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A FUZZY TYPE DETERIORATION INVENTORY MODEL WITH STOCK DEPENDENT SELLING RATE AND PARTIAL BACKLOGGING

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ABSTRACT

In the present paper we considered an inventory model for deteriorating items with stock dependent selling rate and fuzzy type deterioration. Shortages are allowed and completely backlogged. The backlogging rate of unsatisfied demand is assumed to be a decreasing exponential function of waiting time. The purpose of our study is to defuzzify the total profit function by signed distance method and comparing the results of this method with the crisp model. Further a numerical example is also given to demonstrate the developed model and to show the sensitivity analysis of the effect of change of parameters.

INTRODUCTION

In some consumer goods it has been observed that the demand of some items is influenced by the amount of on hand inventory stock. The demand of such goods may be increase or decrease according as on hand inventory stock increases or decreases. This situation arise in a super market where is a lot of stock of goods. The effect of deterioration cannot be avoided in any business organization and deterioration is defined by a process in which the stocked items loose part of their value with passage of time.

In past few years some researchers considered the time dependent demand rate because the demand of newly launched products such as fashionable garments, electronic items, mobile phones increases with time and later it becomes constant. In some real life situations the customers suffer the problem of shortage because there are some customers who wait for the next replenishment while the others do not wait for replenishment and go elsewhere as waiting time increases, but in the recent years some researchers gave their attention towards the stock dependent demand rate.

In some consumer goods it has been observed that the demand of some items is influenced by the amount of on hand inventory stock. The demand of such goods may be increase or decrease according as on hand inventory stock increases or decreases. This situation arises in a super market, when there is a lot of stock of goods. In this area the work has been done by the following researchers. Baker and Urban [1] proposed a deterministic inventory model for deteriorating items with stock level dependent demand rate. Mandal and Phaujdar [2] presented an inventory model for deteriorating items with stock level dependent consumption rate. Vrat and Padmanabhan [3] developed an EOQ model for perishable products with stock dependent selling rate. Mandal and Maiti [4] proposed an inventory model for damageable products with stock dependent demand and variable replenishment rate. Dye and Ouyang [5] developed an EOQ model for perishable items with stock dependent selling rate by allowing shortages. Kao and Hsu [6] developed a single period inventory model with fuzzy demand. Hsieh [7] proposed an inventory model with fuzzy production rate. Yao and Chiang [8] developed an inventory model without backorders and defuzzified the fuzzy holding cost by signed distance method and centroid method. Hou [9] presented an inventory model for deteriorating items with stock dependent consumption rate and shortages. He was also used inflation and time discounting in his inventory model. Sujit *et al.* [10] presented an economic production quantity model with fuzzy type demand rate and deterioration rate. Syed and Aziz [11] considered a fuzzy inventory model without shortages. Valliaththal and Uthayakumar [12] presented an EOQ model for perishable products with stock dependent selling rate and shortages. Roy *et al.* [13] developed an inventory model for deteriorating items with stock dependent demand. He was also considered the fuzzy inflation rate and time discounting over a random planning horizon. Sana [14] proposed a lot size inventory model with time varying deterioration rate and stock dependent demand by allowing shortages. Chang *et al.* [15] determined an optimal replenishment policy for an inventory model of non-instantaneous deteriorating items with stock dependent demand. De and Rawat [16] developed a fuzzy inventory model for deteriorating items without shortages. Nagrare and Dutta [17] developed a continuous review inventory model for perishable products with inventory dependent demand. Sana [18] proposed a control policy for a production system with stock dependent demand.

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The figure 1show the developed model and the figures 2 and 3 show the graphs of total profit function with respect to deterioration parameter and selling rate parameter. The tables 2 and 3 show the variation of total profit function with respect to the parameters θ and β.

In the present paper we consider an inventory model for deteriorating items with stock dependent demand rate. Shortages are allowed and completely backlogged. The backlogging rate of unsatisfied demand is assumed as a decreasing exponential function of waiting time. The purpose of our study is to defuzzify the total profit function by signed distance method and comparing the results of this method with the crisp model. A numerical example is also given to demonstrate the developed model and to shoe the sensitivity analysis of the parameters.

DEFINITIONS AND PRELIMINARIES

When we are considering the fuzzy inventory model then the following definitions are needed.

Definition: A fuzzy set $\tilde{A}$ on the given universal set X is denoted and defined by

$$\tilde{A} = \{(x, \lambda_{\tilde{A}}(x)) : x \in X\}$$

where, $\lambda_{\tilde{A}} : X \to [0,\ 1]$, is called the membership function,

and, $\lambda_{\tilde{A}}(x)$ = degree of x in $\tilde{A}$

Definition: A fuzzy number $\tilde{A}$ is a fuzzy set on the real line R, if its membership function $\lambda_{\tilde{A}}$ has the following properties

1. $\lambda_{\tilde{A}}(x)$ is upper semi continuous.
2. $\lambda_{\tilde{A}}(x)$ =0, outside some interval $[a_1, a_4]$
3. $\exists$ real numbers a_2 and a_3, $a_1 \le a_2 \le a_3 \le a_4$ such that $\lambda_{\tilde{A}}(x)$ is increasing on $[a_1, a_2]$, decreasing on $[a_3, a_4]$ and $\lambda_{\tilde{A}}(x)$ =1, for each x in $[a_2, a_3]$

