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SOME NOTIONS OF NEARLY OPEN SETS IN FUZZY TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce a new class of sets, namely fuzzy α^*-open sets and fuzzy α^*-closed sets. Further we define fuzzy α-interior and fuzzy α*-closure and discuss their properties. Finally, we relate fuzzy α*-open sets and fuzzy α*-closed sets with some other sets in fuzzy topological spaces.*

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Keywords: *fuzzy α*-open set, fuzzy α*-closed set, fuzzy α*-interior, fuzzy α*-closure.*

1 INTRODUCTION

Zadeh, in [7] introduced the concept of fuzzy sets. The study of fuzzy topology was introduced by Chang [4]. In 1991, A.S.Bin shahna [3] introduced α-open sets in fuzzy topological spaces. After Bin shahna's work, many mathematicians turned their attention to generalizing various concepts in fuzzy topology by considering fuzzy α-open sets instead of fuzzy open sets. The concept of fuzzy generalized closed sets was introduced by S.S.Thakur [6]. In this paper, we define a new class of sets, namely fuzzy α^*-open sets and fuzzy α*-closed sets. Further we define fuzzy α*-interior and fuzzy α*-closure and discuss their properties. Finally, we relate fuzzy α*-open sets and fuzzy α*-closed sets with some other sets in fuzzy topological spaces.

2. PRELIMINARIES

Throughout this paper X and Y denote fuzzy topological spaces (X, τ) and (Y, σ) on which no separation axioms are assumed. Let A be a subset of a space X. The closure of A and the interior of A are denoted by Cl(A) and Int(A) respectively. The following concepts are used in the sequel.

Definition 2.1: [3] A subset A of a fuzzy topological space (X, τ) is said to be a **fuzzy pre-open** if $A \leq Int(Cl(A))$ and a **fuzzy pre-closed** if $Cl(Int(A)) \leq A$.

Definition 2.2: [1] A subset A of a fuzzy topological space (X, τ) is said to be a **fuzzy semi-open** if $A \leq Cl(Int(A))$ and a **fuzzy semi-closed** if $Int(Cl(A)) \leq A$.

Definition 2.3: [3] A subset A of a fuzzy topological space (X, τ) is said to be a **fuzzy α-open** if $A \leq Int(Cl(Int(A)))$ and a **fuzzy α-closed** if $Cl(Int(Cl(A))) \leq A$.

Definition 2.4: [6] A subset A of a fuzzy topological space (X, τ) is said to be **fuzzy generalized closed** (briefly g-closed) if $Cl(A) \leq U$ whenever $A \leq U$ and U is fuzzy open in X.

Definition 2.5: [6] A subset A of a fuzzy topological space (X, τ) is said to be **fuzzy generalized open** (briefly g-open) if its complement is g-closed in X.

Definition 2.6: [2] Let A be a subset of a fuzzy topological space (X, τ), then the **fuzzy generalized closure** of A is defined as the intersection of all fuzzy g-closed sets in X containing A and is denoted by $Cl^*(A)$.

Definition 2.7: [2] Let A be a subset of a fuzzy topological space (X, τ), then the **fuzzy generalized interior** of A is defined as the union of all fuzzy g-open sets in X that are contained A and is denoted by $Int^*(A)$.

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Definition 2.8: [5] A subset A of a fuzzy topological space (X, τ) is said to be **fuzzy generalized α-closed** if αCl(A) ≤ U whenever A ≤ U and U is fuzzy α-open in (X, τ).

Definition 2.9: [5] A subset A of a fuzzy topological space (X, τ) is said to be **fuzzy α-generalized closed** if αCl(A) ≤ U whenever A ≤ U and U is fuzzy open in (X, τ).

The **fuzzy α-interior** [3] of a subset A of a fuzzy topological space (X, τ) is the union of all fuzzy open sets contained in A and is denoted by αInt(A). The **fuzzy semi-interior** [1] of A and **fuzzy pre-interior** [3] of A are analogously defined and that are respectively denoted by sInt(A) and pInt(A).

