PSEUDO IRREGULAR FUZZY GRAPHS AND HIGHLY PSEUDO IRREGULAR FUZZY GRAPHS

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ABSTRACT

In this paper, pseudo irregular fuzzy graphs and pseudo totally irregular fuzzy graphs are defined. Comparative study between pseudo irregular fuzzy graph and pseudo totally irregular fuzzy graph is done. Highly pseudo irregular fuzzy graphs and highly pseudo totally irregular fuzzy graphs are defined. Comparative study between pseudo irregular fuzzy graphs and highly pseudo totally irregular fuzzy graphs is done. Some properties of pseudo irregular fuzzy graphs and highly pseudo irregular fuzzy graphs are studied.

Key words: 2-degree, pseudo degree of a vertex in a graph, irregular fuzzy graph, totally irregular fuzzy graph.

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1. INTRODUCTION

In this paper, we consider only finite, simple, connected graphs. We denote the vertex set and the edge set of a graph G by V (G) and E(G) respectively. The degree of a vertex v is the number of edges incident at v, and it is denoted by d(v). A graph G is regular if all its vertices have the same degree. The 2-degree of v \[ t(v) \] is the sum of the degrees of the vertices adjacent to v and it is denoted by \( t(v) \). A pseudo degree of a vertex \( v \) is denoted by \( d_a(v) \) and defined as \( \frac{t(v)}{d^*_G(v)} \), where \( d^*_G(v) \) is the number of edges incident at v.

A graph is called pseudo-regular if every vertex of G has equal pseudo (average) degree [3].

The notion of fuzzy sets was introduced by Zadeh as a way of representing uncertainty and vagueness [18]. The first definition of fuzzy graph was introduced by Haufmann in 1973. In 1975, A. Rosenfeld introduced the concept of fuzzy graphs [8]. The theory of graph is an extremely useful tool for solving combinatorial problems in different areas. Irregular fuzzy graphs plays a central role in combinatorics and theoretical computer science.

2. REVIEW OF LITERATURE


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2. PRELIMINARIES

Definition 2.1: A fuzzy graph \( G: (\sigma, \mu) \) is a pair of functions \( (\sigma, \mu) \), where \( \sigma: V \rightarrow [0,1] \) is a fuzzy subset of a non-empty set \( V \) and \( \mu: VXV \rightarrow [0,1] \) is a symmetric fuzzy relation on \( \sigma \) such that for all \( u,v \) in \( V \), the relation \( \mu(uv) \leq \sigma(u) \wedge \sigma(v) \) is satisfied. A fuzzy graph \( G \) is called complete fuzzy graph if the relation \( \mu(uv) = \sigma(u) \wedge \sigma(v) \) is satisfied.

Definition 2.2: Let \( G: (\sigma, \mu) \) be a fuzzy graph on \( G^*(V, E) \). The degree of a vertex \( u \) in \( G \) is denoted by \( d(u) \) and is defined as \( d(u) = \sum \mu(uv) \), for all \( uv \in E \).

Definition 2.3: Let \( G: (\sigma, \mu) \) be a fuzzy graph on \( G^*(V, E) \). The total degree of a vertex \( u \) in \( G \) is denoted by \( td(u) \) and is defined as \( td(u) = d(u) + \sigma(u) \) for all \( u \in V \).

Definition 2.4: Let \( G: (\sigma, \mu) \) be a fuzzy graph on \( G^*(V, E) \). Then \( G \) is said to be an irregular fuzzy graph, if there is a vertex which is adjacent to vertices with distinct total degrees.

Definition 2.5: Let \( G: (\sigma, \mu) \) be a fuzzy graph on \( G^*(V, E) \). Then \( G \) is said to be a totally irregular fuzzy graph if there is a vertex which is adjacent to vertices with distinct total degrees.

Definition 2.6: Let \( G: (\sigma, \mu) \) be a fuzzy graph on \( G^*(V, E) \). Then \( G \) is said to be a neighbourly irregular fuzzy graph if every two adjacent vertices of \( G \) have distinct degree.

Definition 2.7: Let \( G: (\sigma, \mu) \) be a fuzzy graph on \( G^*(V, E) \). Then \( G \) is said to be a neighbourly total irregular fuzzy graph if every two adjacent vertices have distinct total degrees.

Definition 2.8: Let \( G: (\sigma, \mu) \) be a fuzzy graph on \( G^*(V, E) \). Then \( G \) is said to be a highly irregular fuzzy graph if every vertex of \( G \) is adjacent to vertices with distinct degree.

