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CHROMATIC WEAKLY CONVEX DOMINATION OF SOME FAMILIES OF GRAPH

E. HELENA*[a], R. POOVAZHAKI[b]

[a]Department of Mathematics, Fatima College, Madurai, India.

[b]Department of Mathematics, E.M.G Yadava Women's College, Madurai, India.

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ABSTRACT

A chromatic weakly convex dominating set is defined as follows: A Weakly convex dominating set D of a graph $G = (V, E)$ is said to be a Chromatic weakly convex dominating set if $\chi(G) = \chi(< D >)$. A Chromatic weakly convex domination number $\gamma_{cwc}(G)$ of G is the minimum cardinality of a Chromatic weakly convex dominating set. This parameter was introduced in [5]. This paper further extends the study by obtaining the Chromatic weakly convex domination number of central, middle and total graph of complete bipartite graph.

Keywords: *Central graph, Middle graph, Total graph, Chromatic number, Chromatic Weakly Convex dominating set.*

Mathematics Subject Classification: *05C69, 05C15.*

1. INTRODUCTION

By a graph $G = (V, E)$ we mean a connected, finite, non-trivial, undirected graph with neither loops nor multiple edges. For graph theoretic terminology, we refer to Chartrand and Lesniak [1].

The central graph of G, denoted by $C(G)$ is obtained by subdividing each edge of G exactly once and joining all the non adjacent vertices of G in $C(G)$.

The middle graph of G, denoted by $M(G)$ is defined as follows: The vertex set of $M(G)$ is $V(M(G)) = V(G) \cup E(G)$. Two vertices $u, v \in V(M(G))$ are adjacent in $M(G)$ if either (i) $u, v \in E(G)$ and u, v are adjacent in G or (ii) $u \in V(G), v \in E(G)$ and u, v are incident in G.

The total graph of G, denoted by $T(G)$ is defined as follows: The vertex set of $T(G)$ is $V(T(G)) = V(G) \cup E(G)$. Two vertices $u, v \in V(T(G))$ are adjacent in $T(G)$ if either (i) $u, v \in V(G)$ and u, v are adjacent in G or (ii) $u, v \in E(G)$ and u, v are adjacent in G or (iii) $u \in V(G), v \in E(G)$ and u, v are incident in G.

A dominating set D of a graph $G = (V, E)$ is said to be a weakly convex dominating set if for every $u, v \in D$ there exists a u-v shortest path of G entirely contained in $< D >$. A coloring of a graph G is an assignment of colors to the vertices of G in such a way that no two adjacent vertices receive the same color. The minimum number of colors needed for coloring a graph G is called the chromatic number and is denoted by $\chi(G)$. A weakly convex dominating set D of a graph $G = (V, E)$ is said to be a Chromatic weakly convex dominating set if $\chi(G) = \chi(< D >)$. A Chromatic weakly convex domination number $\gamma_{cwc}(G)$ of G is the minimum cardinality of a Chromatic weakly convex dominating set. The notion of Chromatic weakly convex domination was introduced in [5]. In this paper we extend the study of this notion.

Corresponding Author: E. Helena*[a]
[a]Department of Mathematics, Fatima College, Madurai, India.

2. CHROMATIC WEAKLY CONVEX DOMINATION NUMBER OF CENTRAL GRAPH, MIDDLE GRAPH AND TOTAL GRAPH OF A COMPLETE BIPARTITE GRAPH

Theorem 2.1: For any complete bipartite graph $K_{m,n}, \gamma_{cwc}(C(K_{m,n})) = n+2, 3 \leq m \leq n$

