A. Therefore mfcl(A) = A $\subseteq$ B. Hence A is multi fuzzy rw-closed in multi fuzzy topological space X.

Remark: The converse of the above Theorem need not be true in general.

2.1 Example: Let X = {1, 2, 3}. Define a multi fuzzy set A = {< 1, 0.6, 0.5, 0.4 >, < 2, 0, 0, 0 >, < 3, 0, 0, 0 >}. Let $\Im = \{\underline{\mathbf{1}}, \bar{0}, A\}$. Then $(X, \Im)$ is a multi fuzzy topological space. Define a multi fuzzy set B = {< 1, 0, 0, 0 >, < 2, 0.6, 0.5, 0,4 >, < 3, 0, 0, 0 >}. Then B is a multi fuzzy rw-closed set but it is not a multi fuzzy closed set in multi fuzzy topological space X.

Remark: multi fuzzy generalized closed sets and multi fuzzy rw-closed sets are independent.

2.2 Example: Let X = {1, 2, 3, 4}. Define multi fuzzy sets A, B, C in X by A = {< 1, 1, 1, 1 >, < 2, 0, 0, 0 >, < 3, 0, 0, 0 >, < 4, 0, 0, 0 >}, B = {< 1, 0, 0, 0 >, < 2, 1, 1, 1 >, < 3, 0, 0, 0 >, < 4, 0, 0, 0 >}, C = {< 1, 1, 1, 1 >, < 2, 1, 1, 1 >, < 3, 0, 0, 0 >, < 4, 0, 0, 0 >}. Consider $\Im$ = {0, 1, A, B, C}. Then (X, $\Im$) is a multi fuzzy topological space. In this multi fuzzy topological space X, the multi fuzzy set D is defined by D = {< 1, 0, 0, 0 >, < 2, 0, 0, 0 >, < 3, 1, 1, 1 >, < 4, 0, 0, 0 >}. Then D is a multi fuzzy generalized closed set in multi fuzzy topological space X. In this multi fuzzy topological space, the multi fuzzy set E is defined by E = {<1, 1, 1, 1 >, < 2, 0, 0, 0 >, < 3, 1, 1, 1 >, < 4, 0, 0, 0 >}. Then E is a multi fuzzy regular semiopen set containing D, but E does not contain mfcl(D) which is C^C. Therefore E is not a multi fuzzy rw-clsoed set in multi fuzzy topological space X.

2.3 Example: Let X = I = [0, 1]. Define a multi fuzzy set D in X by $D_1(x) = 0.5$ if x = 2/3 and $D_1(x) = 0$ otherwise, $D_2(x) = 0.4$ if x = 2/3 and $D_2(x) = 0$ otherwise and $D_3(x) = 0.3$ if x = 2/3 and $D_3(x) = 0$ otherwise. Let $\Im = \{ \bar{0}, \bar{1}, D \}$. Then (X, $\Im$) is an multi fuzzy topological space. A multi fuzzy set A is defined as $A_1(x) = 0.3$ if x = 2/3 and $A_1(x) = 0$ otherwise, $A_2(x) = 0.2$ if x = 2/3 and $A_2(x) = 0$ otherwise and $A_3(x) = 0.1$ if x = 2/3 and $A_3(x) = 0$ otherwise. Then A is a multi fuzzy rw-closed set in multi fuzzy topological space X. Now mfcl(A) = D^C and D is a multi fuzzy open set containing A but D does not contain mfcl(A) which is D^C. Therefore A is not a multi fuzzy generalized closed.

Remark: multi fuzzy rw-closed sets and multi fuzzy semi-closed sets are independent.

2.4 Example: consider the multi fuzzy topological space (X, $\Im$) defined in Example 2.1. Then the multi fuzzy set A = {< 1, 1, 1, 1] >, < 2, 0, 0, 0 >, < 3, 0, 0, 0 >} is a multi fuzzy rw-closed but it is not a multi fuzzy semi-closed set in multi fuzzy topological space X.

