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DOMINATION ON STRONG LINE CORPORATE GRAPHS

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ABSTRACT

In this paper, we introduce Strong line corporate graph SLC(G) and strong line corporate dominating set, inverse strong line corporate dominating set, connected strong line corporate dominating set. We establish some properties connecting strong line corporate dominating set and inverse line corporate dominating set and connected strong line corporate dominating set. Further, we prove bounds and properties related to strong line and inverse strong line corporate domination number of G.

Keywords: *connected Strong line corporate dominating set, strong line corporate graph, strong line and inverse strong line corporate dominating sets.*

1. INTRODUCTION

All graphs in this paper are finite, simple and undirected. Let $G = (V, E)$ be a graph where the symbols V and E denote the vertex set and edge set of G. A line graph L(G) is the graph whose vertices correspond to the edges of G and two vertices in L(G) are adjacent if and only if the corresponding edges in G are adjacent. For all other terminology and notations, we follow Harray [1] and the definitions related to dominations are referred from T.W. Haynes & Kulli [2, 3].

M.H. Muddebihal[4] introduced inverse line domination graph. S.Pethanachi Selvam, S.Padmashini [5] introduced inverse complementary domination graph. S.Pethanachi Selvam, S.Padmashini[6] introduced line corporate domination graph.

By the motivation of the papers, we introduce strong line corporate graph, strong line corporate domination number and inverse strong line corporate domination number.

In this paper we prove the bounds and properties related to strong line and inverse strong line corporate domination number of G.

2. PRELIMINARIES

Definition 2.1: Let G_1(= (p, q)) and G_2(= (s, t)) be any two graphs. A graph obtained by joining the graphs G_1 and G_2 by a bridge is called corporate graph and is denoted by C(G).

Definition 2.2: Let G_1(= (p, q)) and G_2(= (s, t)) be any two graphs. A graph obtained by joining the maximum degree vertex of G_1 and the maximum degree vertex of G_2 by a bridge is called
strong corporate graph and is denoted by SC(G) .

Definition 2.3: If we take $G_2 = L(G_1)$ in corporate graph then, it is called line corporate graph and is denoted by LC(G).

Definition 2.4: If we take $G_2 = L(G_1)$ in strong corporate graph then, it is called strong line corporate graph and is denoted by SLC(G).

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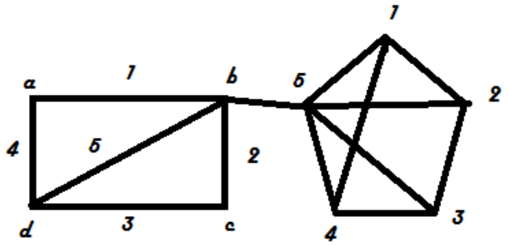
Example 2.5:



The following results will be used to prove the theorems related to the strong line corporate graphs.

Result 2.6:[4] For any connected graph G, $\gamma(G) \le \frac{p}{2}$.

Result 2.7: [4] For any connected graph G, $\gamma(L(G)) + \gamma^{-1}(L(G)) \le p - 1$.

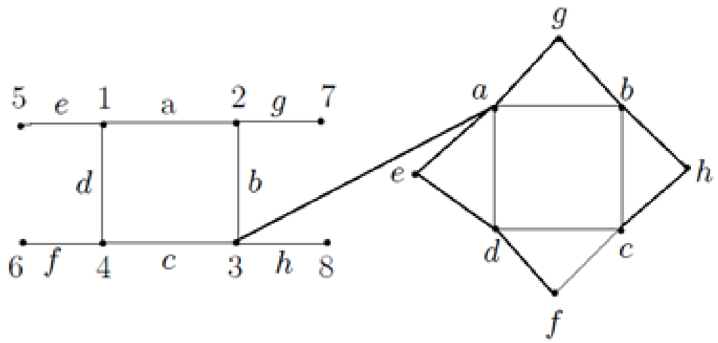
3. STRONG LINE CORPORATE DOMINATION

First we define the connected domination for strong line corporate graph and further we deal with the properties.

Definition 3.1: Let SLC(G) be a strong line corporate graph. A set D $\subseteq$V(SLC(G)) is said to be strong line corporate dominating set (shortly written as SLCDS) if every vertex in V − D is adjacent to some vertex in D. The minimum cardinality of vertices in such a set is called strong line corporate domination number of SLC(G) and is denoted by γ(SLC(G)).

