SOME NOTIONS OF NEARLY OPEN SETS IN FUZZY TOPOLOGICAL SPACES

N. SUDHA*, V. THIRIPURASUNDARI

*1M.Phil Scholar, 2PG and Research Department of Mathematics, Sri S. R. N. M. College, Sattur, Tamil Nadu, India.

(Received On: 10-03-16; Revised & Accepted On: 22-04-16)

ABSTRACT

In this paper, we introduce a new class of sets, namely fuzzy $\alpha^*$-open sets and fuzzy $\alpha^*$-closed sets. Further we define fuzzy $\alpha^*$-interior and fuzzy $\alpha^*$-closure and discuss their properties. Finally, we relate fuzzy $\alpha^*$-open sets and fuzzy $\alpha^*$-closed sets with some other sets in fuzzy topological spaces.

AMS Subject Classification (2010): 54C10, 54C08, 54C05.

Keywords: fuzzy $\alpha^*$-open set, fuzzy $\alpha^*$-closed set, fuzzy $\alpha^*$-interior, fuzzy $\alpha^*$-closure.

1 INTRODUCTION

Zadeh, in [7] introduced the concept of fuzzy sets. The study of fuzzy topology was introduced by Chang [4]. In 1991, A.S.Bin shahna [3] introduced $\alpha$-open sets in fuzzy topological spaces. After Bin shahna’s work, many mathematicians turned their attention to generalizing various concepts in fuzzy topology by considering fuzzy $\alpha$-open sets instead of fuzzy open sets. The concept of fuzzy generalized closed sets was introduced by S.S.Thakur [6]. In this paper, we define a new class of sets, namely fuzzy $\alpha^*$-open sets and fuzzy $\alpha^*$-closed sets. Further we define fuzzy $\alpha^*$-interior and fuzzy $\alpha^*$-closure and discuss their properties. Finally, we relate fuzzy $\alpha^*$-open sets and fuzzy $\alpha^*$-closed sets with some other sets in fuzzy topological spaces.

2. PRELIMINARIES

Throughout this paper X and Y denote fuzzy topological spaces $(X, \tau)$ and $(Y, \sigma)$ on which no separation axioms are assumed. Let $A$ be a subset of a space $X$. The closure of $A$ and the interior of $A$ are denoted by $\text{Cl}(A)$ and $\text{Int}(A)$ respectively. The following concepts are used in the sequel.

Definition 2.1: [3] A subset $A$ of a fuzzy topological space $(X, \tau)$ is said to be a fuzzy pre-open if $A \leq \text{Int}(\text{Cl}(A))$ and a fuzzy pre-closed if $\text{Cl}(\text{Int}(A)) \leq A$.

Definition 2.2: [1] A subset $A$ of a fuzzy topological space $(X, \tau)$ is said to be a fuzzy semi-open if $A \leq \text{Cl}(\text{Int}(A))$ and a fuzzy semi-closed if $\text{Int}(\text{Cl}(A)) \leq A$.

Definition 2.3: [3] A subset $A$ of a fuzzy topological space $(X, \tau)$ is said to be a fuzzy $\alpha$-open if $A \leq \text{Int}(\text{Cl}(\text{Int}(A)))$ and a fuzzy $\alpha$-closed if $\text{Cl}(\text{Int}(\text{Cl}(A))) \leq A$.

Definition 2.4: [6] A subset $A$ of a fuzzy topological space $(X, \tau)$ is said to be fuzzy generalized closed (briefly g-closed) if $\text{Cl}(A) \leq U$ whenever $A \leq U$ and U is fuzzy open in X.

Definition 2.5: [6] A subset $A$ of a fuzzy topological space $(X, \tau)$ is said to be fuzzy generalized open (briefly g-open) if its complement is g-closed in X.

Definition 2.6: [2] Let $A$ be a subset of a fuzzy topological space $(X, \tau)$, then the fuzzy generalized closure of $A$ is defined as the intersection of all fuzzy g-closed sets in X containing $A$ and is denoted by $\text{Cl}^*(A)$.

Definition 2.7: [2] Let $A$ be a subset of a fuzzy topological space $(X, \tau)$, then the fuzzy generalized interior of $A$ is defined as the union of all fuzzy g-open sets in X that are contained $A$ and is denoted by $\text{Int}^*(A)$.

