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ON WEAKLY μg-CONTINUOUS FUNCTIONS IN GENERALIZED TOPOLOGICAL SPACES

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ABSTRACT

In this paper, a new class of functions called $(\mu_1, w\mu g\_\mu_2)$ continuous functions, $(w\mu g\_\mu_1, \mu_2)$ continuous functions and $w\mu g$-irresolute functions in generalized topological space are introduced and studied. These functions are defined by $w\mu g$-open sets. Some of their properties are investigated.

***AMS Subject classification**: 54A05, 54C08.*

***Keywords**: $w\mu g$ - closed sets, (μ_1, μ_2) continuous functions, $(\mu_1, w\mu g\_\mu_2)$ continuous functions, $(w\mu g\_\mu_1, \mu_2)$ continuous functions, $w\mu g$-irresolute function.*

1. INTRODUCTION

In 2002, generalized topological space (GTS) introduced and developed by A. Császár [2] and many authors [3, 4] have studied various types of continuity functions using generalized open sets in GTS.

A generalized topology or simply GT μ [2] on a nonempty set X is a collection of subsets of X such that $\phi \in \mu$ and μ is closed under arbitrary union. Elements of μ are called μ-open sets. A subset A of X is said to be μ-closed if A^c is μ-open. The pair (X,μ) is called a generalized topological space (GTS). If A is a subset of X, then c_μ is the smallest μ-closed set containing A and $i_\mu(A)$ is the largest μ-open set contained in A. A space (X, μ) is said to be strong if $X \in \mu$.

In 2013, Chunfang Cao *et al.* [1] introduced the notions of $(\mu_1, \alpha\mu_2)$ continuous functions, $(\mu_1, \pi\mu_2)$ continuous functions on GTS**.** The purpose of the present paper is to introduce $(\mu_1, w\mu g\_\mu_2)$ continuous functions, $(w\mu g\_\mu_1, \mu_2)$ continuous functions and wμg-irresolute function in GTS and investigate its properties and the relationships among existing continuities.

2. PRELIMINARIES

Throughout this paper X and Y mean GTS's (X,μ_1) and (Y, μ_2) and the function $f:X\to Y$ denotes a single valued function of a space (X,μ_1) into a space (Y, μ_2). We recall the following definitions and results.

Definition 2.1: Let (X,μ) be a GTS and $A\subseteq X$. Then A is said to be

(1) μ-α-open[2] if $A\subseteq i_\mu c_\mu i_\mu(A)$
(2) μ-π-open [2] if $A\subseteq i_\mu c_\mu(A)$

The complement of μ-α-open (resp. μ-π-open, μ-open) is said to be μ-α-closed (resp.μ-π-closed, μ-closed).

Definition 2.2: A subset A of X is said to be weakly μg-closed set (briefly wμg-closed) [5] if $c_\mu i_\mu(A)\subseteq U$ whenever $A\subseteq U$ and U is μ-open. The complement of wμg-closed set is called a wμg-open set.

Let us denote $\mu(X)$ (resp. $\alpha\mu(X)$, $\pi\mu(X)$, $G\mu(X)$, $W\mu G(X)$) the class of all μ-open (resp. μ-α-open, μ-π-open, μg-open wμg-open) sets on X.

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Definition 2.3: A function f: X→Y is said to be

(1) (μ_1,μ_2) continuous functions [2] if $f^{-1}(U)$ is μ_1-open in X for every μ_2-open set U of Y.
(2) $(\mu_1,\alpha\mu_2)$ continuous functions [1] if $f^{-1}(U)$ is μ_1-open in X for every μ_2-α-open set U of Y.
(3) $(\mu_1,\pi\mu_2)$ continuous functions [1] if $f^{-1}(U)$ is μ_1-open in X for every μ_2-π-open set U of Y.