Definition 3.2: Let D be a strong line corporate dominating set (SLCDS). If V (SLC (G)) − D contains another SLCDS namely D^{-1}, then D^{-1} is called the Inverse strong line corporate dominating set (shortly by ISLCDS) w.r.t D. The minimum cardinality of vertices in ISLCDS is called Inverse strong line corporate domination number and is denoted by γ^{-1}(SLC(G)).

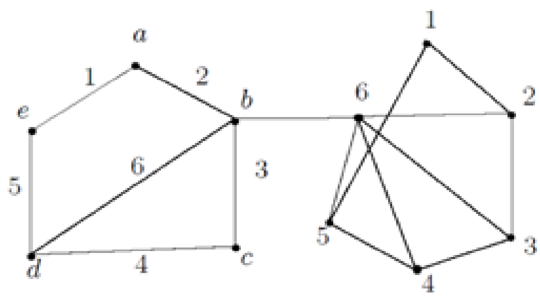
Example 3.3:



Here, the strong line corporate dominating set is {1,2,3,4,a,c} and the inverse strong line corporate dominating set is {5,6,7,8,b,d}

Definition 3.4: Let SLC(G) be a strong line corporate graph. A dominating set K$\subseteq$ V (SLC(G)) is said to be connected strong line corporate dominating set (shortly written as CSLCDS), if the induced subgraph < K > is connected. The connected strong line corporate domination number is the minimum cardinality of connected strong line corporate dominating set and is denoted by γ_c(SLC(G)).

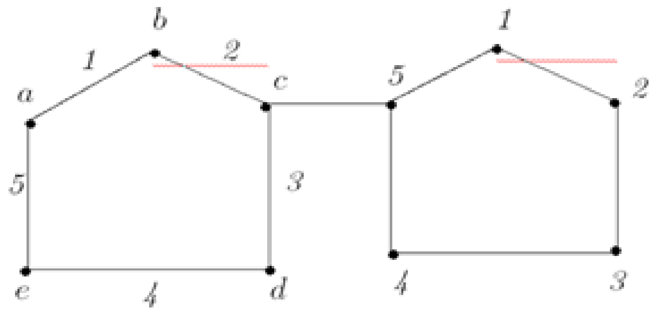
Example 3.5:



Here, the minimal connected strong line corporate dominating set is {d, b, 6, 5} and γ_c(SLC(G)) = 4.

Note 3.6: Every CSLCDS is also SLCDS. But the converse need not be true.

Example 3.7:



Here, CSLCDS is {a, b, c, 1, 2, 5} which is also SLCDS. But SLCDS is {e, c, 2, 4} which is not connected.

4. MAIN RESULTS

In the following theorem, we characterize the graphs having the domination number and the strong line corporate domination number are equal.

Theorem 4.1: Let G be a graph having p vertices, then $\gamma(SLC(G)) = \gamma(LC(G))= \gamma(G)= p$ if and only if G is a null graph.

Proof: Let $G(= (p, q))$ be a graph .

Assume that, $\gamma(SLC(G))= \gamma(LC(G))= \gamma(G)= p$.

We have to prove that, G is a null graph.

Suppose not, then it has at least one vertex of degree greater than or equal to 1.

Consider the following cases.

Case-(i): G is connected.

From the result 2.6, it follows that for any connected graph $\gamma(G) \leq \frac{p}{2} < p$ which is a contradiction to $\gamma(G) = p$.

Case-(ii): G is disconnected.

Since $d(v_i) > 1$, we have $\gamma(G) < p$ which is a contradiction to $\gamma(G) = p$.

Conversely, assume that G be a null graph having p -vertices.

We have to prove that, $\gamma(SLC(G)) = \gamma(LC(G)) = \gamma(G) = p$.

Since each vertex having degree zero, $\gamma(G)= p$.

Also, the line corporate graph and the strong line corporate graph is G.

Then, $\gamma(SLC(G))= \gamma(LC(G))= p$.

Hence , $\gamma(SLC(G))= \gamma(LC(G))= \gamma(G)= p$.

The following theorem relates the bounds of the strong line corporate domination number.