Corresponding Author: N. Sudha*1,
Definition 2.8: [5] A subset A of a fuzzy topological space \((X, \tau)\) is said to be fuzzy generalized \(a\)-closed if \(aCl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is fuzzy \(a\)-open in \((X, \tau)\).

Definition 2.9: [5] A subset A of a fuzzy topological space \((X, \tau)\) is said to be fuzzy \(a\)-generalized closed if \(aCl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is fuzzy open in \((X, \tau)\).

The fuzzy \(a\)-interior [3] of a subset A of a fuzzy topological space \((X, \tau)\) is the union of all fuzzy open sets contained in A and is denoted by \(dInt(A)\). The fuzzy semi-interior [1] of A and fuzzy pre-interior [3] of A are analogously defined and that are respectively denoted by \(sInt(A)\) and \(pInt(A)\).

The fuzzy \(a\)-closure [3] of a subset A of a fuzzy topological space \((X, \tau)\) is the intersection of all fuzzy closed sets containing A and is denoted by \(a\overline{Cl}(A)\). The fuzzy semi-closure [1] of A and fuzzy pre-closure [3] of A are analogously defined and that are respectively denoted by \(sCl(A)\) and \(pCl(A)\).

3. FUZZY \(a\)*-OPEN SETS

Definition 3.1: A subset A of a fuzzy topological space \((X, \tau)\) is called fuzzy \(a\)*-open set if \(A \leq Int^*(Cl^*(A))\). The collection of all fuzzy \(a\)*-open sets in \((X, \tau)\) is denoted by \(a^*O(X, \tau)\).

Lemma 3.2: If there exists fuzzy g-open set V such that \(V \subseteq A \leq Int^*(Cl(V))\), then A is fuzzy \(a\)*-open.

Proof: Since V is fuzzy g-open, \(Int^*(V) = V\). Therefore, \(A \leq Int^*(Cl(V)) = Int^*(Cl(Int^*(V))) \leq Int^*(Cl(Int^*(A)))\). Hence A is fuzzy \(a\)*-open.

Theorem 3.3: Every fuzzy open set is fuzzy \(a\)*-open.

Proof: Let A be a fuzzy open set in X. Every fuzzy open set is fuzzy \(a\)-open. Then \(A \leq Int(Cl(Int(A))) \leq Int^*(Cl(Int^*(A)))\). Hence A is fuzzy \(a\)*-open.

Remark 3.4: The converse of the above theorem need not be true as seen from the following example.

Example 3.5: Let \(X = \{a, b\}\) and \(\tau = \{0, 1, 1a\}_1\). The fuzzy sets are defined as \(a_1(a) = 0.5, a_1(b) = 0.4, a_2(a) = 0.5, a_2(b) = 0.6, a_3(a) = 0.5, a_3(b) = 0.5\). Clearly \(a_3\) is fuzzy \(a\)*-open but not fuzzy open.

Theorem 3.6: Let \(\{A\alpha\}\) be a collection of fuzzy \(a\)*-open sets in a fuzzy topological space X. Then \(\forall A\alpha\) is fuzzy \(a\)*-open.

Proof: Since \(A\alpha\) is fuzzy \(a\)*-open for each \(\alpha\). Then \(A\alpha \leq Int^*(Cl(Int^*(A\alpha)))\). This implies \(\forall A\alpha \leq \forall (Int^*(Cl(Int^*(A\alpha)))) \leq (Int^*(\forall Cl(Int^*(A\alpha)))) \leq (Int^*(Cl(\forall Int^*(A\alpha))))\). Hence \(\forall A\alpha\) is fuzzy \(a\)*-open.

Remark 3.7: The intersection of two fuzzy \(a\)*-open sets need not be fuzzy \(a\)*-open is shown in the following example.

Example 3.8: Let \(X = \{a, b\}\) and \(\tau = \{0, 1, 1a\}_1\). The fuzzy sets are defined as \(a_1(a) = 0.5, a_1(b) = 0.4, a_2(a) = 0.4, a_2(b) = 0.6, a_3(a) = 0.5, a_3(b) = 0.6, a_4(b) = 0.4, a_4(a) = 0.4, a_4(b) = 0.4\). Clearly \(a_1\) and \(a_2\) are fuzzy \(a\)*-open sets but \(a_1 \land a_2 = a_3\) is not fuzzy \(a\)*-open.

Theorem 3.9: Every fuzzy \(a\)-open set is fuzzy \(a\)*-open.