3. ON WEAKLY μg CONTINUITY

Definition 3.1: Let (X,μ_1) and (Y, μ_2) be GTS's. Then a function f: X→Y is said to be

(1) $(\mu_1,w\mu g\_\mu_2)$ continuous if $f^{-1}(U)$ is μ_1-open in X for every wμg -open set U of Y.
(2) $(w\mu g\_\mu_1,\mu_2)$ continuous if $f^{-1}(U)$ is wμg-open in X for every μ_2-open set U of Y.
(3) $w\mu g$-irresolute if $f^{-1}(U)$ is wμg-open in X for every wμg-open set U of Y.

Example 3.2: Let X=Y= {a, b, c} and $\mu_1=\mu_2=\{\phi,\{b\},\{a, b\}\}$. Then W$\mu$G (Y) $=\{\phi,\{a\},\{b\},\{a, b\}\}$. A function f:X→Y defined by f(a) = b = f(b), f(c) = c. Then f is $(\mu_1,w\mu g\_\mu_2)$ continuous.

Example 3.3: Let X=Y={a, b, c} and $\mu_1=\mu_2=\{\phi,\{b\},\{a, b\}\}$. Then W$\mu$G(X)= $=\{\phi,\{a\},\{b\},\{a, b\}\}$. A function f:X→Y defined by f(a)=a, f(b)=b, f(c)=c. Then f is $(w\mu g\_\mu_1,\mu_2)$ continuous.

Example 3.4: Let X={a, b, c}, Y={1, 2, 3} and $\mu_1=\{\phi, \{a, b\}\}$ and $\mu_2=\{\phi, \{2\}, \{1,2\}\}$. Then W$\mu$G(X) $=\{\phi,\{a\}, \{b\}, \{a, b\}\}$ and WμG (Y) $=\{\phi,\{1\},\{2\},\{1,2\}\}$.A function f:X→Y defined by f(a)=1,f(b)=2, f(c)=3. Then f is $w\mu g$ – irresolute.

Remark 3.5: $\mu(X)\subset\alpha\mu(X)\subset\pi\mu(X)\subset W\mu G(X)$

Theorem 3.6: Every $(\mu_1,w\mu g\_\mu_2)$ continuous is $(\mu_1,\pi\mu_2)$ continuous but not conversely.

Proof: Let f: X→Y be a $(\mu_1,w\mu g\_\mu_2)$ continuous. Then for every wμg-open set U in Y, $f^{-1}(U)$ is μ_1-open in X. Since every μ-π-open set is wμg-open, for every μ_2-π-open set U in Y, $f^{-1}(U)$ is μ_1-open in X. Hence f is $(\mu_1,\pi\mu_2)$ continuous.

The converse of the above theorem is not necessarily true as seen from the following example.

Example 3.7: Let X=Y={a, b, c} and $\mu_1=\mu_2=\{\phi, \{b\},\{a, b\}\}$. Then W$\mu$G(Y)= $=\{\phi,\{a\},\{b\},\{a, b\}\}$and $\pi\mu(Y)=\{\phi,\{b\},\{a, b\}\}$. A function f: X→Y defined by f(a) = a, f(b) = b, f(c) = c. Then f is $(\mu_1,\pi\mu_2)$ continuous but not $(\mu_1,w\mu g\_\mu_2)$ continuous.

Corollary 3.8: Every $(\mu_1,w\mu g\_\mu_2)$ continuous is $(\mu_1,\alpha\mu_2)$ continuous but not conversely.

Proof: Follows from theorem.3.6. and the fact that every $(\mu_1,\pi\mu_2)$ continuous map is $(\mu_1,\alpha\mu_2)$ continuous.

Corollary 3.9: Every $(\mu_1,w\mu g\_\mu_2)$ continuous is (μ_1,μ_2) continuous but not conversely.

Proof: Follows from corollary 3.8. and the fact that every $(\mu_1,\alpha\mu_2)$ continuous map is (μ_1,μ_2) continuous.