Definition 2.9: Let \( G: (\sigma, \mu) \) be a fuzzy graph on \( G^*(V, E) \). The 2-degree of a vertex \( v \) is defined as the sum of degrees of vertices incident at \( v \) and it is denoted by \( t(v) \).

Definition 2.10: A pseudo degree of a vertex \( v \) is denoted by \( d_\sigma(v) \) and defined as \( \frac{t(v)}{\sigma(v)} \), where \( d_\sigma'(v) \) is the number of edges incident at \( v \).

Definition 2.11: Let \( G: (\sigma, \mu) \) be a fuzzy graph on \( G^*(V, E) \). The pseudo total degree of a vertex \( v \) in \( G \) is denoted by \( td_\sigma(v) \) and is defined as \( td_\sigma(v) = d_\sigma(v) + \sigma(v) \) for all \( v \in V \).

3. PSEUDO IRREGULAR FUZZY GRAPHS AND TOTALLY PSEUDO IRREGULAR FUZZY GRAPHS

Definition 3.1: Let \( G: (\sigma, \mu) \) be a fuzzy graph on \( G^*(V, E) \). Then \( G \) is said to be a pseudo irregular fuzzy graph if there is a vertex which is adjacent to the vertices with distinct pseudo degree.

Example 3.2: Consider a graph on \( G^*(V, E) \).

Now, \( d_\sigma(u) = 0.4, d_\sigma(v) = 0.5, d_\sigma(w) = 0.45, d_\sigma(x) = 0.4, d_\sigma(y) = 0.45 \). Here, \( v \) and \( y \) are adjacent vertices of \( u \) and they have distinct pseudo degree. Hence the graph is pseudo irregular fuzzy graph.

Definition 3.3: Let \( G: (\sigma, \mu) \) be a fuzzy graph on \( G^*(V, E) \). Then \( G \) is said to be a pseudo totally irregular fuzzy graph if there is a vertex which is adjacent to the vertices with distinct total pseudo degree.
Example 3.4: Consider a graph on $G^*(V, E)$.

Now, $td_a(u) = d_a(u) + \sigma(u) = 0.45 + 0.2 = 0.65$, $td_a(v) = 0.9$, $td_a(w) = 0.95$, $td_a(x) = 1.1$, $td_a(y) = 1.1$, $td_a(z) = 1$. Here, $v$ and $z$ are adjacent vertices of $u$ and they have distinct total pseudo degree. Hence the graph is pseudo totally irregular fuzzy graph.

Remark 3.5: A pseudo irregular fuzzy graph need not be a pseudo totally irregular fuzzy graph.

Example 3.6: Consider a graph on $G^*(V, E)$.

Now, $d_a(u) = 0.5$, $d_a(v) = 0.4$, $d_a(w) = 0.45$, $d_a(x) = 0.5$, $d_a(y) = 0.55$ and $td_a(u) = 1$, $td_a(v) = 1$, $td_a(w) = 1$, $td_a(x) = 1$, $td_a(y) = 1$. Here, all the vertices have distinct pseudo degree and same total pseudo degree. Hence the graph is pseudo irregular fuzzy graph but it is not pseudo totally irregular fuzzy graph.

Remark 3.7: A pseudo totally irregular fuzzy graph need not be a pseudo irregular fuzzy graph.

Example 3.8: Consider a graph on $G^*(V, E)$.

Now, $d_a(u) = 0.5$, $d_a(v) = 0.5$, $d_a(w) = 0.5$, $d_a(x) = 0.5$ and $td_a(u) = 1$, $td_a(v) = 1$, $td_a(w) = 1$, $td_a(x) = 0.7$. Here, all the vertices have same pseudo degree and distinct total pseudo degree. Hence the graph is pseudo totally irregular fuzzy graph but it is not a pseudo irregular fuzzy graph.

Example 3.9: A pseudo irregular fuzzy graph which is pseudo totally irregular fuzzy graph. Consider a graph on $G^*(V, E)$.
Now, \( d_u(u) = 0.533, d_u(v) = 0.7, d_u(x) = 0.466 \) and \( td_u(u) = 1.033, td_u(v) = 1.4, td_u(w) = 1.1, td_u(x) = 1.066 \). Here, the adjacent vertices of \( u \) have distinct pseudo degree and distinct total pseudo degree. Hence the graph is both pseudo irregular and pseudo totally irregular fuzzy graph.

**Theorem 3.10:** Let \( G: (\sigma, \mu) \) be a fuzzy graph on \( G^*(V, E) \). If \( \sigma \) is a constant function then the following conditions are equivalent.