Proof: Let A be a fuzzy \(a\)-open set. Then \(A \leq Int(Cl(Int(A))) \leq Int^*(Cl(Int^*(A)))\). Hence A is fuzzy \(a\)*-open.

Remark 3.10: The converse of the above theorem need not be true as seen from the following example.

Example 3.11: Let \(X = \{a, b\}\) and \(\tau = \{0, 1, 1a\}_1\). The fuzzy sets are defined as \(a_1(a) = 0.5, a_1(b) = 0.4, a_2(a) = 0.5, a_2(b) = 0.6, a_3(a) = 0.5, a_3(b) = 0.5\). Clearly \(a_3\) is fuzzy \(a\)*-open but not fuzzy \(a\)-open.

Theorem 3.12: Every fuzzy g-open set is fuzzy \(a\)*-open.

Proof: Let A be a fuzzy g-open set. Then \(Int^*(A) = A\). Therefore \(Int^*(A) \subseteq Cl(Int^*(A))\).

Then \(Int^*(Int^*(A)) \subseteq Int^*(Cl(Int^*(A))) \Rightarrow Int^*(A) \subseteq Int^*(Cl(Int^*(A))) \Rightarrow Int^*(A) = A \leq Int^*(Cl(Int^*(A)))\). Hence A is fuzzy \(a\)*-open.

Remark 3.13: The converse of the above theorem need not be true as seen from the following example.
Example 3.14: Let $X = \{a, b\}$ and $\tau = \{0, 1, \alpha_1\}$. The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.6$, $\alpha_2(a) = 0.5$, $\alpha_2(b) = 0.6$ and $\alpha_3(a) = 0.6$, $\alpha_3(b) = 0.4$. Clearly $\alpha_2$ is fuzzy $\alpha^*$-open but not fuzzy g-open.

Theorem 3.15: If a subset $A$ is fuzzy $\alpha^*$-open and $B$ is fuzzy open, then $A \cup B$ is fuzzy $\alpha^*$-open.

Proof: Proof follows from theorem 3.3 and theorem 3.6.

Theorem 3.16: If a subset $A$ is fuzzy $\alpha^*$-open and $B$ is fuzzy $\alpha$-open, then $A \cup B$ is fuzzy $\alpha^*$-open.

Proof: Proof follows from theorem 3.9 and theorem 3.6.

Theorem 3.17: If a subset $A$ is fuzzy $\alpha^*$-open and $B$ is fuzzy g-open, then $A \cup B$ is fuzzy $\alpha^*$-open.

Proof: Proof follows from theorem 3.12 and theorem 3.6.

Remark 3.18: The concept of fuzzy $\alpha^*$-open sets and fuzzy semi-open sets are independent as shown in the following examples.

Example 3.19: Let $X = \{a, b\}$ and $\tau = \{0, 1, \alpha_1\}$. The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.6$, $\alpha_2(a) = 0.5$, $\alpha_2(b) = 0.6$, $\alpha_3(a) = 0.6$, $\alpha_3(b) = 0.4$. Clearly $\alpha_1$ is fuzzy $\alpha^*$-open but not fuzzy semi-open.

Example 3.20: Let $X = \{a, b\}$ and $\tau = \{0, 1, \alpha_1\}$. The fuzzy sets are defined as $\alpha_1(a) = 0.5$, $\alpha_1(b) = 0.4$, $\alpha_2(a) = 0.5$, $\alpha_2(b) = 0.6$, $\alpha_3(a) = 0.5$, $\alpha_3(b) = 0.5$. Clearly $\alpha_2$ is fuzzy semi-open but not fuzzy $\alpha^*$-open.

Remark 3.21: The concept of fuzzy $\alpha^*$-open sets and fuzzy $\alpha$-generalized open sets are independent as shown in the following examples.

Example 3.22: Let $X = \{a, b\}$ and $\tau = \{0, 1, \alpha_1\}$. The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.6$, $\alpha_2(a) = 0.5$, $\alpha_2(b) = 0.6$, $\alpha_3(a) = 0.6$, $\alpha_3(b) = 0.4$. Clearly $\alpha_2$ is fuzzy $\alpha^*$-open but not fuzzy $\alpha$-generalized open.