Definition: A triangular fuzzy number is specified by the triplet (a_1, a_2, a_3) where $a_1 \prec a_2 \prec a_3$ and defined by its continuous membership function $\lambda_{\tilde{A}} : X \to [0,\ 1]$ as follows

$$\lambda_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3 \\ 0, & otherwise \end{cases}$$

Definition: Let $\tilde{A}$ be a fuzzy set defined on R, then the signed distance of $\tilde{A}$ is defined as

$$d\left(\tilde{A}, 0\right) = \frac{1}{2}\int_0^1 \left[A_L(\alpha) + A_R(\beta)\right] d_\alpha$$

where, $A_\alpha = [A_L(\alpha) + A_R(\beta)] = \left[a + (b-a)\alpha,\ d - (d-c)\alpha\right],\ \alpha \in [0,1]$ is an α cut of a fuzzy set $\tilde{A}$.

Definition: If $\tilde{A} = (a, b, c)$ is a triangular fuzzy number then the signed distance of $\tilde{A}$ is defined as

$$d\left(\tilde{A}, 0\right) = \frac{1}{4}(a + 2b + c).$$

Assumptions and Notations- We consider the following assumptions and notations

1. The demand rate is $R(t) = \alpha + \beta I(t),$ where α is a positive constant and β is the stock dependent selling rate parameter, $0 \le \beta \le 1$
2. θ is the deterioration parameter.
3. δ is the backlogging parameter.
4. A is the ordering cost per order.
5. h_c is the holding cost per unit per unit time.
6. s_c is the shortages cost per unit per unit time.
7. c is the purchase cost per unit.
8. p is the selling price per unit, where.
9. c_3 is the opportunity cost per unit due to lost sales.
10. $\tilde{\theta}$ is the fuzzy deterioration parameter.
11. T is the length of order cycle.
12. T_1 is the time at which shortage starts.
13. $TP(T_1, T)$ is the total profit per unit time.
14. $\tilde{TP}(T_1, T)$ is the total fuzzy profit per unit time.
15. $I(t)$ is the inventory level at any time in $[0, T]$.
16. The time horizon T is infinite.
17. The lead time is zero.
18. The replenishment rate is infinite.

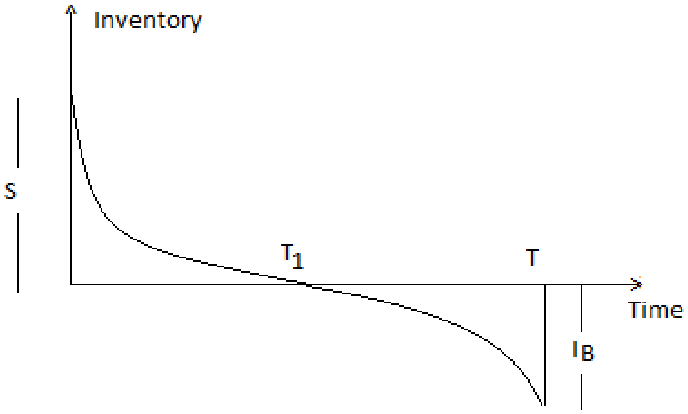

Figure-1

CRISP MODEL

Mathematical Formulation- Suppose an inventory system consists S units of the product in the beginning of each cycle. Due to demand and deterioration the inventory level decreases in $[0, T_1]$ and becomes zero at $t = T_1$. The interval $[T_1, T]$ is the shortages interval. During the shortages interval $[T_1, T]$ the unsatisfied demand is backlogged at a rate of $e^{-\delta t}$, where δ is the backlogging parameter and t is the waiting time.

The instantaneous inventory level at any time t in [0, T] are governed by the following differential equations

$$\frac{dI}{dt} + \theta I = -(\alpha + \beta I), \qquad 0 \le t \le T_1 \tag{1}$$

with boundary condition I(0) = S

$$\frac{dI}{dt} = -\alpha e^{-\delta t}, \qquad T_1 \le t \le T \tag{2}$$

with boundary condition $I(T_1) = 0$

The solution of equation (1) is

$$I = \frac{\alpha}{(\theta+\beta)}[e^{(\theta+\beta)t} - 1] + Se^{(\theta+\beta)t}$$

$$I = S + \{\alpha + S(\theta+\beta)\}t + (\theta+\beta)\{\alpha + S(\theta+\beta)\}\frac{t^2}{2} \tag{3}$$

The solution of equation (2) is

$$I = \frac{\alpha}{\delta}[e^{-\delta(T-T_1)} - e^{-\delta(T-t)}]$$

$$I = \alpha(T_1 - t) + \frac{\alpha\delta}{2}(T_1^2 - t^2 - 2TT_1 + 2Tt) \tag{4}$$