(i) \( G \) is a pseudo irregular fuzzy graph.

(ii) \( G \) is a pseudo totally irregular fuzzy graph.

**Proof:** Assume that \( \sigma \) is a constant function. Let \( \sigma(u) = c \), for all \( u \in V \). Suppose \( G \) is a pseudo irregular fuzzy graph. Then at least one vertex of \( G \) which is adjacent to the vertices with distinct pseudo degree. Let \( u_1 \) and \( u_2 \) be the adjacent vertices of \( u_3 \) with distinct pseudo degrees \( k_1 \) and \( k_2 \) respectively. Then \( k_1 \neq k_2 \). Suppose \( G \) is not a pseudo totally irregular fuzzy graph. Then every vertex of \( G \) which is adjacent to the vertices with same pseudo total degree \( \Rightarrow td(u_1) = td(u_2) \Rightarrow d(u_1) + \sigma(u_1) = d(u_2) + \sigma(u_2) \Rightarrow k_1 + c = k_2 + c \Rightarrow k_1 = k_2 \), which is a contradiction to \( k_1 \neq k_2 \). Hence \( G \) is pseudo totally irregular fuzzy graph. Thus (i) \( \Rightarrow \) (ii) is proved.

Now, suppose \( G \) is a pseudo totally irregular fuzzy graph. Then at least one vertex of \( G \) which is adjacent to the vertices with distinct total pseudo degree. Let \( u_1 \) and \( u_2 \) be the adjacent vertices of \( u_3 \) with distinct pseudo total degrees \( k_1 \) and \( k_2 \) respectively. Now, \( td(u_1) \neq td(u_2) \Rightarrow d(u_1) + \sigma(u_1) \neq d(u_2) + \sigma(u_2) \Rightarrow k_1 + c \neq k_2 + c \Rightarrow k_1 \neq k_2 \). Hence \( G \) is pseudo irregular fuzzy graph. Thus (ii) \( \Rightarrow \) (i) is proved.

Hence (i) and (ii) are equivalent.

**4. HIGHLY PSEUDO IRREGULAR FUZZY GRAPHS AND HIGHLY PSEUDO TOTALLY IRREGULAR FUZZY GRAPHS**

**Definition 4.1:** Let \( G: (\sigma, \mu) \) be a fuzzy graph on \( G^*(V, E) \). Then \( G \) is said to be a highly pseudo irregular fuzzy graph if every vertex of \( G \) which is adjacent to the vertices with distinct pseudo degree.

**Example 4.2:** Consider a graph on \( G^*(V, E) \).

![Figure-6](image)

Now, \( d_u(u) = 0.9, d_u(v) = 0.85, d_u(w) = 0.766, d_u(x) = 0.8, d_u(y) = 1.1 \). Here, every vertex of \( G \) is adjacent to the vertices with distinct pseudo degree. Hence the graph is highly pseudo irregular fuzzy graph.

**Definition 4.3:** Let \( G: (\sigma, \mu) \) be a fuzzy graph on \( G^*(V, E) \). Then \( G \) is said to be a highly pseudo totally irregular fuzzy graph if every vertex of \( G \) which is adjacent to the vertices with distinct total pseudo degree.

**Example 4.4:** Consider a graph on \( G^*(V, E) \).

![Figure-7](image)

Now, \( d_u(u) = 0.7, d_u(v) = 0.7, d_u(w) = 0.7, d_u(x) = 0.7 \) and \( td_u(u) = 1.1, td_u(v) = 1.4, td_u(w) = 1.2, td_u(x) = 1 \). Here every vertex of \( G \) is adjacent to vertices with distinct total pseudo degree. Hence the graph is highly pseudo totally irregular fuzzy graph.
Remark 4.5: A highly pseudo irregular fuzzy graph need not be a highly pseudo totally irregular fuzzy graph.

Example 4.6: Consider a graph on $G^*(V,E)$.

Now, $d_a(u)=1$, $d_a(v)=0.6$, $d_a(w)=0.95$, $d_a(x)=0.85$, $d_a(y)=0.8$ and $td_a(u)=1.5$, $td_a(v)=1.2$, $td_a(w)=1.45$, $td_a(x)=1.45$, $td_a(y)=1.3$. Here, every two adjacent vertices have distinct pseudo degree and the adjacent vertices $w$ and $x$ have same total pseudo degree. Hence the graph is highly pseudo irregular fuzzy graph but it is not a highly pseudo totally irregular fuzzy graph.