Example 3.23: Let $X = \{a, b\}$ and $\tau = \{0, 1, \alpha_2\}$. The fuzzy sets are defined as $\alpha_1(a) = 0.3$, $\alpha_1(b) = 0.4$, $\alpha_2(a) = 0.3$, $\alpha_2(b) = 0.6$, $\alpha_3(a) = 0.4$, $\alpha_3(b) = 0.7$, $\alpha_3(c) = 0.6$. Clearly $\alpha_1$ is fuzzy $\alpha$-generalized open but not fuzzy $\alpha^*$-open.

Remark 3.24: The concept of fuzzy $\alpha^*$-open sets and fuzzy generalized $\alpha$-open sets are independent as shown in the following examples.

Example 3.25: Let $X = \{a, b\}$ and $\tau = \{0, 1, \alpha_2\}$. The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.6$, $\alpha_2(a) = 0.5$, $\alpha_2(b) = 0.6$, $\alpha_3(a) = 0.6$, $\alpha_3(b) = 0.4$. Clearly $\alpha_2$ is fuzzy $\alpha^*$-open but not fuzzy generalized $\alpha$-open.

Example 3.26: Let $X = \{a, b\}$ and $\tau = \{0, 1, \alpha_2\}$. The fuzzy sets are defined as $\alpha_1(a) = 0.3$, $\alpha_1(b) = 0.4$, $\alpha_1(c) = 0.4$, $\alpha_2(a) = 0.3$, $\alpha_2(b) = 0.6$, $\alpha_2(c) = 0.4$, $\alpha_3(a) = 0.7$, $\alpha_3(b) = 0.6$, $\alpha_3(c) = 0.6$. Clearly $\alpha_3$ is fuzzy generalized $\alpha$-open but not fuzzy $\alpha^*$-open.

Remark 3.27: From the above theorems and remarks, we have the following implication diagram.
Definition 3.28: Let A be a subset of a fuzzy topological space \((X, \tau)\), then **fuzzy \(\alpha^*\)-interior** of A is defined as the union of all fuzzy \(\alpha^*\)-open sets in X that are contained in A and is denoted by \(\alpha^*\text{Int}(A)\).

**Theorem 3.29:** If \(A\) is any subset of a fuzzy topological space \((X, \tau)\), \(\alpha^*\text{Int}(A)\) is fuzzy \(\alpha^*\)-open. Infact, \(\alpha^*\text{Int}(A)\) is the largest fuzzy \(\alpha^*\)-open set contained in A.

**Proof:** Proof follows from the definition 3.28 and theorem 3.3.

**Theorem 3.30:** Let A be a subset of a fuzzy topological space \((X, \tau)\). Then \(A\) is fuzzy \(\alpha^*\)-open if and only if \(\alpha^*\text{Int}(A) = A\).

**Proof:** If \(A\) is fuzzy \(\alpha^*\)-open then \(\alpha^*\text{Int}(A) = A\). Conversely, let \(\alpha^*\text{Int}(A) = A\), by theorem 3.29, \(\alpha^*\text{Int}(A)\) is fuzzy \(\alpha^*\)-open. Hence A is fuzzy \(\alpha^*\)-open.

**Theorem 3.31:** Let A and B are subsets of a fuzzy topological space \((X, \tau)\), then the following conditions are hold:

a) \(\alpha^*\text{Int}(\emptyset) = \emptyset\)

b) \(\alpha^*\text{Int}(X) = X\)

c) \(\alpha^*\text{Int}(A) \leq A\)

d) If \(A \leq B\), then \(\alpha^*\text{Int}(A) \leq \alpha^*\text{Int}(B)\)

e) \(A \leq \text{Int}(A) \leq \text{clInt}(A) \leq \alpha^*\text{Int}(A)\)

f) \(\alpha^*\text{Int}(A) \lor \alpha^*\text{Int}(B) \leq \alpha^*\text{Int}(A \lor B)\)

g) \(\alpha^*\text{Int}(A) \land \alpha^*\text{Int}(B) \geq \alpha^*\text{Int}(A \land B)\)

**Proof:** a), b), c), d) follows from the definition 3.28 and e) follows from theorem 3.9. From d) \(\alpha^*\text{Int}(A) \leq \alpha^*\text{Int}(AVB)\) and \(\alpha^*\text{Int}(B) \leq \alpha^*\text{Int}(AVB)\).

\[
\Rightarrow \alpha^*\text{Int}(A) \lor \alpha^*\text{Int}(B) \leq \alpha^*\text{Int}(A \lor B).
\]

Hence f) follows.