The ordering cost per cycle is

$$O_C = \frac{A}{T} \tag{5}$$

The holding cost per cycle is

$$H_C = \frac{h_c}{T}\int_0^{T_1} I(t)\,dt$$

$$H_C = \frac{h_c}{T}\int_0^{T_1}\left[S + \{\alpha + S(\theta+\beta)\}t + (\theta+\beta)\{\alpha + S(\theta+\beta)\}\frac{t^2}{2}\right]$$

$$H_C = \frac{h_c}{T}\left[ST_1 + \{\alpha + S(\theta+\beta)\}\frac{T_1^2}{2} + (\theta+\beta)\{\alpha + S(\theta+\beta)\}\frac{T_1^3}{6}\right] \tag{6}$$

The shortage cost per cycle is

$$S_C = \frac{s_c}{T}\int_{T_1}^{T} -I(t)dt$$

$$S_C = -\frac{s_c}{T}\left[\alpha\delta\left(\frac{T^3}{3} - \frac{T_1^3}{3} + TT_1^2 - T_1T^2\right) + \alpha TT_1 - \frac{\alpha T^2}{2} - \frac{\alpha T_1^2}{2}\right] \tag{7}$$

Due to lost sales the opportunity cost per cycle in $[T_1, T]$ is

$$O_{PC} = \frac{\alpha c_3}{T}\int_{T_1}^{T}[1 - e^{-\delta(T-t)}]dt$$

$$O_{PC} = \frac{\alpha\delta c_3}{2}[T^2 + T_1^2 - 2TT_1 + \frac{\delta}{3}(T^3 - T_1^3 - 3T_1T^2 + 3TT_1^2)] \tag{8}$$

The purchase cost per cycle in $[T_1, T]$ is

$$P_C = cS + c \times back\,ordered\,quantity$$

$$P_C = c[S + \alpha(T_1 - T) + \frac{\alpha\delta}{2}(T_1^2 - T^2 - 2TT_1 + 2T^2] \tag{9}$$

The sales revenue per cycle is

$$S_R = p[\int_0^{T_1}(demand)\,dt + \int_{T_1}^{T}(demand)dt]$$

$$S_R = p\beta\left[S\,T_1 + \{\alpha+S(\theta+\beta)\}\frac{T_1^2}{2}+(\theta+\beta)\{\alpha+S(\theta+\beta)\}\frac{T_1^3}{6}\right]+p\alpha T-\frac{p\alpha\delta}{2}(T-T_1)^2+\frac{p\alpha\delta^2}{6}(T-T_1)^3 \tag{10}$$

Therefore the total profit per unit time is

$$TP(T_1,T)=\frac{1}{T}[S_R-O_C-H_C-S_C-O_{PC}-P_C]$$

$$\begin{aligned}TP(T_1,T)=\frac{1}{T}[&-(A+cS)+(p+c)\alpha T+\{p\beta S-Sh_c-\alpha c\}T_1+\{\alpha s_C-\alpha\delta p-\alpha\delta c_3-\alpha\delta c\}\frac{T^2}{2}\\
&+\{p\beta(\alpha+S(\theta+\beta))-\alpha\delta p+\alpha s_c-h_c(\alpha+S(\theta+\beta))-\alpha\delta c_3-\alpha\delta c\}\frac{T_1^2}{2}\\
&+\{\alpha\delta p-\alpha s_c+\alpha\delta c_3+\alpha\delta c\}TT_1+\{\alpha\delta^2 p-2\alpha\delta s_c-\alpha\delta^2 c_3\}\frac{T^3}{6}\\
&+\{p\beta(\theta+\beta)(\alpha+S(\theta+\beta))-\alpha\delta^2 p+2\alpha\delta s_c-h_c(\theta+\beta)(\alpha+S(\theta+\beta))+\alpha\delta^2 c_3\}\frac{T_1^3}{6}\\
&+\{2\alpha\delta s_c-\alpha\delta^2 p+\alpha\delta^2 c_3\}\frac{T^2T_1}{2}+\{\alpha\delta^2 p-2\alpha\delta s_c-\alpha\delta^2 c_3\}\frac{TT_1^2}{2}]\end{aligned} \tag{11}$$

The necessary condition for $TP(T_1,T)$ to be maximum is that

$\frac{\partial TP(T_1,T)}{\partial T_1}=0$ *and* $\frac{\partial TP(T_1,T)}{\partial T}=0$, on solving these two equations we find the optimum values of T_1 *and* T say T_1^* *and* T^* for which profit is maximum and the sufficient condition is

$$\left(\frac{\partial^2 TP(T_1,T)}{\partial T_1^2}\right)\left(\frac{\partial^2 TP(T_1,T)}{\partial T^2}\right)-\left\{\frac{\partial^2 TP(T_1,T)}{\partial T_1\partial T}\right\}^2\succ 0 \text{ and } \left(\frac{\partial^2 TP(T_1,T)}{\partial T_1^2}\right)\prec 0 \tag{12}$$