Remark 4.7: A highly pseudo totally irregular fuzzy graph need not be a highly pseudo irregular fuzzy graph.

Example 4.8: Consider a graph on $G^*(V,E)$.

Now, $d_a(u)=0.6$, $d_a(v)=0.6$, $d_a(w)=0.6$, $d_a(x)=0.6$ and $td_a(u)=1.733$, $td_a(v)=1$, $td_a(w)=0.9$, $td_a(x)=1.1$. Here, every vertex of $G$ is adjacent to the vertices with distinct total pseudo degree and all the vertices have same pseudo degree. Hence the graph is highly pseudo totally irregular fuzzy graph and it is not a highly pseudo irregular fuzzy graph.

Example 4.9: A highly pseudo irregular fuzzy graph which is highly pseudo totally irregular fuzzy graph. Consider a graph on $G^*(V,E)$.

Now, $d_a(u)=1.333$, $d_a(v)=0.966$, $d_a(w)=1.5$, $d_a(x)=1.133$, $d_a(y)=1.2$ and $td_a(u)=1.733$, $td_a(v)=1.466$, $td_a(w)=2.1$, $td_a(x)=1.833$, $td_a(y)=2.1$. Here, every two adjacent vertices have distinct pseudo degree and distinct total pseudo degree. Hence the graph is both highly pseudo irregular fuzzy graph and highly pseudo totally irregular fuzzy graph.
**Theorem 4.10:** Let \( G((\sigma, \mu)) \) be a fuzzy graph on \( G^*(V, E) \). Then \( G \) is neighbourly pseudo irregular and highly pseudo irregular fuzzy graph if and only if the pseudo degrees of all vertices of \( G \) are distinct.

**Proof:** Assume that the pseudo degrees of all vertices of \( G \) are distinct. Then clearly \( G \) is neighbourly pseudo irregular fuzzy graph and highly pseudo irregular fuzzy graph.

Conversely, assume that \( G \) is neighbourly pseudo irregular fuzzy graph and highly pseudo irregular fuzzy graph. Let \( G \) be a fuzzy graph with \( n \) vertices \( v_1, v_2, v_3, \ldots, v_n \). Let \( v_1 \) be adjacent with the vertices of \( v_2, v_3, \ldots, v_n \) with pseudo degrees \( k_2, k_3, \ldots, k_n \) respectively. Then \( k_2 \neq k_3 \neq \cdots \neq k_n \) and the pseudo degree of \( v_1 \) cannot be any one of \( k_2, k_3, \ldots, k_n \). Hence the pseudo degrees of all vertices of \( G \) are distinct.

**Remark 4.11:** If every pseudo regular fuzzy graph \( G \) with membership values of all the vertices are distinct, then \( G \) is highly pseudo totally irregular fuzzy graph.

**Remark 4.12:** With membership values of adjacent vertices of atleast one vertex are distinct, then \( G \) is highly pseudo totally irregular fuzzy graph.

**Theorem 4.13:** Let \( G((\sigma, \mu)) \) be a fuzzy graph on \( G^*(V, E) \). Then \( G \) is neighbourly pseudo total irregular and highly pseudo total irregular fuzzy graph if and only if the total pseudo degrees of all vertices of \( G \) are distinct.

**Proof:** Proof is similar to the above theorem.

**Theorem 4.14:** Let \( G((\sigma, \mu)) \) be a fuzzy graph on \( G^*(V, E) \). If \( \sigma \) is a constant function then the following conditions are equivalent.

(i) \( G \) is a highly pseudo irregular fuzzy graph.

(ii) \( G \) is a highly pseudo totally irregular fuzzy graph.

**Proof:** Assume that \( \sigma \) is a constant function. Let \( \sigma(u) = c \), for all \( u \in V \). Suppose \( G \) is a highly pseudo irregular fuzzy graph. Then every vertex of \( G \) is adjacent to the vertices with distinct pseudo degree. Let \( u_1 \) and \( u_2 \) be the adjacent vertices of \( u_3 \) with \( d_a(u_1) = k_1 \) and \( d_a(u_2) = k_2 \). Then \( k_1 \neq k_2 \). Suppose \( G \) is not a highly pseudo totally irregular fuzzy graph. Then every vertex of \( G \) which is adjacent to the vertices with same total pseudo degree. Suppose, \( td_a(u_1) = t d_a(u_2) \Rightarrow d_a(u_1) + \sigma(u_1) = d_a(u_2) + \sigma(u_2) \Rightarrow k_1 + c = k_2 + c \Rightarrow k_1 - k_2 = c - c = 0 \Rightarrow k_1 = k_2 \), which is a contradiction to \( k_1 \neq k_2 \). Hence \( G \) is a highly pseudo totally irregular fuzzy graph. Thus (i) \( \Rightarrow \) (ii) is proved.