Theorem 4.2: For any graph G, having $\Delta(SLC(G))\geq 2$. Then $1 \leq \gamma(SLC(G))\leq p + q - 1$.

Proof: Let $\{v_1, v_2, ..., v_{p+q}\}$ be the vertices of $SLC(G)$ and let $D \subseteq V(SLC(G))$ be the strong line corporate dominating set of $SLC(G)$.

First we have to prove that , $\gamma(SLC(G)) \leq p + q - 1$.

Consider the following cases.

Case-(i): G is connected.

Since $\gamma(G) \leq \frac{p}{2}$ and $\gamma(SLC(G)) \leq \gamma(G)+\gamma(L(G))$, we have
$\gamma(SLC(G)) \leq \frac{p+q}{2} \leq p + q - 1.$

Case-(ii): G is not connected.

Suppose $\gamma\ (SLC(G)) > p + q - 1$, then $SLC(G)$ is a null graph which is a contradiction to $\Delta(SLC(G)) \geq 2$.

Then, G must have at least one edge.

Hence, $\gamma\ (SLC(G)) \leq p + q - 1$.

Next, we have to prove $1 \leq \gamma(SLC(G))$

Consider the following cases.

Case-(i): Let $SLC(G)$ be a strong line corporate graph having $p + q \leq 2$ vertices. Then $\gamma(SLC(G))= 1$,

Since $\Delta(SLC(G)) \geq 2$.

Case-(ii): If SLC(G) having $p + q \geq 3$ vertices, then $\gamma(SLC(G)) > 1$.

From the above two cases, $\gamma(SLC(G)) \geq 1$.

Hence, $1 \leq \gamma(SLC(G)) \leq p + q - 1$.

The following theorem relates the upper bound for the connected strong line corporate domination number.

Theorem 4.3: Let $SLC(G)$ be a strong line corporate graph. Then $\gamma_c(SLC(G)) \leq p + q - 2$.

Proof: Let G be a graph having p vertices and q edges.

Let $\{v_1, v_2, ..., v_{p+q}\}$ be the vertices of $SLC(G)$ and let $D \subseteq V\ (SLC(G))$ be the connected strong line corporate dominating set of $SLC(G)$ in which the induced subgraph $<D>$ is connected.

Take $p = 2$, then $SLC(G) \cong P_3$ and $\gamma_c(SLC(G)) = 1$.

Next, to prove $\gamma_c(SLC(G)) < p + q - 2$.

Suppose not, then D does not contain the minimum cardinality among the *CSLCDS* which is a contradiction *to* $\gamma_c(SLC(G))$.

Hence, $\gamma_c(SLC(G)) \leq p + q - 2$.

Theorem 4.4: For any connected graph $SLC(G)$,

(i) $\gamma_c(SLC(G)) = \gamma_c(G) + \gamma_c(L(G))$
(ii) $\gamma(SLC(G) \leq \gamma_c(SLC(G)$

Proof: (i) Let $\{v_1, v_2, ..., v_{p+q}\}$ be the vertices of $SLC(G)$ and let $D \subseteq V\ (SLC(G))$ be the connected strong line corporate dominating set of $SLC(G)$ and $H \subseteq V\ (G)$ is one of the connected dominating sets of G and $K \subseteq V\ (L(G))$ is one of the connected dominating sets of $L(G)$.

First, we have to prove $\gamma_c(SLC(G)) \leq \gamma_c(G) + \gamma_c(L(G))$.

Suppose $v_i \in D$, then by the definition of *CSLCDS*, v_i be the vertex contained in *H or K.*

That is, $v_i \in H$ *or* $v_i \in K$.

Then, $\gamma_c(SLC(G)) \leq \gamma_c(G) + \gamma_c(L(G))$ ………………….(1)

Next, we have to prove $\gamma_c(G) + \gamma_c(L(G)) \leq \gamma_c(SLC(G))$.

Suppose $v_i \in H$ *or* $v_i \in K$.

Consider the following cases.

Case-(i): Suppose $v_i \in H$, then v_i is one of the vertices in H having at most $p-2$ vertices.

Thus $v_i \in D$, since by the definition of *CSLCDS*.

Case-(ii): Suppose $v_i \in K$, then similarly as in *case (i)*, $v_i \in D$.