2.5 Example: Consider the multi fuzzy topological space (X, $\Im$) defined in Example 2.2. In this multi fuzzy topological space X, the multi fuzzy set E is define by E = {< 1,1, 1, 1 >, < 2, 0, 0, 0 >, < 3, 1, 1, 1 >, < 4, 0, 0, 0 >}. Then E is a multi fuzzy semi-closed in multi fuzzy topological space X. E is also multi fuzzy regular semiopen set containing E which is does not contain mfcl(E) = B^c = { < 1, 1, 1, 1 >, < 2, 0, 0, 0 >, < 3, 1, 1, 1 >, < 4, 1, 1, 1 >}. Therefore E is not a multi fuzzy rw-closed set in multi fuzzy topological space X.

2.2 Theorem: Every multi fuzzy w-closed set is multi fuzzy rw-closed.

Proof: The proof follows from the Definition 1.19 and the fact that every multi fuzzy regular semi open set is multi fuzzy semi open.

Remark: The converse of Theorem 2.2 need not be true as from the following example.

2.6 Example: Let X = {1, 2} and $\Im = \{ \bar{0}, \bar{1}, A\}$ be a multi fuzzy topology on X, where A = {< 1, 0.7, 0.7, 0.6 >, < 2, 0.6, 0.6, 0.5 >}. Then the multi fuzzy set B = {< 1, 0.7, 0.7, 0.6 >, < 2, 0.8, 0.8, 0.7 >} is multi fuzzy rw-closed but it is not multi fuzzy w-closed.

2.3 Theorem: Every multi fuzzy rw-closed set is multi fuzzy rg-closed.

Proof: The proof follows from the Definition 1.19 and the fact that every multi fuzzy regular open set is multi fuzzy regular semi open.

Remark: The converse of Theorem 2.3 need not be true as from the following example.

2.7 Example: Let X = {1, 2, 3, 4} and multi fuzzy sets A, B, C, D defined as follows A = {< 1, 0.9, 0.9, 0.8 >, < 2, 0, 0, 0 >, < 3, 0, 0, 0 >, < 4, 0, 0, 0 >}, B = {< 1, 0, 0, 0 >, < 2, 0.8, 0.8, 0.7 >, < 3, 0, 0, 0 >, < 4, 0, 0, 0 >}, C = {< 1, 0.9, 0.9, 0.8 >, < 2, 0.8, 0.8, 0.7 >, < 3, 0, 0, 0 >, < 4, 0, 0, 0 >}, D = {< 1, 0.9, 0.9, 0.8 >, < 2, 0.8, 0.8, 0.7 >, < 3, 0.7, 0.7, 0.6 >, < 4, 0, 0, 0 >}, $\Im = \{ \bar{1}, \bar{0}, A, B, C, D\}$ be a multi fuzzy topology on X. Then the multi fuzzy set E = {<1, 0, 0, 0>, < 2, 0, 0, 0>, < 3, 0.7, 0.7, 0.6>, < 4, 0, 0, 0>} is multi fuzzy rg-closed but it is not multi fuzzy rw-closed.

2.4 Theorem: Every multi fuzzy rw-closed set is multi fuzzy gpr-closed.

Proof: Let A is a multi fuzzy rw-closed set in multi fuzzy topological space (X, $\Im$). Let $A \subseteq O$, where O is multi fuzzy regular open in X. Since every multi fuzzy regular open set is multi fuzzy regular semi open and A is multi fuzzy rw-closed set, we have mfcl(A) $\subseteq$ O. Since every multi fuzzy closed set is multi fuzzy pre closed, mfpcl(A) $\subseteq$ mfcl(A). Hence mfpcl(A) $\subseteq$ O which implies that A is multi fuzzy gpr-closed.

Remark: The converse of Theorem 2.4 need not be true as from the following example.