Proof: Let $K_{m,n}$ be the complete bipartite graph with partite sets $\{v_1, v_2, ..., v_{m,}\}$ and $\{u_1, u_2, ..., u_n\}$ and with edge set $\{e_{11}, ..., e_{1n}, ..., e_{m1}, ..., e_{mn}\}$. By the definition of central graph, $C(K_{m,n})$ is obtained by subdividing each edge $e_{ij}, 1 \leq i \leq m$ *and* $1 \leq j \leq n$ of $K_{m,n}$ exactly once by the vertex x_{ij}, $1 \leq i \leq m$ *and* $1 \leq j \leq n$ in $C(K_{m,n})$ and joining all the non adjacent vertices of $K_{m,n}$. Let $V_1 = \{v_1, v_2, ..., v_m\}$, $V_2 = \{u_1, u_2, ..., u_n\}$, $V_3 = \{x_{11}, x_{12}, ..., x_{1n}, x_{21}, x_{22}, ..., x_{2n}, ..., x_{m1}, x_{m2}, ...x_{mn}\}$ .Then $V(C(K_{m,n})) = V_1 \cup V_2 \cup V_3$. Here $< V_1 >$ and $< V_2 >$ are complete in $C(K_{m,n})$ and the set V_3 is independent. Also $C(K_{m,n})$ contains a clique of order n. Hence n colors are required to color the vertices of V_2. Let u_i be colored by color $i, 1 \leq i \leq n$. Since no two vertices in V_1 and V_2 are adjacent, the vertices v_i can be colored by color i, $1 \leq i \leq m, 3 \leq m \leq n$. Also the vertices x_{ij} can be colored by any color $k, 1 \leq k \leq n$ other than i & j. Hence all the vertices of $C(K_{m,n})$ can be colored using n colors. That is, $\chi(C(K_{m,n})) = n$. Since $C(K_{m,n})$ contains a clique of order n, it should be included in any chromatic weakly convex dominating set. Let $D = \{u_1, u_2, ..., u_n\}$. Here the vertex u_j dominates the vertex $x_{ij}, 1 \leq i \leq m$ & $1 \leq j \leq n$. Hence all the vertices of V_3 are dominated by D. Since no vertex in V_1 are dominated and $< V_1 >$ is complete, $D_1 = \{u_1, u_2, ..., u_n\} \cup \{v_1\}$ is a dominating set but not weakly convex. Thus $D_1 = \{u_1, u_2, ..., u_n\} \cup \{v_1, x_{11}\}$ is a weakly convex dominating set. Also by the coloring of vertices of $V(C(K_{m,n}))$, $\chi(< D_1 >) = n$. Hence D_1 is a chromatic weakly convex dominating set which is of minimum cardinality. For, removal of any vertex from D_1 cannot be either a weakly convex dominating set or a chromatic weakly convex dominating set. Thus $\gamma_{cwc}(C(K_{m,n})) = n+2, \quad 3 \leq m \leq n$.

Theorem 2.2: For any complete bipartite graph $K_{m,n}, \quad \gamma_{cwc}(M(K_{m,n})) = m+n, \quad 3 \leq m \leq n.$

Proof: Let $K_{m,n}$ be the complete bipartite graph with partite sets $\{v_1, v_2, ..., v_{m,}\}$ and $\{u_1, u_2, ..., u_n\}$ and with edge set $\{e_{11}, ..., e_{1n}, ..., e_{m1}, ..., e_{mn}\}$. By the definition of middle graph, $M(K_{m,n})$ is obtained by subdividing each edge $e_{ij}, 1 \leq i \leq m$ *and* $1 \leq j \leq n$ of $K_{m,n}$ exactly once by the vertex x_{ij}, $1 \leq i \leq m$ *and* $1 \leq j \leq n$ in $M(K_{m,n})$ and joining all these newly added middle vertices x_{ij}, $1 \leq i \leq m$ *and* $1 \leq j \leq n$ of adjacent edges of $K_{m,n}$. For m = 3 and n = 4, the required middle graph is given in Fig 1. Let $V_1 = \{v_1, v_2, ..., v_m\}$, $V_2 = \{u_1, u_2, ..., u_n\}$ $V_3 = \{x_{11}, x_{12}, ..., x_{1n}, x_{21}, x_{22}, ..., x_{2n}, ..., x_{m1}, x_{m2}, ...x_{mn}\}$ .Then $V(M(K_{m,n})) = V_1 \cup V_2 \cup V_3$. The middle graph $M(K_{m,n})$ contains a clique of order $n+1$. Hence $n+1$ colors are required to color the vertices $\{x_{i1}, x_{i2}, ..., x_{in}, v_i\}$. Hence the vertices $\{x_{11}, x_{12}, ..., x_{1n}\}$ are colored by colors $1,2,...,n$ respectively and $\{x_{21}, x_{22}, ..., x_{2n}\}$ are colored by colors $2,...,n, 1$ respectively and so on. Hence the set $\{x_{m1}, x_{m2}, ..., x_{mn}\}$ can be colored by colors $m, m+1..., n,1,2,...m-1$ respectively. Hence all the vertices of V_3 are properly colored using n colors. Since V_1 and V_2 are independent sets and no v_i is adjacent to any of the u_j, all the vertices in V_1 and V_2 can be colored with the color $n+1$ .Hence $\chi(M(K_{m,n})) = n+1$. Since $M(K_{m,n})$ contains a clique of order $n+1$, it should be included in any chromatic weakly convex dominating set. Let $D = \{x_{11}, x_{12}, ..., x_{1n}, v_1\}$. Here the vertex x_{1j} dominates the vertex $u_j, 1 \leq i \leq m$ & $1 \leq j \leq n$. Hence all the vertices of V_2 are dominated. Also all the vertices of V_3 are dominated by D . Since V_1 is independent and no vertices of $v_2, v_3..., v_m$ are dominated by D, $m-1$ vertices are needed to dominate $v_2, v_3..., v_m$ .Thus $x_{21}, x_{31}, ..., x_{m1}$ dominates $v_2, v_3..., v_m$. Since $d(x_{11}, x_{i1}) = 1$ for $2 \leq i \leq m,$

$D_1 = \{x_{11},...,x_{1n},v_1\} \cup \{x_{21},...,x_{m1}\}$ is a weakly convex dominating set. Also by the coloring of vertices of $V(M(K_{m,n}))$, $\chi(<D_1>) = n+1$. Hence D_1 is a chromatic weakly convex dominating set which is of minimum cardinality. For, removal of any vertex from D cannot be either a weakly convex dominating set or a chromatic weakly convex dominating set. Thus $\gamma_{cwc}(M(K_{m,n})) = m+n$.