Hence, $\gamma_c(G) + \gamma_c(L(G)) \leq \gamma_c(SLC(G))$ ……………..(2)

From equation (1) and (2), we have $\gamma_c(SLC(G)) = \gamma_c(G) + \gamma_c(L(G))$.

(ii) Let *SLC(G)* be the strong line corporate graph.

Since $\gamma(SLC(G)) \leq \gamma(G) + \gamma(L(G)) \leq \gamma_c(G) + \gamma_c(L(G))$ *and* $\gamma_c(SLC(G)) = \gamma_c(G) + \gamma_c(L(G))$,

We have $\gamma(SLC(G)) \leq \gamma_c(SLC(G))$.

The following theorem relates the inverse strong line corporate domination number and the connected strong line corporate domination number of *G*.

Theorem 4.5: For any connected graph *SLC(G)*, $\gamma^{-1}(SLC(G)) + \gamma_c(SLC(G)) \leq \left\lceil \frac{3p+3q-4}{2} \right\rceil$.

Proof: Let $\{v_1, v_2, \ldots, v_{p+q}\}$ be the vertices of SLC(G).

From the Theorem 4.3, it follows that the connected strong line corporate domination number can not exceed $p + q - 2$.............(a)

As $\gamma(SLC(G)) + \gamma^{-1}(SLC(G)) \leq p + q$ *and* $\gamma(SLC(G)) \leq \gamma^{-1}(SLC(G))$, $2(\gamma(SLC(G))) \leq p + q$.

If p + q is even, then $\gamma^{-1}(SLC(G)) \leq \frac{p+q}{2} \leq \left\lceil \frac{p+q}{2} \right\rceil$

If p + q is odd, then $\gamma^{-1}(SLC(G)) \leq \left\lceil \frac{p+q}{2} \right\rceil$

Thus, $\gamma^{-1}(SLC(G)) \leq \left\lceil \frac{p+q}{2} \right\rceil$ ………….(b)

From (a) and (b), we have

$\gamma^{-1}(SLC(G)) + \gamma_c(SLC(G)) \leq \left\lceil \frac{p+q}{2} \right\rceil + p+q-2$

Consider the following cases.

Case-(i): If p + q is even, then

$$\gamma^{-1}(SLC(G)) + \gamma_c(SLC(G)) \leq \left\lceil \frac{p+q}{2} \right\rceil + p+q-2 \leq \frac{3p+3q-4}{2} \leq \left\lceil \frac{3p+3q-4}{2} \right\rceil$$

Case-(ii): If p + q is odd, then

$$\begin{aligned}\gamma^{-1}(SLC(G)) + \gamma_c(SLC(G)) &\leq \left\lceil \frac{p+q}{2} \right\rceil + p+q-2 \\ &\leq p + q - 2 + \frac{p+q+1}{2} \\ &\leq p + q - \frac{3}{2} + \frac{p+q}{2} \\ &\leq p + q + \frac{-3+1-1}{2} + \frac{p+q}{2} \\ &\leq \frac{3p+3q}{2} + \frac{-4+1}{2} \\ &\leq \left\lceil \frac{3p+3q-4}{2} \right\rceil\end{aligned}$$

Hence, $\gamma^{-1}(SLC(G)) + \gamma_c(SLC(G)) \leq \left\lceil \frac{3p+3q-4}{2} \right\rceil$

Theorem 4.6: Let SLC(G) be a strong line corporate graph without isolated vertices and $d(v_i) > 0$, $v_i \in V(G)$, then $\gamma(SLC(G)) = 2q - 1$ if and only if $G \cong qK_2$.

Proof: Let $G \cong qK_2$ be a graph having p vertices and $q = \frac{p}{2}$ edges.

Then, L(G) has $\frac{p}{2}$ isolated vertices.

Since SLC(G) connects the maximum degree vertex of G and the maximum degree vertex by a bridge and since $q=\frac{p}{2}$,

we have, $\gamma(SLC(G)) = \frac{p}{2} + q - 1 = q+q-1 = 2q-1$.

Conversely, assume that $\gamma(SLC(G)) = 2q - 1$ with $d(v_i) > 0, v_i \in V(G)$.

We have to prove that $G \cong qK_2$.

Suppose not, consider the following cases.

Case-(i): Suppose G is connected.

Since G is connected, SLC(G) is connected.