$$\begin{aligned}\frac{\partial TP(T_1,T)}{\partial T_1}=&(p\beta S-Sh_c-\alpha c)+\{p\beta(\alpha+S(\theta+\beta))-\alpha\delta p+\alpha s_c-h_c(\alpha+S(\theta+\beta))-\alpha\delta c_3\\
&-\alpha\delta c\}T_1+\{\alpha\delta p-\alpha s_c+\alpha\delta c+\alpha\delta c_3\}T+\{p\beta(\theta+\beta)(\alpha+S(\theta+\beta))-\alpha\delta^2 p\\
&+2\alpha\delta s_c-h_c(\theta+\beta)(\alpha+S(\theta+\beta))+\alpha\delta^2 c_3\}\frac{T_1^2}{2}+\{2\alpha\delta s_c\\
&-\alpha\delta^2 p+\alpha\delta^2 c_3\}\frac{T^2}{2}+\{\alpha\delta^2 p-2\alpha\delta s_c-\alpha\delta^2 c_3\}TT_1]\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\partial^2 TP(T_1,T)}{\partial T_1^2}=\frac{1}{T}[&\{p\beta(\alpha+S(\theta+\beta))-\alpha\delta p+\alpha s_c-\alpha\delta c-\alpha\delta c_3-h_c(\alpha+S(\theta+\beta))\}\\
&+\{p\beta(\theta+\beta)(\alpha+S(\theta+\beta))-\alpha\delta^2 p+2\alpha\delta s_c-h_c(\theta+\beta)(\alpha+S(\theta+\beta))\\
&+\alpha\delta^2 c_3\}T_1+\{\alpha\delta^2 p-2\alpha\delta s_c-\alpha\delta^2 c_3\}T]\end{aligned} \tag{14}$$

$$\begin{aligned}\frac{\partial TP}{\partial T}=\frac{1}{T}[&(p+c)\alpha+\{\alpha s_c-\alpha p\delta-\alpha\delta c_3-\alpha\delta c\}T+\{\alpha p\delta-\alpha s_c+\alpha\delta c_3+\alpha\delta c\}T_1\\
&+\{\alpha\delta^2 p-2\alpha\delta s_c-\alpha\delta^2 c_3\}\frac{T^2}{2}+\{2\alpha\delta s_c-\alpha\delta^2 p+\alpha\delta^2 c_3\}TT_1\end{aligned}$$

$$+\{\alpha\delta^2 p - 2\alpha\delta s_c - \alpha\delta^2 c_3\}\frac{T_1^2}{2}] - \frac{1}{T^2}[-(A+CS) + (p+C)\alpha T + \{p\beta S - Sh_c - \alpha c\}T_1$$
$$+\{\alpha s_c - \alpha\delta p - \alpha\delta c_3 - \alpha\delta c\}\frac{T^2}{2} + \{p\beta(\alpha + S(\theta+\beta)) - \alpha\delta p + \alpha s_c - \alpha\delta c_3$$
$$-\alpha\delta c - h_c(\alpha + S(\theta+\beta))\}\frac{T_1^2}{2} + \{\alpha\delta p - \alpha s_c + \alpha\delta c_3 + \alpha\delta c\}TT_1$$
$$+\{\alpha\delta^2 p - 2\alpha\delta s_c - \alpha\delta^2 c_3\}\frac{T^3}{6} + \{p\beta(\theta+\beta)(\alpha + S(\theta+\beta)) - \alpha\delta^2 p + 2\alpha\delta s_c$$
$$-h_c(\theta+\beta)(\alpha + S(\theta+\beta)) - \alpha\delta^2 c_3\}\frac{T_1^3}{6} + \{2\alpha\delta s_c - \alpha\delta^2 p + \alpha\delta^2 c_3\}\frac{T^2 T_1}{2}$$
$$+\{\alpha\delta^2 p - 2\alpha\delta s_c - \alpha\delta^2 c_3\}\frac{TT_1^2}{2}] \tag{15}$$

$$\frac{\partial^2 TP}{\partial T^2} = \frac{1}{T}[\{\alpha s_c - \alpha\delta p - \alpha\delta c_3 - \alpha\delta c\} + \{\alpha\delta^2 p - 2\alpha\delta s_c - \alpha\delta^2 c_3\}T + \{2\alpha\delta s_c - \alpha\delta^2 p + \alpha\delta^2 c_3\}T_1]$$
$$-\frac{1}{T^2}[(p+C)\alpha + \{\alpha s_c - \alpha\delta p - \alpha\delta c_3 - \alpha\delta c\}T + \{\alpha\delta p - \alpha s_c + \alpha\delta c_3 + \alpha\delta c\}T_1$$
$$+\{\alpha\delta^2 p - 2\alpha\delta s_c - \alpha\delta^2 c_3\}\frac{T^2}{2} + \{2\alpha\delta s_c - \alpha\delta^2 p + \alpha\delta^2 c_3\}TT_1 + \{\alpha\delta^2 p - 2\alpha\delta s_c$$
$$-\alpha\delta^2 c_3\}\frac{T_1^2}{2}] + \frac{2}{T^3}[-(A+CS) + (p+C)\alpha T + \{p\beta S - Sh_c - \alpha c\}T_1 + \{\alpha s_c - \alpha\delta p$$
$$-\alpha\delta c_3 - \alpha\delta c\}\frac{T^2}{2} + \{p\beta(\alpha + S(\theta+\beta)) - \alpha\delta p + \alpha\delta c - \alpha\delta c_3 - \alpha\delta c$$
$$-h_c(\alpha + S(\theta+\beta))\}\frac{T_1^2}{2} + \{p\beta(\theta+\beta)(\alpha + S(\theta+\beta)) - \alpha\delta^2 p + 2\alpha\delta s_c$$
$$-h_c(\theta+\beta)(\alpha + S(\theta+\beta)) - \alpha\delta^2 c_3\}\frac{T_1^3}{6} + \{2\alpha\delta s_c - \alpha\delta^2 p + \alpha\delta^2 c_3\}\frac{T^2 T_1}{2}$$
$$+\{\alpha\delta^2 p - 2\alpha\delta s_c - \alpha\delta^2 c_3\}\frac{TT_1^2}{2}] \tag{16}$$