The fuzzy α-closure [3] of a subset A of a fuzzy topological space (X, τ) is the intersection of all fuzzy closed sets containing A and is denoted by αCl(A). The **fuzzy semi-closure** [1] of A and **fuzzy pre-closure** [3] of A are analogously defined and that are respectively denoted by sCl(A) and pCl(A).

3. FUZZY α*-OPEN SETS

Definition 3.1: A subset A of a fuzzy topological space (X, τ) is called **fuzzy** α***-open set** if A ≤ Int*(Cl(Int*(A))). The collection of all fuzzy α*-open sets in (X, τ) is denoted by α*O(X, τ).

Lemma 3.2: If there exists fuzzy g-open set V such that V ≤ A ≤ Int*(Cl(V)), then A is fuzzy α*-open.

Proof: Since V is fuzzy g-open, Int*(V) = V. Therefore, A ≤ Int*(Cl(V)) = Int*(Cl(Int*(V))) ≤ Int*(Cl(Int*(A))). Hence A is fuzzy α*-open.

Theorem 3.3: Every fuzzy open set is fuzzy α*-open.

Proof: Let A be a fuzzy open set in X. Every fuzzy open set is fuzzy α-open. Then
A ≤ Int(Cl(Int(A))) ≤ Int*(Cl(Int*(A))). Hence A is fuzzy α*-open.

Remark 3.4: The converse of the above theorem need not be true as seen from the following example.

Example 3.5: Let X = {a, b} and τ = {0, 1, α_1}. The fuzzy sets are defined as $\alpha_1(a) = 0.5$, $\alpha_1(b) = 0.4$, $\alpha_2(a) = 0.5$, $\alpha_2(b) = 0.6$, $\alpha_3(a) = 0.5$, $\alpha_3(b) = 0.5$. Clearly α_3 is fuzzy α*-open but not fuzzy open.

Theorem 3.6: Let {Aα} be a collection of fuzzy α*-open sets in a fuzzy topological space X. Then ∨Aα is fuzzy α*-open.

Proof: Since Aα is fuzzy α*-open for each α. Then Aα ≤ Int*(Cl(Int*(Aα))). This implies ∨Aα ≤ ∨(Int*(Cl(Int*(Aα)))) ≤ (Int*(∨Cl(Int*(Aα)))) ≤ (Int*(Cl(∨Int*(Aα)))) ≤ (Int*(Cl(Int*(∨Aα)))). Hence ∨Aα is fuzzy α*-open.

Remark 3.7: The intersection of two fuzzy α*-open sets need not be fuzzy α*-open is shown in the following example.

Example 3.8: Let X = {a, b} and τ = {0, 1, α_1}. The fuzzy sets are defined as $\alpha_1(a) = 0.5$, $\alpha_1(b) = 0.4$, $\alpha_2(a) = 0.4$, $\alpha_2(b) = 0.6$, $\alpha_3(a) = 0.5$, $\alpha_3(b) = 0.6$, $\alpha_4(a) = 0.6$, $\alpha_4(b)=0.4$, $\alpha_5(a) = 0.4$, $\alpha_5(b) = 0.4$. Clearly $\alpha_1$and α_2 are fuzzy α*-open sets but $\alpha_1 \wedge \alpha_2 = \alpha_5$ is not fuzzy α*-open.

Theorem 3.9: Every fuzzy α-open set is fuzzy α*-open.

Proof: Let A be a fuzzy α-open set. Then A ≤ Int(Cl(Int(A))) ≤ Int*(Cl(Int*(A))). Hence A is fuzzy α*-open.

Remark 3.10: The converse of the above theorem need not be true as seen from the following example.

Example 3.11: Let X = {a, b} and τ = {0, 1, α_1}. The fuzzy sets are defined as $\alpha_1(a) = 0.5$, $\alpha_1(b) = 0.4$, $\alpha_2(a) = 0.5$, $\alpha_2(b) = 0.6$, $\alpha_3(a) = 0.5$, $\alpha_3(b) = 0.5$. Clearly α_3 is fuzzy α*-open but not fuzzy α-open.