Then, $\gamma(SLC(G)) \leq \left\lceil \frac{p+q}{2} \right\rceil \neq 2q - 1$ which is a contradiction.

Case-(ii): Suppose G is not connected.

Consider the disconnected graph having $d(v_i) > 0, v_i \in V(G)$.

Then the strong line corporate domination number is not equal to $2q - 1$ which is a contradiction.

Hence, $G \cong qK_2$.

Theorem 4.7: If G be a graph having $d(v_i) = 0$ *and* $1\ \forall\ v_i \in V(G)$, then $\gamma(SLC(G)) = p - 1$.

Proof: Let $G(= (p, q))$ be a graph.

Let $\{v_1, v_2, ..., v_{p+q}\}$ be the vertices of SLC(G).

Let S = $\{v_1, v_2, ..., v_s\}$ be the set of all vertices having degree one and T = $\{v_1, v_2, ..., v_t\}$ be the set of all isolated vertices. Then, $p = s + t$.

By the definition of *SLCDS*, $\gamma(SLC(G)) = \frac{s}{2} + t + q - 1$.

Since the number of edges in G is half of the number of vertices in the set S.

That is, $q = \frac{s}{2}$, then $\gamma(SLC(G)) = \frac{s}{2} + t + \frac{s}{2} - 1$
$= s + t - 1 = p - 1$.

The following theorem relates the upper bound of sum of the inverse domination number and the connected strong line corporate domination number.

Theorem 4.8: For any connected graph SLC(G), $\gamma^{-1}(G)+\gamma_c(SLC(G)) \leq p+q+\gamma(G)-\delta(SLC(G))$, for every $\delta(SLC(G)) \geq 1$.

Proof: Let S = $\{v_1, v_2, ..., v_{p+q}\}$ be the set of all vertices in $SLC(G)$ and $V = \{v_1, v_2, ..., v_p\}$ be the set of all vertices in G.

Then, there exists a minimal dominating set of G namely, $D = \{v_1, v_2, ..., v_k\} \subseteq V(G)$.

Since $\delta(SLC(G)) \geq 1$, G has another dominating set namely D^{-1} such that $|D| \leq |D^{-1}|$.

Also, there exists the minimal strong line corporate dominating set $K \subseteq V(SLC(G))$.

Consider the following cases.

Case-(i): Suppose $< K >$ has exactly one component. Then K itself is a connected strong line corporate dominating set of G.

Case-(ii): Suppose $< K >$ has more than one component .Then attach the minimum set of vertices K' of S −K which are in every u − v path such that $K_1 = K \cup K'$.

Clearly, K_1 forms connected strong line corporate dominating set of G.

Since for any graph SLC(G), there exists at least one vertex $v \in V(SLC(G))$ with $deg(v) = \delta(SLC(G))$ and $|K_1| \leq p + q - 4$, it follows that,
$| D^{-1} | \cup |K_1| \leq |V(SLC(G))| \cup |D| - \delta(SLC(G))$.

Hence, $\gamma^{-1}(G) + \gamma_c(SLC(G)) \leq p + q + \gamma(G) - \delta(SLC(G))$.

The following theorem relates the bounds of the inverse strong line corporate domination number.

Theorem 4.9: Let SLC(G) be the connected graph, then

$$\left\lceil \frac{p+q}{\Delta(G)+\Delta(L(G))+1} \right\rceil \leq \gamma{-1}(SLC(G)) \leq \left\lceil \frac{p+q}{2} \right\rceil$$

Proof: Let $SLC(G)$ be a connected graph having $p+q$ vertices and $q + r + 1$ edges.

Let X = $\{v_1, v_2, ..., v_{p+q}\}$ be the set of vertices of SLC(G) and let A = $\{v_1, v_2, ..., v_p\}$ be the set of all vertices of G.

Let $S \subseteq A$ such that $N[v_i] = V(G)\ \ \forall v_i \in S,\ 1 \leq i \leq k$ and let $B = \{v_{p+1}, v_{p+2}, ..., v_{p+q}\}$ be the set of all vertices in L(G) corresponding to the edges which are incident to the vertices of S in G.

Suppose D ⊆ X is strong line corporate dominating set of G, then there exists a minimal set of vertices $D^{-1} \subseteq V$ (SLC(G) – D which covers all the vertices in SLC(G).