2.8 Example: Let X = {1, 2, 3, 4, 5} and multi fuzzy sets A, B, C defined as follows A = {< 1, 0.9, 0.9, 0.8 >, < 2, 0.8, 0.8, 0.7 >, < 3, 0, 0, 0 >, < 4, 0, 0, 0 >, < 5, 0, 0, 0 >}, B = {< 1, 0, 0, 0 >, < 2, 0, 0, 0 >, < 3, 0.8, 0.8, 0.7 >, < 4, 0.7, 0.7, 0.6 >, < 5, 0, 0, 0 >}, C = {< 1, 0.9, 0.9, 0.8 >, < 2, 0.8, 0.8, 0.7 >, < 3, 0.8, 0.8, 0.7 >, < 4, 0.7, 0.7, 0.6 >, < 5, 0, 0, 0 > }. Let $\Im$ = { $\bar{1}$, $\bar{0}$, A, B, C } be a multi fuzzy topology on X. Then the multi fuzzy set D = {< 1, 0.9, 0.9, 0.8>, < 2, 0, 0, 0>, < 3, 0, 0, 0>, < 4, 0, 0, 0>, < 5, 0, 0, 0 >} is multi fuzzy gpr-closed but it is not multi fuzzy rw-closed.

2.5 Theorem: If A is a multi fuzzy regular open and multi fuzzy rg-closed in multi fuzzy topological space (X, $\Im$), then A is multi fuzzy rw-closed in X.

Proof: Let A is a multi fuzzy regular open and multi fuzzy rg-closed in X. We prove that A is a multi fuzzy rw-closed in X. Let U be any multi fuzzy regular semi open set in X such that A $\subseteq$ U. Since A is multi fuzzy regular open and multi fuzzy rg-closed, we have mfcl(A) $\subseteq$ A. Then mfcl(A) $\subseteq$ A$\subseteq$ U. Hence A is multi fuzzy rw-closed in X.

2.6 Theorem: If A and B are multi fuzzy rw-closed sets in multi fuzzy topological space X, then union of A and B is multi fuzzy rw-closed set in multi fuzzy topological space X.

Proof: Let C be a multi fuzzy regular semiopen set in multi fuzzy topological space X such that (A$\cup$B) $\subseteq$ C. Now A $\subseteq$ C and B $\subseteq$ C. Since A and B are multi fuzzy rw-closed sets in multi fuzzy topological space X, mfcl(A) $\subseteq$ C and mfcl(B) $\subseteq$ C. Therefore (mfcl(A) $\cup$ mfcl(B)) $\subseteq$ C. But (mfcl(A) $\cup$ mfcl(B)) = mfcl(A$\cup$B). Thus mfcl(A$\cup$B) $\subseteq$ C. Hence A$\cup$B is a multi fuzzy rw-closed set in multi fuzzy topological space X.

2.7 Theorem: If A and B are multi fuzzy rw-closed sets in multi fuzzy topological space X, then the intersection of A and B need not be a multi fuzzy rw-closed set in multi fuzzy topological space X.

Proof: Consider the multi fuzzy topological space (X, $\Im$) defined in Example 2.2. In this multi fuzzy topological space X, the multi fuzzy sets G_1, G_2 are defined by G_1 = { < 1, 0, 0, 0 >, < 2, 0, 0, 0 >, < 3, 1, 1, 1 >, < 4, 1, 1, 1 > } and G_2 = {< 1, 1, 1, 1 >, < 2, 1, 1, 1 >, < 3, 1, 1, 1 >, < 4, 0, 0, 0 >}. Then G_1 and G_2 are the multi fuzzy rw-closed sets in multi fuzzy topological space X. Let D = $G_1 \cap G_2$. Then D = {< 1, 0, 0, 0 >, < 2, 0, 0, 0 >, < 3, 1, 1, 1 >, < 4, 0, 0, 0 >}. Then D = $G_1 \cap G_2$ is not a multi fuzzy rw-closed set in multi fuzzy topological space X.