Theorem 3.12: Every fuzzy g-open set is fuzzy α*-open.

Proof: Let A be a fuzzy g-open set. Then Int*(A) = A. Therefore Int*(A) ≤ Cl(Int*(A)).

Then Int*(Int*(A)) ≤ Int*(Cl(Int*(A))) ⟹ Int*(A) ≤ Int*(Cl(Int*(A))) ⟹ Int*(A) = A ≤ Int*(Cl(Int*(A))). Hence A is fuzzy α*-open.

Remark 3.13: The converse of the above theorem need not be true as seen from the following example.

Example 3.14: Let X = {a, b} and $\tau = \{0, 1, \alpha_1\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.6$, $\alpha_2(a) = 0.5$, $\alpha_2(b) = 0.6$ and $\alpha_3(a) = 0.6$, $\alpha_3(b) = 0.4$. Clearly α_2 is fuzzy α*-open but not fuzzy g-open.

Theorem 3.15: If a subset A is fuzzy α*-open and B is fuzzy open, then A∨B is fuzzy α*-open.

Proof: Proof follows from theorem 3.3 and theorem 3.6.

Theorem 3.16: If a subset A is fuzzy α*-open and B is fuzzy α-open, then A ∨ B is fuzzy α*-open.

Proof: Proof follows from theorem 3.9 and theorem 3.6.

Theorem 3.17: If a subset A is fuzzy α*-open and B is fuzzy g-open, then A ∨ B is fuzzy α*-open.

Proof: Proof follows from theorem 3.12 and theorem 3.6.

Remark 3.18: The concept of fuzzy α*-open sets and fuzzy semi-open sets are independent as shown in the following examples.

Example 3.19: Let X = {a, b} and $\tau = \{0, 1, \alpha_2\}$. The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.6$, $\alpha_2(a) = 0.5$, $\alpha_2(b) = 0.6$, $\alpha_3(a) = 0.6$, $\alpha_3(b) = 0.4$. Clearly α_1 is fuzzy α*-open but not fuzzy semi-open.

Example 3.20: Let X = {a, b} and $\tau = \{0, 1, \alpha_1\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.5$, $\alpha_1(b) = 0.4$, $\alpha_2(a) = 0.5$, $\alpha_2(b) = 0.6$, $\alpha_3(a) = 0.5$, $\alpha_3(b) = 0.5$. Clearly α_2 is fuzzy semi open but not fuzzy α*-open.

Remark 3.21: The concept of fuzzy α*-open sets and fuzzy α-generalized open sets are independent as shown in the following examples.

Example 3.22: Let X = {a, b} and $\tau = \{0, 1, \alpha_2\}$. The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.6$, $\alpha_2(a) = 0.5$, $\alpha_2(b) = 0.6$, $\alpha_3(a) = 0.6$, $\alpha_3(b) = 0.4$. Clearly α_2 is fuzzy α*-open but not fuzzy α-generalized open.

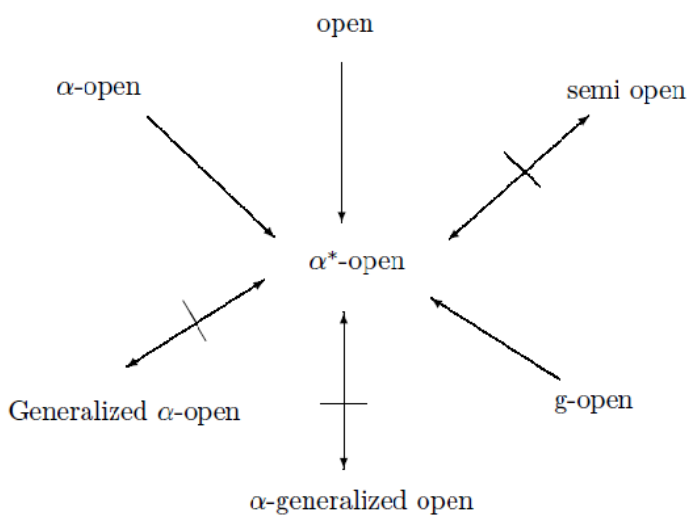
Example 3.23: Let X = {a, b} and $\tau = \{0, 1, \alpha_2\}$. The fuzzy sets are defined as $\alpha_1(a) = 0.3$, $\alpha_1(b) = 0.4$, $\alpha_1(c) = 0.4$, $\alpha_2(a) = 0.3$, $\alpha_2(b) = 0.6$, $\alpha_2(c) = 0.4$, $\alpha_3(a) = 0.7$, $\alpha_3(b) = 0.6$, $\alpha_3(c) = 0.6$. Clearly α_3 is fuzzy α-generalized open but not fuzzy α*-open.