Again from d) \(\alpha^*\text{Int}(A) \geq \alpha^*\text{Int}(A \land B)\) and \(\alpha^*\text{Int}(B) \geq \alpha^*\text{Int}(A \land B)\).

\[
\Rightarrow \alpha^*\text{Int}(A) \land \alpha^*\text{Int}(B) \geq \alpha^*\text{Int}(A \land B).\]

Hence g) follows.

4. \(\alpha^*\)-CLOSED SETS

**Definition 4.1:** A subset A of a fuzzy topological space \((X, \tau)\) is called **fuzzy \(\alpha^*\)-closed set** if its complement is \(\alpha^*\)-open. The collection of all fuzzy \(\alpha^*\)-closed sets in \((X, \tau)\) is denoted by \(\alpha^*\text{C}(X, \tau)\).

**Lemma 4.2:** If there exists an fuzzy g-closed set F such that \(\text{Cl}*(\text{Int}(F)) \leq A \leq F\), then A is fuzzy \(\alpha^*\)-closed.

**Proof:** Since F is fuzzy g-closed, \(\text{Cl}*(\text{Int}(F)) = F\). Therefore, \(\text{Cl}*(\text{Int}(\text{Cl}*(A))) \leq \text{Cl}*(\text{Int}(\text{Cl}*(F))) = \text{Cl}*(\text{Int}(F)) \leq A\). Hence A is fuzzy \(\alpha^*\)-closed.

**Theorem 4.3:** A subset A of a fuzzy topological space \((X, \tau)\) is fuzzy \(\alpha^*\)-closed if and only if \(\text{Cl}*(\text{Int}(\text{Cl}*(A))) \leq A\).

**Proof:** Let A be a fuzzy \(\alpha^*\)-closed set. Then \(1 - A\) is fuzzy \(\alpha_\text{-open}\). By definition \(1 - A \leq \text{Int}(\text{Cl}*(1 - A))\). That is \(1 - A \leq 1 - \text{Cl}*(\text{Int}(*A))\). Hence \(\text{Cl}*(\text{Int}(\text{Cl}*(A))) \leq A\). Conversely, suppose \(\text{Cl}*(\text{Int}(\text{Cl}*(A))) \leq A\).

Then \(1 - A \leq 1 - \text{Cl}*(\text{Int}(\text{Cl}*(A)))\). That is \(1 - A \leq \text{Int}(\text{Cl}*(1 - A))\). This shows that \(1 - A\) is fuzzy \(\alpha^*\)-open. Then A is fuzzy \(\alpha^*\)-closed.

**Theorem 4.4:** If \(\{A\_\alpha\}\) is a collection of fuzzy \(\alpha^*\)-closed sets in fuzzy topological space \((X, \tau)\), then \(\Lambda\_\alpha\) is fuzzy \(\alpha^*\)-closed.

**Proof:** Let \(\Lambda\_\alpha\) be a fuzzy \(\alpha^*\)-closed in X \(\Rightarrow 1 - \Lambda\_\alpha\) is fuzzy \(\alpha^*\)-open in X \(\Rightarrow \) By theorem 3.6, \(\forall (1 - \Lambda\_\alpha)\) is fuzzy \(\alpha^*\)-open in X \(\Rightarrow \) 1 - \(\Lambda\_\alpha\) is fuzzy \(\alpha^*\)-open in X. Hence \(\Lambda\_\alpha\) is fuzzy \(\alpha^*\)-closed.

**Remark 4.5:** The union of fuzzy \(\alpha^*\)-closed sets need not be \(\alpha^*\)-closed as seen from the following example.

**Example 4.6:** Let X = \(\{a, b\}\) and \(\tau = \{0, 1, a_0, a_\alpha, a_{\alpha}\}\). The fuzzy sets are defined as \(a_0(a) = 0.3, a_0(b) = 0.6, a_\alpha(a) = 0.4, a_\alpha(b) = 0.5, a_{\alpha_\alpha}(a) = 0.6, a_{\alpha_\alpha}(b) = 0.6, a_{\alpha_\alpha}(a) = 0.6, a_{\alpha_\alpha}(b) = 0.7, a_{\alpha_\alpha}(a) = 0.6, a_{\alpha_\alpha}(b) = 0.4\).