Clearly, D^{-1} forms inverse strong line corporate dominating set of G.

Since for any graph SLC(G) , there exists atleast one vertex of maximum degree in G and L(G) .

It follows that, $\left\lceil \frac{p+q}{\Delta(G)+\Delta(L(G))+1} \right\rceil \leq | D^{-1} |$.

Thus, $\left\lceil \frac{p+q}{\Delta(G)+\Delta(L(G))+1} \right\rceil \leq \gamma^{-1}(SLC(G))$

Next, we have to prove, $\gamma^{-1}(SLC(G)) \leq \left\lceil \frac{p+q}{2} \right\rceil$

Consider the following cases.

Case-(i): If p + q is even, then $\gamma(SLC(G)) \leq \frac{p+q}{2}$.

Since $\gamma(SLC(G)) + \gamma^{-1}(SLC(G)) \leq p + q$, we have $\gamma^{-1}(SLC(G)) \leq p + q - \frac{p+q}{2} \leq \left\lceil \frac{p+q}{2} \right\rceil$

Case-(ii): If p + q is odd, then $\gamma(SLC(G)) \leq \left\lfloor \frac{p+q}{2} \right\rfloor$

Thus, $\gamma^{-1}(SLC(G)) \leq \left\lceil \frac{p+q}{2} \right\rceil$

The following theorem relates bound for the product of strong line corporate domination number and the inverse strong line corporate domination number.

Theorem 4.10: For any connected graph $G,\ 2 \leq \gamma(SLC(G)).\gamma^{-1}(SLC(G)) \leq (p + q)^2$.

Proof: Let $SLC(G)$ be a strong line corporate graph.

Let $D \subseteq V(SLC(G))$ be the dominating set of $SLC(G)$ and D^{-1} be another dominating set which covers all the vertices of $SLC(G)$.

To prove the lower bound, consider the following cases.

Case-(i): If $p + q \leq 3$, then $SLC(G) \cong P_3$.

Thus, $\gamma(SLC(G)).\gamma^{-1}(SLC(G)) = 2$.

Case-(ii): If $p + q > 3$, then $\gamma(SLC(G)).\gamma^{-1}(SLC(G)) > 3$ which is obvious.

Thus, $2 \leq \gamma(SLC(G)).\gamma^{-1}(SLC(G))$.

To prove the upper bound, consider the following cases.

Case-(i): $p = q$, then $\gamma(SLC(G)).\gamma^{-1}(SLC(G)) \leq \frac{p+q}{2}.\left\lceil\frac{p+q}{2}\right\rceil$

$$\leq \frac{2p}{2}.\left\lceil\frac{2p}{2}\right\rceil \leq p^2 \leq (p+1)^2$$

Case-(ii): $p > q$, then $\gamma(SLC(G)).\gamma^{-1}(SLC(G)) \leq \frac{p+q}{2}.\left\lceil\frac{p+q}{2}\right\rceil$

$$\leq (p+1)^2$$

Case-(iii): If p<q, then it is enough to prove that $\gamma(L(G)) \leq \frac{p}{2}$

Let G be a graph having p vertices and q edges and let L(G) be its line graph having q vertices and r edges.

From the result 2.7, it follows that for any connected graph G, $\gamma(L(G)) + \gamma^{-1}(L(G)) \leq p - 1$,

We have, $2\gamma(L(G)) + \gamma^{-1}(L(G)) \leq p - 1 + \gamma(L(G)) \leq p - 1 + \gamma^{-1}(L(G))$

Thus, $\gamma(L(G)) \leq \frac{p-1}{2} \leq \frac{p}{2}$ *and* $\gamma^{-1}(L(G)) \leq \frac{p-1}{2} \leq \frac{p}{2}$

Also, $\gamma(SLC(G)).\gamma^{-1}(SLC(G)) \leq (\gamma(G)+\gamma(L(G))).(\gamma^{-1}(G)+\gamma^{-1}(L(G)) \leq (\frac{p}{2}+\frac{p}{2}).(\left\lceil\frac{p}{2}\right\rceil+\frac{p}{2}) \leq (p+1)^2$

Hence, $2 \leq \gamma(SLC(G)).\gamma^{-1}(SLC(G)) \leq (p+q)^2$.

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