Now, suppose \( G \) is a highly pseudo total irregular fuzzy graph. Then every vertex of \( G \) is adjacent to the vertices with distinct total pseudo degree. Let \( u_1 \) and \( u_2 \) be adjacent vertices of \( u_3 \) with \( d_a(u_1) = k_1 \) and \( d_a(u_2) = k_2 \). Now, \( td_a(u_1) \neq td_a(u_2) \Rightarrow d_a(u_1) + \sigma(u_1) \neq d_a(u_2) + \sigma(u_2) \Rightarrow k_1 + c \neq k_2 + c \Rightarrow k_1 \neq k_2 \). Therefore, every vertex of \( G \) is adjacent to the vertices with distinct pseudo degree. Hence \( G \) is a highly pseudo irregular fuzzy graph. Thus (ii) \( \Rightarrow \) (i) is proved. Hence (i) and (ii) are equivalent.

**Remark 4.15:** The converse of the above theorem need not be true.

**Example 4.16:** Consider a graph on \( G^*(V, E) \).

![Graph](image)

Now, \( d_a(u)=0.6, d_a(v)=0.466, d_a(w)=0.5, d_a(x)=0.566, d_a(y)=0.45 \) and \( td_a(u)=0.9, td_a(v)=1.166, td_a(w)=1, td_a(x)=0.766, td_a(y)=1.25 \). Here, every vertex of \( G \) is adjacent to the vertices with distinct pseudo degree and distinct total pseudo degree. Hence the graph is both highly pseudo irregular and highly pseudo totally irregular fuzzy graph. But \( \sigma \) is not a constant function.
Theorem 4.17: Every highly pseudo irregular fuzzy graph is pseudo irregular fuzzy graph.

Proof: Let $G$ be a highly pseudo irregular fuzzy graph. Then every vertex of $G$ is adjacent to the vertices with distinct pseudo degrees $\Rightarrow$ there is a vertex which is adjacent to the vertices with distinct pseudo degrees. Hence the graph $G$ is pseudo irregular fuzzy graph.

Theorem 4.18: Let $G: (\sigma, \mu)$ be a fuzzy graph on $G=(V,E)$, a cycle $C_n, n \geq 5$ vertices. If the membership values of the edges are $c_1, c_2, c_3, \ldots, c_n$ such that $c_1 < c_2 < c_3 < \ldots < c_n$. Then $G$ is highly pseudo irregular fuzzy graph and pseudo irregular fuzzy graph.

Proof: Let $G: (\sigma, \mu)$ be a fuzzy graph on a cycle $G=(V,E)$ of length $n$ and $n \geq 5$. Let $e_1, e_2, e_3, \ldots, e_n$ be the edges of the cycle $C_n$ in that order. Let the membership values of the edges $e_1, e_2, e_3, \ldots, e_n$ be $c_1, c_2, c_3, \ldots, c_n$ such that $c_1 < c_2 < c_3 < \ldots < c_n$.

Now, $d(v_i) = \begin{cases} c_n + c_1 & \text{if } i = 1 \\ c_{i-1} + c_1 & \text{if } i = 2, 3, \ldots, n \end{cases}$

$\Rightarrow d_a(v_i) = \begin{cases} \frac{d(v_2)+d(v_n)}{2} & \text{if } i = 1 \\ \frac{d(v_{i-1})+d(v_{i+1})}{2} & \text{if } i = 2, 3, \ldots, n-1 \\ \frac{d(v_{n-1})+d(v_1)}{2} & \text{if } i = n \end{cases}$

$\Rightarrow d_a(v_i) = \begin{cases} \frac{c_2+c_3+c_n+c_1}{2} & \text{if } i = 1 \\ \frac{c_n+c_{i-1}+c_{i+1}}{2} & \text{if } i = 2 \\ \frac{c_{i-2}+c_{i-1}+c_{i+1}+c_{n-1}}{2} & \text{if } i = 3, 4, \ldots, n-1 \\ \frac{c_1+c_n+c_{n-1}+c_{n-2}}{2} & \text{if } i = n \end{cases}$

Also since $c_1 < c_2 < c_3 < \ldots < c_n$, we have every vertex of $G$ is adjacent to the vertices with distinct pseudo degree. Hence the graph $G$ is highly pseudo irregular fuzzy graph and pseudo irregular fuzzy graph.

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