Clearly \(a_\alpha\) and \(a_{\alpha_\alpha}\) are fuzzy \(\alpha^*\)-closed sets but \(a_\alpha \lor a_{\alpha_\alpha}\) is not fuzzy \(\alpha^*\)-closed.
Theorem 4.7: Every fuzzy closed set is fuzzy $\alpha^*$-closed

Proof: Let $A$ be a fuzzy closed set in $X$. Then $1 - A$ is fuzzy open in $X$. By theorem 3.3, $1 - A$ is fuzzy $\alpha^*$-open

$\Rightarrow A$ is fuzzy $\alpha^*$-closed.

Remark 4.8: The converse of the above theorem need not be true as seen from the following example.

Example 4.9: Let $X = \{a, b\}$ and $\tau = \{0, 1, \alpha_2\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.3$, $\alpha_1(b) = 0.4$, $\alpha_2(a) = 0.6$, $\alpha_2(b) = 0.5$. Clearly $\alpha_1$ is fuzzy $\alpha^*$-closed but not fuzzy closed.

Theorem 4.10: If a subset $A$ of a fuzzy topological space $X$ is fuzzy $\alpha^*$-closed and $B$ is fuzzy closed then $A \land B$ is fuzzy $\alpha^*$-closed

Proof: Proof follows from theorem 4.7 and theorem 4.4

Theorem 4.11: Every fuzzy $\alpha$-closed set is fuzzy $\alpha^*$-closed

Proof: Let $A$ be a fuzzy $\alpha$-closed. Then $1 - A$ is fuzzy $\alpha$-open. By theorem 3.9, $1 - A$ is fuzzy $\alpha^*$-open. Hence $A$ is fuzzy $\alpha^*$-closed.

Remark 4.12: The converse of the above theorem need not be true as seen from the following example.

Example 4.13: Let $X = \{a, b\}$ and $\tau = \{0, 1, \alpha_1\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.6$, $\alpha_1(b) = 0.5$, $\alpha_2(a) = 0.7$, $\alpha_2(b) = 0.8$. Clearly $\alpha_2$ is fuzzy $\alpha^*$-closed but not fuzzy $\alpha$-closed.

Theorem 4.14: Every fuzzy $g$-closed set is fuzzy $\alpha^*$-closed

Proof: Proof follows from theorem 4.11 and theorem 4.4

Theorem 4.17: If a subset $A$ of a fuzzy topological space $X$ is fuzzy $\alpha^*$-closed and $B$ is fuzzy $\alpha^*$-closed, then $A \land B$ is fuzzy $\alpha^*$-closed

Proof: Proof follows from theorem 4.11 and theorem 4.4

Remark 4.19: The concept of fuzzy $\alpha^*$-closed sets and fuzzy semi-closed sets are independent as shown in the following examples.

Example 4.20: Let $X = \{a, b\}$ and $\tau = \{0, 1, \alpha_1\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.5$, $\alpha_2(a) = 0.6$, $\alpha_2(b) = 0.5$. Clearly $\alpha_2$ is fuzzy $\alpha^*$-closed but not fuzzy semi-closed.

Example 4.21: Let $X = \{a, b\}$ and $\tau = \{0, 1, \alpha_1\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.4$, $\alpha_2(a) = 0.1$, $\alpha_2(b) = 0.9$. Clearly $\alpha_1$ is fuzzy semi closed but not fuzzy $\alpha^*$-closed.

Remark 4.22: The concept of fuzzy $\alpha^*$-closed sets and fuzzy $\alpha$-generalized closed sets are independent as shown in the following examples.

Example 4.23: Let $X = \{a, b\}$ and $\tau = \{0, 1, \alpha_1\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.5$, $\alpha_2(a) = 0.6$, $\alpha_2(b) = 0.3$, $\alpha_3(a) = 0.7$, $\alpha_3(b) = 0.5$. Clearly $\alpha_2$ is fuzzy $\alpha^*$-closed but not fuzzy $\alpha$-generalized closed.
Example 4.24: Let $X = \{a, b\}$ and $\tau = \{0, 1, \alpha_2\}$. The fuzzy sets are defined as $\alpha_1(a) = 0.3$, $\alpha_1(b) = 0.4$, $\alpha_1(c) = 0.4$, $\alpha_2(a) = 0.3$, $\alpha_2(b) = 0.6$, $\alpha_2(c) = 0.4$, $\alpha_3(a) = 0.7$, $\alpha_3(b) = 0.6$, $\alpha_3(c) = 0.6$. Clearly $\alpha_1$ is fuzzy $\alpha_*$-generalized closed but not fuzzy $\alpha^*$-closed.

