COMMUTATIVITY OF ASSOCIATIVE RINGS WITH $(X, Y^2) - (Y^2, X)$, $YX^2Y = XY^2X$

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ABSTRACT

In this paper we have mainly focused on some theorems related to commutativity of associative and non associative rings. We prove that if $R$ is an associative ring with unity satisfying $(x, y^2) - (y^2, x)$, $\forall x, y \in R$, $n \geq 2$ and $xy^3 = y^2xy$ $\forall x, y \in R$, $n \geq 2$. Then $R$ is commutative ring and also I have mainly obtained two principles for a non associative ring to be a commutative ring.

Key words: Ring with unity, Associative ring, Non-associative ring.

INTRODUCTION

The object of this note to investigate the commutativity of the associative and non associative rings satisfying condition '.' Such that $y(yx) = y(xy)$ $\forall x, y \in R$ and $(yx)x = (xy)x$ $\forall x, y \in R$.

PRELIMINARIES

Definition:

(i) A non empty set $R$ together with two binary operations $+$ and $.$ is said to be a ring (Associative ring) if $(R, +)$ is an abelian group and $(R, .)$ is a semi group satisfying distributive laws.

(ii) In a ring $R$ if there exists an element ‘1’ in $R$ such that $a.1 = 1.a = a$ for all $a \in R$ then $R$ is said to be a ring with unity.

Theorem 1: $R$ is an Associative Ring with unity 1 then $R$ is Commutative such that $(x, y^2) - (y^2, x)$ belongs to $z(R)$.

Proof: Given that $xy^2 = y^2x$ 
Replacing $y$ by $y+1$, $x(y+1)^2 = (y+1)^2x \in Z(R)$ 
$x(y^2+2y+1) = (y^2+2y+1)x \in Z(R)$ 
$xy^2+2xy+x = y^2x+2yx+x \in Z(R)$ 
$2xy = 2yx \forall x, y \in Z(R)$ 
$yx = xy \forall x, y \in R$ 
Hence $R$ is Commutative ring.

Theorem 2: Let $R$ is an Associative Ring with unity and $R$ is Commutative then $xy^3 = y^2xy$.

Proof: Given that $xy^3 = y^2xy$ 
Replacing $y$ by $y+1$, $x(y+1)^3 = (y+1)^2x (y+1) \in Z(R)$ 
$x(y+1)(y^2+2y+1) = (y^2+2y+1)(xy+x)$ 
$(xy+y)(y^2+2y+1) = y^2xy+2xy+y \in Z(R)$ 
$xy^3+2xy^2+xy+xy^2+2xy+x = y^2xy+2xy+y \in Z(R)$ 
$2xy = 2yx \ [from \ 1, xy^2 = y^2x]$ 
$yx = xy \forall x, y \in R$ 
Hence $R$ is Commutative ring.

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Theorem 3: Let R be a prime ring with $yx^2y = xy^2x$ in $Z(R)$ for every $x, y$ in R. Then R is a commutative ring.

Proof: Given that, $yx^2y = xy^2x$
Replacing $x$ by $x+1$,
$y(x+1)y = (x+1)(y^2)(x+1) \epsilon Z(R)$
$y(x^2+2x+1)y = (xy^2+y^2)(x+1) \epsilon Z(R)$
$yx^2y + 2yxy + y^2 = xy^2x + xy^2 + y^2 + y^2$
$2yxy = xy^2 + yx$
(From the theorem $y^nx = y^{n-1}xy$)
$yxy = xy^2 \epsilon Z(R)$
Replacing $y$ by $y+1$,
$(y+1)(xy+x) = x(y^2+2y+1)$
We get,
$yxy + yx + xy + x = xy^2 + 2xy + x$
yx = xy \forall x, y, \epsilon R$
Hence R is Commutative ring.

Theorem 4: If R is an Associative Ring with unity 1 then R is Commutative if and only if $x^3yx = x^4y$ for all $x, y$ belongs to R.

Proof: Given that, $x^3yx = x^4y$
Replacing $x$ by $x+1$,
$(x+1)^3y(x+1) = (x+1)^4y \epsilon Z(R)$
$(x^3+3x^2y+x^2y)(x+1) = (x^4+4x^3y+6x^2y+4xy^2+y^2)(x+1) \epsilon Z(R)$
$(x^3y+3x^2xy+3xy^2+y^2)(x+1) = (x^4y+4x^3y+6x^2y+4xy^2+y^2)
3x^3y+3x^2y+yx = 3x^4y+3x^3y+xy$
(by the theorem, $x^ny = x^{n-1}yx$)
yx = xy \forall x, y, \epsilon R$
Hence R is Commutative ring.

Theorem 5: If R is ring with unity 1 satisfying $[(xy)^2 - xy, x] = 0$ then R is commutative.

Proof: Given that, $[(xy)^2 - xy, x] = 0$
$[(xy)^2 - xy]x = x[(xy)^2 - xy]$
Replacing $x$ by $x+1$, $[(x+1)y]y(x+1) = (x+1)[(xy)^2 - xy] \epsilon Z(R)$
$[(xy)^2 - xy]y(x+1) = (x+1) [(xy)^2 - xy]$
$[(xy)^2 - xy]y(x+1) = (x+1) [(xy + xy) - (xy + y)]$
$(xy)(xy) + y(xy)x - yx = x(xy) + (xy) (xy) - yx \epsilon Z(R)$
Replacing $x$ by $x+1$,
$[(x+1)y]y(x+1) = (x+1)(xy+y)((x+1)y+y) - (x+1)y$
$(xy)^2 + yxy = (xy)^2 + y(xy)$
By replacing $y$ by $y+1$, we get $(xy)^2 + yxy = (xy)^2 + yxy \epsilon Z(R)$
yx = xy \forall x, y, \epsilon R$
Hence R is Commutative ring.

Theorem 6: If R be a non-associative ring with unity 1 satisfying $y(yx) = y(xy)$ for all $x, y$ belongs to R then R is commutative.

Proof: Given that, $y(yx) = y(xy)$
Replacing $y$ by $y+1$, $(y+1)[xy+x] = (y+1) [xy+x] \epsilon Z(R)$
y(xy) + xy + xy + x = y(xy) + xy + xy + x
yx = xy \forall x, y, \epsilon R$
Hence R is Commutative ring.

Theorem 7: If R be a non-associative ring with unity 1 satisfying $(yx)x = (xy)x$ for all $x, y$ belongs to R then R is commutative.
Proof: Given that, \((yx)x = (xy)x\)

Replacing \(x\) by \(x+1\), \([y(x+1)](x+1) = [(x+1)y](x+1) \in \mathbb{Z}(R)\)

\((yx+y)(x+1) = (xy+y)(x+1) \in \mathbb{Z}(R)\)

\((yx)x+yx+yx+y) = (xy)x+xy+yx+y \in \mathbb{Z}(R)\)

\(yx = xy \quad \forall x, y, \in \mathbb{R}\)

Hence \(R\) is Commutative ring

REFERENCES


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