Remark 3.24: The concept of fuzzy α*-open sets and fuzzy generalized α-open sets are independent as shown in the following examples.

Example 3.25: Let X = {a, b} and $\tau = \{0, 1, \alpha_2\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.6$, $\alpha_2(a) = 0.5$, $\alpha_2(b) = 0.6$, $\alpha_3(a) = 0.6$, $\alpha_3(b) = 0.4$. Clearly α_2 is fuzzy α*-open but not fuzzy generalized α-open.

Example 3.26: Let X = {a, b} and $\tau = \{0, 1, \alpha_2\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.3$, $\alpha_1(b) = 0.4$, $\alpha_1(c) = 0.4$, $\alpha_2(a) = 0.3$, $\alpha_2(b) = 0.6$, $\alpha_2(c) = 0.4$, $\alpha_3(a) = 0.7$, $\alpha_3(b) = 0.6$, $\alpha_3(c) = 0.6$. Clearly α_3 is fuzzy generalized α-open but not fuzzy α*-open.

Remark 3.27: From the above theorems and remarks, we have the following implication diagram.



Definition 3.28: Let A be a subset of a fuzzy topological space (X, τ), then **fuzzy α*-interior** of A is defined as the union of all fuzzy α*-open sets in X that are contained in A and is denoted by α*Int(A).

Theorem 3.29: If A is any subset of a fuzzy topological space (X, τ), α*Int(A) is fuzzy α*-open. Infact, α*Int(A) is the largest fuzzy α*-open set contained in A.

Proof: Proof follows from the definition 3.28 and theorem 3.3.

Theorem 3.30: Let A be a subset of a fuzzy topological space (X, τ). Then A is fuzzy α*-open if and only if α*Int(A) = A.

Proof: If A is fuzzy α*-open then α*Int(A) = A. Conversely, let α*Int(A) = A, by theorem 3.29, α*Int(A) is fuzzy α*-open. Hence A is fuzzy α*-open.

Theorem 3.31: Let A and B are subsets of a fuzzy topological space (X, τ), then the following conditions are hold:

a) $\alpha^*Int(\varphi) = \varphi$
b) $\alpha^*Int(X) = X$
c) $\alpha^*Int(A) \leq A$
d) If $A \leq B$, then $\alpha^*Int(A) \leq \alpha^*Int(B)$
e) $A \leq Int(A) \leq \alpha Int(A) \leq \alpha^*Int(A)$
f) $\alpha^*Int(A) \vee \alpha^*Int(B) \leq \alpha^*Int(A \vee B)$
g) $\alpha^*Int(A) \wedge \alpha^*Int(B) \geq \alpha^*Int(A \wedge B)$

Proof: a), b), c), d) follows from the definition 3.28 and e) follows from theorem 3.9. From d) $\alpha^*Int(A) \leq \alpha^*Int(A\vee B)$ and $\alpha^*Int(B) \leq \alpha^*Int(A\vee B)$.

$\Rightarrow \alpha^*Int(A) \vee \alpha^*Int(B) \leq \alpha^*Int(A \vee B)$. Hence f) follows.

Again from d) $\alpha^*Int(A) \geq \alpha^*Int(A \wedge B)$ and $\alpha^*Int(B) \geq \alpha^*Int(A \wedge B)$.

$\Rightarrow \alpha^*Int(A) \wedge \alpha^*Int(B) \geq \alpha^*Int(A \wedge B)$. Hence g) follows.