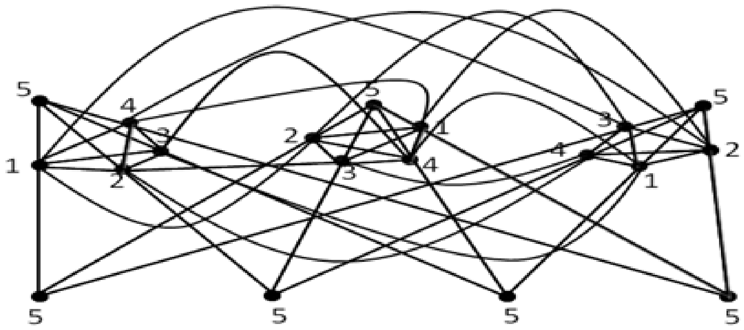

Fig 1 : Middle graph of $K_{3,4}$

Theorem 2.3: For any complete bipartite graph $K_{m,n}, \gamma_{cwc}(T(K_{m,n})) = n+2, \quad 3 \le m \le n$

Proof: Let $K_{m,n}$ be the complete bipartite graph with partite sets $\{v_1, v_2,..., v_{m,}\}$ and $\{u_1, u_2,..., u_n\}$ and with edge set $\{e_{11},...,e_{1n},...,e_{m1},...,e_{mn}\}$. By the definition of total graph, $T(K_{m,n})$ is obtained by subdividing each edge $e_{ij}, 1 \le i \le m \; and \; 1 \le j \le n$ of $K_{m,n}$ exactly once by the vertex $x_{ij}, \; 1 \le i \le m \; and \; 1 \le j \le n$ in $T(K_{m,n})$ and joining all these newly added middle vertices of adjacent edges of $K_{m,n}$ and also joining the adjacent vertices $v_i \; and \; u_j, \; 1 \le i \le m \; and \; 1 \le j \le n$. Let $V_1 = \{v_1, v_2,..., v_m\}$, $V_2 = \{u_1, u_2,..., u_n\}$ $V_3 = \{x_{11}, x_{12},..., x_{1n}, x_{21}, x_{22},..., x_{2n},..., x_{m1}, x_{m2},..., x_{mn}\}$. Then $V(T(K_{m,n})) = V_1 \cup V_2 \cup V_3$. The total graph $T(K_{m,n})$ contains a clique of order $n+1$. Hence $n+1$ colors are required to color the vertices $\{x_{i1}, x_{i2},..., x_{in}, v_i\}$. Hence the vertices $\{x_{11}, x_{12},..., x_{1n}\}$ are colored by colors $1,2,...,n$ respectively and $\{x_{21}, x_{22},..., x_{2n}\}$ are colored by colors $2,...,n, 1$ respectively and so on. Hence the set $\{x_{m1}, x_{m2},..., x_{mn}\}$ can be colored by colors $m, m+1..., n,1,2,...m-1$ respectively. Hence all the vertices of V_3 are properly colored using n colors. Since V_1 is independent, all the vertices in V_1 can be colored with the color $n+1$. Since all the vertices in V_1 are adjacent to all the vertices in V_2, no vertex in V_2 can be colored by color $n+1$. Since each vertex of V_2 is adjacent to exactly m vertices of V_3 and $m \le n$, $\{u_1, u_2,..., u_n\}$ can be colored by colors $m+1..., n,1,2,...m-1, m$ respectively. Hence $\chi(T(K_{m,n})) = n+1$. Since $T(K_{m,n})$ contains a clique of order $n+1$, it should be included in any chromatic weakly convex dominating set. Let $D_1 = \{x_{11}, x_{12},..., x_{1n}, v_1\}$. Here the vertex x_{1j} dominates the vertex $u_j, 1 \le i \le m \; \& \; 1 \le j \le n$. Hence all the vertices of V_2 are dominated. Also all the vertices of V_3 are dominated by D. Since no vertices of $v_2, v_3..., v_m$ are dominated by D_1 and by the definition of total graph each u_j is adjacent to each v_i, $D_1 = \{x_{11},..., x_{1n}, v_1\} \cup \{u_1\}$ is a weakly convex dominating set. Also by the coloring of vertices of $V(T(K_{m,n}))$, $\chi(<D>) = n+1$. Hence D_1 is a chromatic weakly convex dominating set which is of minimum cardinality. For, removal of any vertex from D_1 cannot be either a weakly convex dominating set or a chromatic weakly convex dominating set. Thus $\gamma_{cwc}(T(K_{m,n})) = n+2$.

3. CONCLUSION

In this paper, we obtained the chromatic weakly convex domination number of some families of graph of complete bipartite graph. This paper can be further extended by studying this parameter for some families of graphs of hypercube.

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