Theorem 3.10: Every (μ_1,μ_2) continuous is $(w\mu g\_\mu_1,\mu_2)$ continuous but not conversely.

Proof: Let f: X→Y be a (μ_1,μ_2) continuous. Then for every μ_2-open set U in Y, $f^{-1}(U)$ is μ_1-open in X. Since every μ-open set is wμg-open, for every μ_2-open set U in Y, $f^{-1}(U)$ is wμg-open in X. Hence f is $(w\mu g\_\mu_1,\mu_2)$ continuous.

The converse of the above theorem is not necessarily true as seen from the following example.

Example 3.11: Let X={a, b, c}, Y={1, 2, 3} and $\mu_1=\{\phi, \{a\},\{a, b\},\{b, c\}, X\}$ and $\mu_2=\{\phi,\{1\},\{2,.3\},\{1,3\}, Y\}$. Then W$\mu$G (X) $=\{\phi,\{a\},\{b\},\{a, b\}, \{b, c\}, \{a, c\}, X\}$.A function f:X→Y defined by f(a)=1, f(b)=2, f(c)=3. Then f is $(w\mu g\_\mu_1,\mu_2)$ continuous. But f is not (μ_1,μ_2) continuous.

Corollary 3.12: Every $(\mu_1,w\mu g\_\mu_2)$ continuous (resp. $(\mu_1,\alpha\mu_2)-continuous,\left(\mu_1,\pi\mu_2\right)-continuous)$is $(w\mu g\_\mu_1,\mu_2)$ continuous but not conversely.

Proof: Follows from theorem.3.10. and the fact that every $(\mu_1,w\mu g\_\mu_2)$ continuous (resp. $(\mu_1,\alpha\mu_2)-continuous,\left(\mu_1,\pi\mu_2\right)-continuous)$ is (μ_1,μ_2) continuous.

Remark 3.13: From the above discussions, we get the relationship $(\mu_{1,}w\mu g\_\mu_2)$ continuous $\rightarrow$ $(\mu_{1,}\pi\mu_{2,})$continuous$\rightarrow$ $(\mu_{1,}\alpha\mu_{2,})$continuous$\rightarrow$$(\mu_{1,}\mu_{2,})$ continuous$\rightarrow$$(w\mu g_\mu_{1,}\mu_{2,})$ continuous

Theorem 3.14: Let f :(X, μ_1)$\rightarrow$(Y, μ_2) be a function. Then the following are equivalent.

(1) f is $(\mu_{1,}w\mu g\_\mu_2)$ -continuous;
(2) The inverse image of each wμg-open set in Y is μ_1-open in X;
(3) The inverse image of each wμg-closed set in Y is μ_1-closed in X;

Proof:
(1)$\Rightarrow$ (2): It is obviously by definition.

(2)$\Rightarrow$(3): Let U be any wμg-closed set in Y. Then Y\U is wμg-open set in Y. By (2) f^{-1}(Y\U) is μ_1-open. But f^{-1}(Y\ U) = X\f^{-1}(U) which is μ_1-open. Therefore f^{-1}(U) is μ_1-closed. This proves (2)$\Rightarrow$(3).

(3)$\Rightarrow$(1): Let G be wμg-open in Y. Then G^c is wμg-closed in Y. By (3) f^{-1} (G^c) is μ_1-closed in X. But $f^{-1}(G^c) = (f^{-1}(G))^c$ which is μ_1-closed in X. Therefore f^{-1}(G) is μ_1-open in X. This proves (3)$\Rightarrow$(1).

Theorem 3.15: Let X be a strong GTS. Let f:X$\rightarrow$Y be a function and h:X$\rightarrow$X$\times$Y be the graph function defined by h(x)=(x, f(x)) for each x$\in$X. If h is $(\mu_{1,}w\mu g\_\mu_2)$ – continuous then f is $(w\mu g - \mu_{1,}\mu_2)$ –continuous.