4. α *-CLOSED SETS

Definition 4.1: A subset A of a fuzzy topological space (X, τ) is called **fuzzy α*-closed set** if its complement is α*-open. The collection of all fuzzy α*-closed sets in (X, τ) is denoted by α*C(X, τ).

Lemma 4.2: If there exists an fuzzy g-closed set F such that $Cl^*(Int(F)) \leq A \leq F$, then A is fuzzy α*-closed.

Proof: Since F is fuzzy g-closed, $Cl^*(F) = F$. Therefore, $Cl^*(Int(Cl^*(A))) \leq Cl^*(Int(Cl^*(F))) = Cl^*(Int(F)) \leq A$. Hence A is fuzzy α*-closed.

Theorem 4.3: A subset A of a fuzzy topological space (X, τ) is fuzzy α*-closed if and only if $Cl^*(Int(Cl^*(A))) \leq A$.

Proof: Let A be a fuzzy α*-closed set. Then 1−A is fuzzy α_-open. By definition $1 - A \leq Int^*(Cl(Int^*(1 - A)))$. That is $1 - A \leq 1 - Cl^*(Int(Cl^*(A)))$. Hence $Cl^*(Int(Cl^*(A))) \leq A$. Conversely, suppose $Cl^*(Int(Cl^*(A))) \leq A$.

Then $1 - A \leq 1 - Cl^*(Int(Cl^*(A)))$. That is $1 - A \leq Int^*(Cl(Int^*(1 - A)))$. This shows that 1 − A is fuzzy α*-open. Then A is fuzzy α*-closed.

Theorem 4:4. If {Aα} is a collection of fuzzy α*-closed sets in fuzzy topological space (X, τ), then ∧Aα is fuzzy α*-closed.

Proof: Let Aα be a fuzzy α*-closed in X $\Rightarrow$ 1 − Aα is fuzzy α*-open in X $\Rightarrow$ By theorem 3.6, ∨(1 − Aα) is fuzzy α*-open in X $\Rightarrow$ 1 − ∧Aα is fuzzy α*-open in X. Hence ∧Aα is fuzzy α*-closed.

Remark 4.5: The union of fuzzy α*-closed sets need not be α*-closed as seen from the following example.

Example 4.6: Let X = {a, b} and $\tau = \{0, 1, \alpha_2, \alpha_5, \alpha_6\}$. The fuzzy sets are defined as $\alpha_1(a) = 0.3$, $\alpha_1(b) = 0.6$, $\alpha_2(a) = 0.4$, $\alpha_2(b) = 0.5$, $\alpha_3(a) = 0.6$, $\alpha_3(b) = 0.5$, $\alpha_4(a) = 0.6$, $\alpha_4(b) = 0.6$, $\alpha_5(a) = 0.6$, $\alpha_5(b) = 0.7$, $\alpha_6(a) = 0.6$, $\alpha_6(b) = 0.4$.

Clearly α_1 and α_3 are fuzzy α*-closed sets but $\alpha_1 \vee \alpha_3 = \alpha_4$ is not fuzzy α*-closed.

Theorem 4.7: Every fuzzy closed set is fuzzy α^*-closed

Proof: Let A be a fuzzy closed set in X. Then $1 - A$ is fuzzy open in X. By theorem 3.3, $1 - A$ is fuzzy α^*-open

$\Rightarrow$ A is fuzzy α^*-closed.

Remark 4.8: The converse of the above theorem need not be true as seen from the following example.

Example 4.9: Let X = {a, b} and $\tau = \{0, 1, \alpha_2\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.3$, $\alpha_1(b) = 0.4$, $\alpha_2(a) = 0.6$, $\alpha_2(b) = 0.5$. Clearly α_1 is fuzzy α^*-closed but not fuzzy closed.

Theorem 4.10: If a subset A of a fuzzy topological space X is fuzzy α^*-closed and B is fuzzy closed then $A \wedge B$ is fuzzy α^*-closed.

Proof: Proof follows from theorem 4.7 and theorem 4.4

Theorem 4.11: Every fuzzy α-closed set is fuzzy α^*-closed.