$$\frac{\partial^2 TP}{\partial T \partial T_1} = \frac{1}{T}[(\alpha\delta p - \alpha s_c + \alpha\delta c_3 + \alpha\delta c) + (2\alpha\delta s_c - \alpha\delta^2 p + \alpha\delta^2 c_3)T + \{\alpha\delta^2 p - 2\alpha\delta s_c$$
$$-\alpha\delta^2 c_3\}T_1] - \frac{1}{T^2}[\{p\beta S - Sh_c - \alpha c\} + \{p\beta(\alpha + S(\theta+\beta)) - \alpha\delta p + \alpha s_c - \alpha\delta c_3$$
$$-\alpha\delta c - h_c(\alpha + S(\theta+\beta))\}T_1 + \{\alpha\delta p - \alpha s_c + \alpha\delta c_3 + \alpha\delta c\}T$$
$$+\{p\beta(\theta+\beta)(\alpha + S(\theta+\beta)) - \alpha\delta^2 p + 2\alpha\delta s_c - h_c(\theta+\beta)(\alpha + S(\theta+\beta))$$
$$-\alpha\delta^2 c_3\}\frac{T_1^2}{2} + \{2\alpha\delta s_c - \alpha\delta^2 p + \alpha\delta^2 c_3\}\frac{T^2}{2} + \{\alpha\delta^2 p - 2\alpha\delta s_c - \alpha\delta^2 c_3\}TT_1] \tag{17}$$

FUZZY MODEL

Let us consider the inventory model in fuzzy environment due to uncertainty in parameter θ let us assume that the parameter θ may change within some limits.

Let $\theta = (\theta_1, \theta_2, \theta_3)$ be a triangular fuzzy numbers then the total profit per unit time in fuzzy sense is

$$\tilde{TP}(T_1,T) = \frac{1}{T}[-(A+cS)+(p+c)\alpha T+\{p\beta S - Sh_c - \alpha c\}T_1 + \{\alpha s_c - \alpha\delta p - \alpha\delta c_3 - \alpha\delta c\}\frac{T^2}{2}$$

$$+\{p\beta(\alpha+S(\tilde{\theta}+\beta)) - \alpha\delta p + \alpha s_c - h_c(\alpha+S(\tilde{\theta}+\beta)) - \alpha\delta c_3 - \alpha\delta c\}\frac{T_1^2}{2}$$

$$+\{\alpha\delta p - \alpha s_c + \alpha\delta c_3 - \alpha\delta c\}TT_1 + \{\alpha\delta^2 p - 2\alpha\delta s_c - \alpha\delta^2 c_3\}\frac{T^3}{6}$$

$$+\{p\beta(\tilde{\theta}+\beta)(\alpha+S(\tilde{\theta}+\beta)) - \alpha\delta^2 p + 2\alpha\delta s_c - h_c(\tilde{\theta}+\beta)(\alpha+S(\tilde{\theta}+\beta)) + \alpha\delta^2 c_3\}\frac{T_1^3}{6}$$

$$+\{2\alpha\delta s_c - \alpha\delta^2 p + \alpha\delta^2 c_3\}\frac{T^2T_1}{2} + \{\alpha\delta^2 p - 2\alpha\delta s_c - \alpha\delta^2 c_3\}\frac{TT_1^2}{2}] \quad (18)$$

Now we defuzzify the total profit $\tilde{TP}(T_1,T)$ by signed distance method

SIGNED DISTANCE METHOD

By signed distance method the total profit per unit time is

$$\tilde{TP}(T_1,T) = \frac{1}{4T}[\tilde{TP}_1(T_1,T) + 2\tilde{TP}_2(T_1,T) + \tilde{TP}_3(T_1,T)]$$

where

$$\tilde{TP}_1(T_1,T) = \frac{1}{T}[-(A+cS)+(p+c)\alpha T+\{p\beta S - Sh_c - \alpha c\}T_1 + \{\alpha s_c - \alpha\delta p - \alpha\delta c_3 - \alpha\delta c\}\frac{T^2}{2}$$