Remark 4.25: The concept of fuzzy $\alpha^*$-closed sets and fuzzy generalized $\alpha_*$-closed sets are independent as shown in the following examples.

Example 4.26: Let $X = \{a, b\}$ and $\tau = \{0, 1, \alpha_2\}$. The fuzzy sets are defined as $\alpha_1(a) = 0.6$, $\alpha_1(b) = 0.5$, $\alpha_2(a) = 0.7$, $\alpha_2(b) = 0.8$. Clearly $\alpha_2$ is fuzzy $\alpha^*$-closed but not fuzzy generalized $\alpha_*$-closed.

Example 4.27: Let $X = \{a, b\}$ and $\tau = \{0, 1, \alpha_2\}$. The fuzzy sets are defined as $\alpha_1(a) = 0.3$, $\alpha_1(b) = 0.4$, $\alpha_1(c) = 0.4$, $\alpha_2(a) = 0.3$, $\alpha_2(b) = 0.6$, $\alpha_2(c) = 0.4$, $\alpha_3(a) = 0.7$, $\alpha_3(b) = 0.6$, $\alpha_3(c) = 0.6$. Clearly $\alpha_1$ is fuzzy generalized $\alpha_*$-closed but not fuzzy $\alpha^*$-closed.

Remark 4.28: From the above theorems and remarks, we have the following implication diagram.

**Definition 4.29:** Let $A$ be a subset of a fuzzy topological space $(X, \tau)$. Then **fuzzy $\alpha^*$-closure** of $A$ is defined as the intersection of all fuzzy $\alpha^*$-closed sets containing $A$ and denoted by $\alpha^*\text{Cl}(A)$.

**Theorem 4.30:** Let $A$ be a subset of a fuzzy topological space $(X, \tau)$. Then $A$ is fuzzy $\alpha^*$-closed if and only if $\alpha^*\text{Cl}(A) = A$.

**Proof:** Suppose $A$ is fuzzy $\alpha^*$-closed. Then by definition 4.29, $\alpha^*\text{Cl}(A) = A$. Conversely, suppose $\alpha^*\text{Cl}(A) = A$. Then by theorem 4.4, $A$ is fuzzy $\alpha^*$-closed.

**Theorem 4.31:** Let $A$ and $B$ are subsets of a fuzzy topological space $(X, \tau)$, then the following conditions are hold:

a) $\alpha^*\text{Cl}(\emptyset) = \emptyset$

b) $\alpha^*\text{Cl}(X) = X$

c) $A \leq \alpha^*\text{Cl}(A)$

d) If $A \leq B$, then $\alpha^*\text{Cl}(A) \leq \alpha^*\text{Cl}(B)$

e) $A \leq \alpha^*\text{Cl}(A) \leq \alpha\text{Cl}(A) \leq \text{Cl}(A)$

f) $\alpha^*\text{Cl}(A) \lor \alpha^*\text{Cl}(B) \leq \alpha^*\text{Cl}(A \lor B)$

g) $\alpha^*\text{Cl}(A) \land \alpha^*\text{Cl}(B) \geq \alpha^*\text{Cl}(A \land B)$

**Proof:** a), b), c), d) follows from the definition 4.29 and e) follows from theorem 4.7.

From d) $\alpha^*\text{Cl}(A) \leq \alpha^*\text{Cl}(A \lor B)$ and $\alpha^*\text{Cl}(B) \leq \alpha^*\text{Cl}(A \lor B)$

$\Rightarrow \alpha^*\text{Cl}(A) \lor \alpha^*\text{Cl}(B) \leq \alpha^*\text{Cl}(A \lor B)$. Hence f) follows.

Again from d) $\alpha^*\text{Cl}(A) \geq \alpha^*\text{Cl}(A \land B)$ and $\alpha^*\text{Cl}(B) \geq \alpha^*\text{Cl}(A \land B)$

$\Rightarrow \alpha^*\text{Cl}(A) \land \alpha^*\text{Cl}(B) \geq \alpha^*\text{Cl}(A \land B)$. Hence g) follows.
REFERENCES


Source of support: Nil, Conflict of interest: None Declared

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