Proof: Let A be a fuzzy α-closed. Then $1-A$ is fuzzy α-open. By theorem 3.9, $1 - A$ is fuzzy α^*-open. Hence A is fuzzy α^*-closed.

Remark 4.12: The converse of the above theorem need not be true as seen from the following example.

Example 4.13: Let X = {a, b} and $\tau = \{0, 1, \alpha_1\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.6$, $\alpha_1(b) = 0.5$, $\alpha_2(a) = 0.7$, $\alpha_2(b) = 0.8$. Clearly α_2 is fuzzy α^*-closed but not fuzzy α-closed.

Theorem 4.14: Every fuzzy g-closed set is fuzzy α^*-closed.

Proof: Let A be a fuzzy g-closed set. Then $1-A$ is fuzzy g-open set. By theorem 3.12, $1 - A$ is fuzzy α^*-open

$\Rightarrow$ A is fuzzy α^*-closed.

Remark 4.15: The converse of the above theorem need not be true as seen from the following example.

Example 4.16: Let X = {a, b} and $\tau = \{0, 1, \alpha_3\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.6$, $\alpha_2(a) = 0.6$, $\alpha_2(b) = 0.5$, $\alpha_3(a) = 0.4$, $\alpha_3(b) = 0.5$. Clearly α_3 is fuzzy α^*-closed but not fuzzy g-closed.

Theorem 4.17: If a subset A of a fuzzy topological space X is fuzzy α^*-closed and B is fuzzy α-closed, then $A \wedge B$ is fuzzy α^*-closed.

Proof: Proof follows from theorem 4.11 and theorem 4.4 .

Theorem 4.18: If a subset A of a fuzzy topological space X is fuzzy α^*-closed and B is fuzzy g-closed, then $A \wedge B$ is fuzzy α^*-closed.

Proof: Proof follows from theorem 4.14 and theorem 4.4 .

Remark 4.19: The concept of fuzzy α^*-closed sets and fuzzy semi-closed sets are independent as shown in the following examples.

Example 4.20: Let X = {a, b} and $\tau = \{0, 1, \alpha_1\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.4$, $\alpha_2(a) = 0.1$, $\alpha_2(b) = 0.9$. Clearly α_2 is fuzzy α^*-closed but not fuzzy semi-closed.

Example 4.21: Let X = {a, b} and $\tau = \{0, 1, \alpha_1\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.4$, $\alpha_2(a) = 0.1$, $\alpha_2(b) = 0.9$. Clearly α_1 is fuzzy semi closed but not fuzzy α^*-closed.

Remark 4.22: The concept of fuzzy α^*-closed sets and fuzzy α-generalized closed sets are independent as shown in the following examples.

Example 4.23: Let X = {a, b} and $\tau = \{0, 1, \alpha_1\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.5$, $\alpha_1(c) = 0.6$, $\alpha_2(a) = 0.3$, $\alpha_2(b) = 0.3$, $\alpha_2(c) = 0.5$, $\alpha_3(a) = 0.7$, $\alpha_3(b) = 0.5$, $\alpha_3(c) = 0.5$. Clearly α_2 is fuzzy α^*-closed but not fuzzy α-generalized closed.

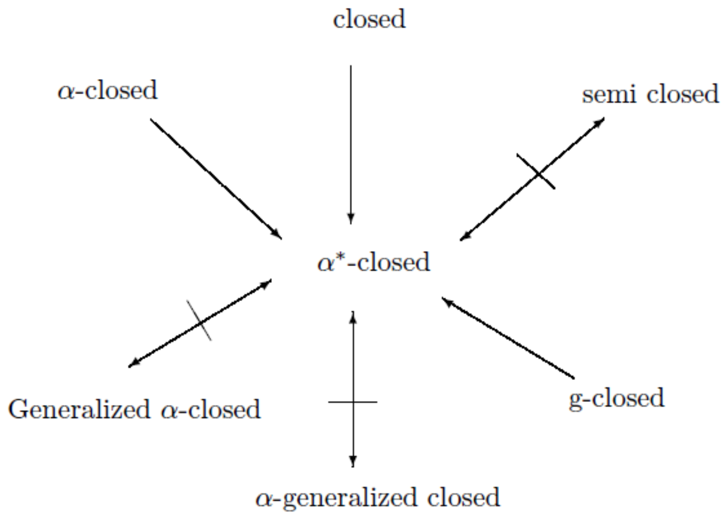
Example 4.24: Let X = {a, b} and $\tau = \{0, 1, \alpha_2\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.3$, $\alpha_1(b) = 0.4$, $\alpha_1(c) = 0.4$, $\alpha_2(a) = 0.3$, $\alpha_2(b) = 0.6$, $\alpha_2(c) = 0.4$, $\alpha_3(a) = 0.7$, $\alpha_3(b) = 0.6$, $\alpha_3(c) = 0.6$. Clearly α_1 is fuzzy α- generalized closed but not fuzzy α*-closed.