$$+\{p\beta(\alpha+S(\tilde{\theta}_1+\beta)) - \alpha\delta p + \alpha s_c - h_c(\alpha+S(\tilde{\theta}_1+\beta)) - \alpha\delta c_3 - \alpha\delta c\}\frac{T_1^2}{2}$$

$$+\{\alpha\delta p - \alpha s_c + \alpha\delta c_3 + \alpha\delta c\}TT_1 + \{\alpha\delta^2 p - 2\alpha\delta s_c - \alpha\delta^2 c_3\}\frac{T^3}{6}$$

$$+\{p\beta(\tilde{\theta}_1+\beta)(\alpha+S(\tilde{\theta}_1+\beta)) - \alpha\delta^2 p + 2\alpha\delta s_c - h_c(\tilde{\theta}_1+\beta)(\alpha+S(\tilde{\theta}_1+\beta)) + \alpha\delta^2 c_3\}\frac{T_1^3}{6}$$

$$+\{2\alpha\delta s_c - \alpha\delta^2 p + \alpha\delta^2 c_3\}\frac{T^2T_1}{2} + \{\alpha\delta^2 p - 2\alpha\delta s_c - \alpha\delta^2 c_3\}\frac{TT_1^2}{2}]$$

$$\tilde{TP}_2(T_1,T) = \frac{1}{T}[-(A+cS)+(p+c)\alpha T+\{p\beta S - Sh_c - \alpha c\}T_1 + \{\alpha s_c - \alpha\delta p - \alpha\delta c_3 - \alpha\delta c\}\frac{T^2}{2}$$

$$+\{p\beta(\alpha+S(\tilde{\theta}_2+\beta)) - \alpha\delta p + \alpha s_c - h_c(\alpha+S(\tilde{\theta}_2+\beta)) - \alpha\delta c_3 - \alpha\delta c\}\frac{T_1^2}{2}$$

$$+\{\alpha\delta p - \alpha s_c + \alpha\delta c_3 + \alpha\delta c\}TT_1 + \{\alpha\delta^2 p - 2\alpha\delta s_c - \alpha\delta^2 c_3\}\frac{T^3}{6}$$

$$+\{p\beta(\tilde{\theta}_2+\beta)(\alpha+S(\tilde{\theta}_2+\beta)) - \alpha\delta^2 p + 2\alpha\delta s_c - h_c(\tilde{\theta}_2+\beta)(\alpha+S(\tilde{\theta}_2+\beta)) + \alpha\delta^2 c_3\}\frac{T_1^3}{6}$$

$$+\{2\alpha\delta s_c - \alpha\delta^2 p + \alpha\delta^2 c_3\}\frac{T^2T_1}{2} + \{\alpha\delta^2 p - 2\alpha\delta s_c - \alpha\delta^2 c_3\}\frac{TT_1^2}{2}]$$

$$\tilde{TP}_3(T_1,T) = \frac{1}{T}[-(A+cS)+(p+c)\alpha T+\{p\beta S - Sh_c - \alpha c\}T_1 + \{\alpha s_c - \alpha\delta p - \alpha\delta c_3 - \alpha\delta c\}\frac{T^2}{2}$$

$$+\{p\beta(\alpha+S(\tilde{\theta}_3+\beta)) - \alpha\delta p + \alpha s_c - h_c(\alpha+S(\tilde{\theta}_3+\beta)) - \alpha\delta c_3 - \alpha\delta c\}\frac{T_1^2}{2}$$

$$+\{\alpha\delta p-\alpha s_c+\alpha\delta c_3+\alpha\delta c\}TT_1+\{\alpha\delta^2 p-2\alpha\delta s_c-\alpha\delta^2 c_3\}\frac{T^3}{6}$$
$$+\{p\beta(\tilde{\theta}_3+\beta)(\alpha+S(\tilde{\theta}_3+\beta))-\alpha\delta^2 p+2\alpha\delta s_c-h_c(\tilde{\theta}_3+\beta)(\alpha+S(\tilde{\theta}_3+\beta))+\alpha\delta^2 c_3\}\frac{T_1^3}{6}$$
$$+\{2\alpha\delta s_c-\alpha\delta^2 p+\alpha\delta^2 c_3\}\frac{T^2 T_1}{2}+\{\alpha\delta^2 p-2\alpha\delta s_c-\alpha\delta^2 c_3\}\frac{TT_1^2}{2}]$$