2.8 Theorem: If a multi fuzzy subset A of multi fuzzy topological space X is both multi fuzzy regular open and multi fuzzy rw-closed, then A is a multi fuzzy regular closed set in multi fuzzy topological space X.

Proof: Suppose a multi fuzzy subset A of multi fuzzy topological space X is both multi fuzzy regular open and multi fuzzy rw-closed. As every multi fuzzy regular open set is a multi fuzzy regular semiopen set and A $\subseteq$ A, we have mfcl(A) $\subseteq$ A. Also A $\subseteq$ mfcl(A). Therefore mfcl(A) = A. That is A multi fuzzy closed. Since A is multi fuzzy regular open, mfint (A) = A. Now mfcl(mfint (A)) = mfcl(A) = A. Therefore A is a multi fuzzy regular closed set in multi fuzzy topological space X.

2.9 Theorem: If a multi fuzzy subset A of a multi fuzzy topological space X is both multi fuzzy regular semiopen and multi fuzzy rw-closed, then A is a multi fuzzy closed set in multi fuzzy topological space X.

Proof: Suppose a multi fuzzy subset A of a multi fuzzy topological space X is both multi fuzzy regular semiopen and multi fuzzy rw-closed. Now A $\subseteq$ A, we have mfcl(A) $\subseteq$A. Also A $\subseteq$ mfcl(A). Therefore mfcl(A) = A and hence A is a multi fuzzy closed set in multi fuzzy topological space X.

2.1 Corollary: If A is a multi fuzzy regular semi open and multi fuzzy rw-closed in multi fuzzy topological space (X, $\Im$). Suppose that F is multi fuzzy closed in X then A$\cap$F is multi fuzzy rw-closed in X.

Proof: Suppose A is both multi fuzzy regular semi open and multi fuzzy rw- closed set in X and F is multi fuzzy closed in X. By Theorem 2.9**,** A is multi fuzzy closed in X. So A$\cap$F is multi fuzzy closed in X. Hence A$\cap$F is multi fuzzy rw-closed in X.

2.10 Theorem: If a multi fuzzy subset A of a multi fuzzy topological space X is both multi fuzzy open and multi fuzzy generalized closed, then A is a multi fuzzy rw-closed set in multi fuzzy topological space X.

Proof: Suppose a multi fuzzy subset A of a multi fuzzy topological space X is both multi fuzzy open and multi fuzzy generalized closed. Now A$\subseteq$A, by hypothesis we have mfcl(A) $\subseteq$ A. Also A $\subseteq$ cl(A). Therefore cl(A) = A. That is A is a multi fuzzy closed set and hence A is a multi fuzzy rw-closed set in multi fuzzy topological space X, as every multi fuzzy closed set is a multi fuzzy rw-closed set.

2.11 Theorem: If a multi fuzzy subset C is both multi fuzzy regular open and multi fuzzy rw-closed set in a multi fuzzy topological space X, then C need not be a multi fuzzy generalized closed set in multi fuzzy topological space X.

Proof: Consider the example, let X = {1, 2, 3} and the multi fuzzy sets A, B, C be defined as A = {< 1, 1, 1, 1 >, < 2, 0, 0, 0 >, < 3, 0, 0, 0 >}, B = {< 1, 0, 0, 0 >, < 2, 1, 1, 1>, <3, 0, 0, 0 >} and C = {< 1, 1, 1, 1 >, < 2, 1, 1, 1] >, < 3, 0, 0, 0 >}. Consider $\Im = \{ \bar{0}, \bar{1}, A, B, C\}$. Then $(X, \Im)$ is a multi fuzzy topological space. In this multi fuzzy topological space X, C is both multi fuzzy open and multi fuzzy rw-closed set in multi fuzzy topological space X but it is not multi fuzzy generalized closed.

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