Remark 4.25: The concept of fuzzy α*-closed sets and fuzzy generalized α- closed sets are independent as shown in the following examples.

Example 4.26: Let X = {a, b} and $\tau = \{0, 1, \alpha_1\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.6$, $\alpha_1(b) = 0.5$, $\alpha_2(a) = 0.7$, $\alpha_2(b) = 0.8$. Clearly α_2 is fuzzy α*-closed but not fuzzy generalized α-closed.

Example 4.27: Let X = {a, b} and $\tau = \{0, 1, \alpha_2\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.3$, $\alpha_1(b) = 0.4$, $\alpha_1(c) = 0.4$, $\alpha_2(a) = 0.3$, $\alpha_2(b) = 0.6$, $\alpha_2(c) = 0.4$, $\alpha_3(a) = 0.7$, $\alpha_3(b) = 0.6$, $\alpha_3(c) = 0.6$. Clearly α_1 is fuzzy generalized α-closed but not fuzzy α*-closed.

Remark 4.28: From the above theorems and remarks, we have the following implication diagram.



Definition 4.29: Let A be a subset of a fuzzy topological space (X, τ). Then **fuzzy α*-closure** of A is defined as the intersection of all fuzzy α*-closed sets containing A and denoted by α*Cl(A).

Theorem 4.30: Let A be a subset of a fuzzy topological space (X, τ). Then A is fuzzy α*-closed if and only if α*Cl(A) = A.

Proof: Suppose A is fuzzy α*-closed. Then by definition 4.29, α*Cl(A) = A. Conversely, suppose α*Cl(A) = A. Then by theorem 4.4, A is fuzzy α*-closed.

Theorem 4.31: Let A and B are subsets of a fuzzy topological space (X, τ), then the following conditions are hold:

a) $\alpha^*Cl(\varphi) = \varphi$
b) $\alpha^*Cl(X) = X$
c) $A \le \alpha^*Cl(A)$
d) If $A \le B$, then $\alpha^*Cl(A) \le \alpha^*Cl(B)$
e) $A \le \alpha^*Cl(A) \le \alpha Cl(A) \le Cl(A)$
f) $\alpha^*Cl(A) \vee \alpha^*Cl(B) \le \alpha^*Cl(A \vee B)$
g) $\alpha^*Cl(A) \wedge \alpha^*Cl(B) \ge \alpha^*Cl(A \wedge B)$

Proof: a), b), c), d) follows from the definition 4.29 and e) follows from theorem 4.7.

From d) $\alpha^*Cl(A) \le \alpha^*Cl(A \vee B)$ and $\alpha^*Cl(B) \le \alpha^*Cl(A \vee B)$

$\Rightarrow \alpha^*Cl(A) \vee \alpha^*Cl(B) \le \alpha^*Cl(A \vee B)$. Hence f) follows.

Again from d) $\alpha^*Cl(A) \ge \alpha^*Cl(A \wedge B)$ and $\alpha^*Cl(B) \ge \alpha^*Cl(A \wedge B)$

$\Rightarrow \alpha^*Cl(A) \wedge \alpha^*Cl(B) \ge \alpha^*Cl(A \wedge B)$. Hence g) follows.

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