Therefore by the signed distance method the total profit per unit time is

$$\begin{aligned}\tilde{TP}(T_1,T)=\frac{1}{4T}[&-4(A+cS)+4\alpha(p+c)T+4\{p\beta S-Sh_C-\alpha c\}T_1+2\{\alpha s_C-\alpha\delta p-\alpha\delta c_3\\&-\alpha\delta c\}T^2+2\{p\alpha\beta-\alpha\delta p+\alpha s_C-\alpha h_C-\alpha\delta c_3-\alpha\delta c\}T_1^2+\{p\beta S(\theta_1+2\theta_2+\theta_3+4\beta)\\&-Sh_C(\theta_1+2\theta_2+\theta_3+4\beta)\}\frac{T_1^2}{2}+4\{\alpha\delta p-\alpha s_C+\alpha\delta c_3+\alpha\delta c\}TT_1+\{\alpha\delta^2 p-2\alpha\delta s_C\\&-\alpha\delta^2 c_3\}\frac{2T^3}{3}+\{2\alpha\delta s_C+\alpha\delta^2 c_3-\alpha\delta^2 p\}\frac{2T_1^3}{3}+\{p\alpha\beta(\theta_1+2\theta_2+\theta_3+4\beta)\\&+p\beta S(\theta_1^2+2\theta_2^2+\theta_3^2+4\beta^2+2\theta_1\beta+4\theta_2\beta+2\theta_3\beta)-\alpha h_C(\theta_1+2\theta_2+\theta_3+4\beta)\\&-Sh_C(\theta_1^2+2\theta_2^2+\theta_3^2+4\beta^2+2\theta_1\beta+4\theta_2\beta+2\theta_3\beta)\}\frac{T_1^3}{6}+2\{2\alpha\delta s_C-\alpha\delta^2 p\\&+\alpha\delta^2 c_3\}T_1T^2+2\{\alpha\delta^2 p-2\alpha\delta s_C-\alpha\delta^2 c_3\}TT_1^2]\end{aligned} \tag{19}$$

The necessary condition for $\tilde{TP}(T_1,T)$ to be maximum is that

$\frac{\partial \tilde{TP}(T_1,T)}{\partial T_1}=0$ *and* $\frac{\partial \tilde{TP}(T_1,T)}{\partial T}=0$, and solving these equations we find the optimum values of T_1 *and* T say T_1^* *and* T^* for which profit is maximum and the sufficient condition is

$$\left(\frac{\partial^2\tilde{TP}(T_1,T)}{\partial T_1^2}\right)\left(\frac{\partial^2\tilde{TP}(T_1,T)}{\partial T^2}\right)-\left\{\frac{\partial^2\tilde{TP}(T_1,T)}{\partial T_1\partial T}\right\}^2\succ 0 \ and \left(\frac{\partial^2\tilde{TP}(T_1,T)}{\partial T_1^2}\right)\prec 0$$

$$\begin{aligned}\frac{\partial\tilde{TP}(T_1,T)}{\partial T_1}=\frac{1}{4T}[&4(p\beta S-Sh_C-\alpha c)+4(p\alpha\beta-\alpha\delta p+\alpha s_C-\alpha h_C-\alpha\delta c_3-\alpha\delta c)T_1\\&+\{p\beta S(\theta_1+2\theta_2+\theta_3+4\beta)-Sh_C(\theta_1+2\theta_2+\theta_3+4\beta)\}T_1+4(\alpha\delta p-\alpha s_C\\&+\alpha\delta c_3+\alpha\delta c)T+2(2\alpha\delta s_C+\alpha\delta^2 c_3-\alpha\delta^2 p)T_1^2+\{p\alpha\beta(\theta_1+2\theta_2+\theta_3+4\beta)\\&+p\beta S(\theta_1^2+2\theta_2^2+\theta_3^2+4\beta^2+2\theta_1\beta+4\theta_2\beta+2\theta_3\beta)-\alpha h_C(\theta_1+2\theta_2+\theta_3+4\beta)\\&-Sh_C(\theta_1^2+2\theta_2^2+\theta_3^2+4\beta^2+2\theta_1\beta+4\theta_2\beta+2\theta_3\beta)\}\frac{T_1^2}{2}+2(2\alpha\delta s_C+\alpha\delta^2 c_3\\&-\alpha\delta^2 p)T^2+4(\alpha\delta^2 p-2\alpha\delta s_C-\alpha\delta^2 c_3)TT_1]\end{aligned} \tag{20}$$

$$\begin{aligned}\frac{\partial\tilde{TP}(T_1,T)}{\partial T}=\frac{1}{4T}[&4\alpha(p+c)+4(\alpha s_C-\alpha\delta p-\alpha\delta c_3-\alpha\delta c)T+4(\alpha\delta p-\alpha s_C+\alpha\delta c_3+\alpha\delta c)T_1\\&+2(\alpha\delta^2 p-2\alpha\delta s_C-\alpha\delta^2 c_3)T^2+4(2\alpha\delta s_C-\alpha\delta^2 p+\alpha\delta^2 c_3)TT_1+2(\alpha\delta^2 p\\&-2\alpha\delta s_C-\alpha\delta^2 c_3)T_1^2]-\frac{1}{4T^2}[-4(A+cS)+4\alpha(p+c)T+4(p\beta S-Sh_C-\alpha c)T_1\end{aligned}$$