Proof: Since every $(\mu_{1,}w\mu g\_\mu_2)$ – continuous is$(\mu_{1,}\alpha\mu_{2,})$continuous, f is $(\mu_{1,}\alpha\mu_{2,})$continuous. by theorem 3.3[1]. Hence f is $(w\mu g - \mu_{1,}\mu_2)$ –continuous.

Theorem 3.16: If f : (X,μ_1) $\rightarrow$ (Y,μ_2) is ($\mu_{1,}$ wμg-μ_2) continuous and g: (Y,μ_2)$\rightarrow$ (Z,μ_3) is ($\mu_{2,}$ wμg-μ_3) continuous the gof: (X,μ_1) $\rightarrow$ (Z,μ_3) is ($\mu_{1,}$ wμg-μ_3) continuous.

Proof: Let U be wμg-open set in Z. Since g is ($\mu_{2,}$ wμg-μ_3) continuous, g^{-1}(U) is μ_2-open in Y. Hence g^{-1}(U) is wμg-open in Y. Since f is ($\mu_{1,}$ wμg-μ_2) continuous, f^{-1} (g^{-1}(U)) is μ_1-open in X. Hence gof: (X,μ_1) $\rightarrow$ (Z,μ_3) is ($\mu_{1,}$ wμg-μ_3) continuous.

Theorem 3.17: Every wμg-irresolute function is (wμg_μ_1,μ_2)-continuous but converse is not necessarily true.

Proof: Suppose f:X$\rightarrow$Y is wμg -irresolute. Let V be any μ_{2-} open set of Y; Then V is wμg-open set in Y. Since f is #wμg-irresolute, f^{-1}(V) is wμg-open in X. Hence f is (wμg_μ_1,μ_2)-continuous.

The converse of the theorem need not be true as seen from the following example.

Example 3.18: Let X={a, b, c}=Y and μ_1={ϕ,{a},{a, b},{b, c},X} and μ_2={ϕ,{a},{a, c},{b, c}}. Then WμG(X) = {ϕ,{a},{b},{a, b},{b, c},{a, c},X}and WμG(Y)={ϕ,{a},{c},{a, b},{b, c},{a, c},Y}.A function f:X$\rightarrow$Y defined by f(a) = a, f(b) = b, f(c) = c. Then clearly f is (wgμ-μ_1, μ_2) continuous but not $w\mu g$ – irresolute.

Theorem 3.19: Let f: (X, μ_1) $\rightarrow$ (Y, μ_2) and g: (Y, μ_2)$\rightarrow$ (Z,μ_3) be any two functions. Let h = gof. Then

(i) h is (wμg_μ_1,μ_3)-continuous if f is wμg-irresolute and g is (wμg_μ_2, μ_3)--continuous.
(ii) h is wμg-irresolute if both f and g are wμg-irresolute and
(iii) h is (wμg_μ_1,μ_3) continuous if g is ($\mu_{2,}\mu_3$)continuous and f is (wμg_μ_1,μ_2)-continuous.

Proof: Let V be μ_3 -open in Z.

(i) Suppose f is wμg-irresolute and g is (wμg_μ_2 ,μ_3)-continuous. Since g is (wμg_μ_2 ,μ_3) -continuous, g^{-1}(V) is wμg-open in Y. Since f is wμg-irresolute, f^{-1}(g^{-1}(V)) is wμg-open in X. This proves (i).
(ii) Let f and g be wμg-irresolute. Then g^{-1}(V) is wμg-open in Y. Since f is wμg-irresolute, using f^{-1}(g^{-1}(V)) is wμg-open in X. This proves (ii).
(iii) (iii)Let g be ($\mu_{2,}\mu_3$)continuous and f be (wμg_μ_1,μ_2)-continuous. Then g^{-1}(V) is μ_2 -open in Y. Since f is (wμg_μ_1,μ_2)--continuous, f^{-1}(g^{-1}(V)) is wμg-open in X. This proves (iii).

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