$$+2(\alpha s_C-\alpha\delta p-\alpha\delta c_3-\alpha\delta C)T^2+2\{p\alpha\beta-\alpha\delta p+\alpha s_C-\alpha h_C-\alpha\delta c_3-\alpha\delta c\}T_1^2$$
$$+\{p\beta S(\theta_1+2\theta_2+\theta_3+4\beta)-Sh_C(\theta_1+2\theta_2+\theta_3+4\beta)\}\frac{T_1^2}{2}+4(\alpha\delta p-\alpha s_C+\alpha\delta c_3$$
$$+\alpha\delta c)TT_1+(\alpha\delta^2p-2\alpha\delta s_C-\alpha\delta^2c_3)\frac{2T^3}{3}+(2\alpha\delta s_C+\alpha\delta^2c_3-\alpha\delta^2p)\frac{2T_1^3}{3}$$
$$+\{p\alpha\beta(\theta_1+2\theta_2+\theta_3+4\beta)+p\beta S(\theta_1^2+2\theta_2^2+\theta_3^2+4\beta^2+2\theta_1\beta+4\theta_2\beta+2\theta_3\beta)$$
$$-\alpha h_C(\theta_1+2\theta_2+\theta_3+4\beta)-Sh_C(\theta_1^2+2\theta_2^2+\theta_3^2+4\beta^2+2\theta_1\beta+4\theta_2\beta+2\theta_3\beta)\}\frac{T_1^3}{6}$$
$$+2(2\alpha\delta s_C-\alpha\delta^2p+\alpha\delta^2c_3)T_1T^2+2(\alpha\delta^2p-2\alpha\delta s_C-\alpha\delta^2c_3)TT_1^2 \tag{21}$$

Numerical example-Let us consider the following parameters in appropriate units
$\alpha=200,\ \beta=0.5,\ \theta=0.005,\ \delta=2,\ A=100,\ h_c=3,\ s_c=4,\ c_3=5,\ c=6,\ p=10,\ S=500$

Table-1: (variation of total profit with respect to the parameter θ)

θ	T_1	T	TP
0.005	36.3938	818.2250	7.5046×10^7
0.010	36.3942	818.3942	7.5050×10^7
0.015	36.3945	818.2710	7.5055×10^7
0.020	36.3949	818.2940	7.5059×10^7

As we increase the parameter θ then the values of the parameters T_1, T and TP increases.

Table-2: (variation of total profit with respect to the parameter β)

β	T_1	T	TP
0.5	36.3938	818.2250	7.5046×10^7
1.0	36.6280	832.8690	8.1140×10^7
2.0	38.1901	938.5680	1.0084×10^8
3.0	41.1326	1174.5800	1.6250×10^8

As we increase the parameter β then the values of the parameters, T_1, T and TP increases.

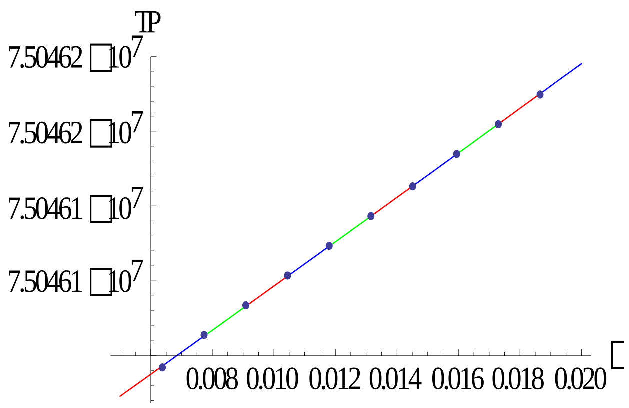

Figure-2: variation of θ

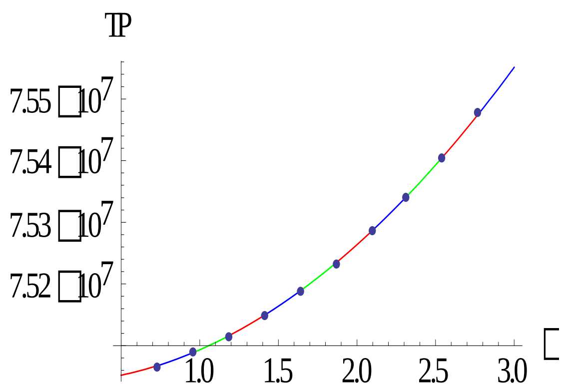

Figure-3: variation of β

Fuzzy Model- Let $\theta = (0.005,\ 0.010,\ 0.020)$ be a triangular fuzzy number then the solution of the fuzzy inventory model by signed distance method is

$\theta = (\theta_1, \theta_2, \theta_3)$	T_1	T	$\tilde{TP}(T_1, T)$
$\theta = (0.005,\ 0.010,\ 0.020)$	12.6106	13.4230	$6.88343 * 10^{12}$
$\theta = (0.030,\ 0.060,\ 0.120)$	10.8719	11.5681	$3.18618 * 10^{12}$

CONCLUSION

In this paper we considered an inventory model for deteriorating items with stock dependent selling rate. Shortages are allowed and completely backlogged. The backlogging rate of unsatisfied demand is assumed to be a decreasing exponential function of waiting time. From the tables 1 and 2 we see that as we increase the deterioration parameter θ, selling rate parameters β and the cycle time, time at which shortages start increases and total profit is also increases. Thus the parameter β is more sensitive than the parameter θ in the crisp model and in the fuzzy sense the total profit decreases as we increase the deterioration parameter. In future this model can be generalized by considering time dependent deterioration, holding and shortage cost.

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