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STUDY OF DUFOUR EFFECT ON UNSTEADY FREE CONVECTION MHD FLOW PAST AN IMPULSIVELY STARTED MOVING VERTICAL PLATE WITH VARIABLE TEMPERATURE AND CONSTANT MASS DIFFUSION IN AN INCLINED MAGNETIC FIELD

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ABSTRACT

Study of Dufour effect on unsteady free convection MHD flow past an impulsively started moving vertical plate with variable temperature and constant mass diffusion in an inclined magnetic field is studied here. Fluid Considered is electrically conducting. The Laplace transform technique has been used to find the solutions for the velocity profile. The results obtained are discussed with the help of graphs drawn for different parameters like thermal Grashof number, mass Grashof number, Prandtl number, the Hartmann number, Schmidt number, Dufour number, time and inclination of magnetic field. The numerical values of Skin-friction, Nusselt number and Sherwood number have been tabulated.

Keywords: *MHD, Dufour effect, free convection, Heat transfer, Mass transfer.*

INTRODUCTION

The effect of magnetic field on viscous, incompressible and electrically conducting fluid plays very important role in many applications such as glass manufacturing, control processing, paper industry, textile industry, magnetic materials processing and purification of crude oil etc. Bejan and Khair [1] have studied heat and mass transfer by natural convection in a porous medium. Nelson and Wood [2] have studied combined heat and mass transfer natural convection between vertical parallel plates with uniform heat flux boundary conditions. MHD flow between two parallel plates with heat transfer have been studied by Attia and Katb [4]. Rajput and Kumar [12] have studied MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion. Hossain and Shayo [3] have discussed the skin-friction in the unsteady free convection flow past an exponentially accelerated plate. Das and Jana [10] have studied heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in porous medium. Rajesh [11] has studied MHD effects on free convection and mass transform flow through a porous medium with variable temperature. Unsteady MHD Poiseuille flow between two infinite parallel plates through porous medium in an inclined magnetic field with heat and mass transfer have been discussed by Rajput and Kumar [16]. Sandeep and Sugunamma [14] have discussed effect of inclined magnetic field on unsteady free convection flow of a dusty viscous fluid between two infinite flat plates filled by a porous medium. Postelnicu [5] has studied influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Alam *et al.* [6] have studied Dufour and Soret effects on unsteady MHD free convection and mass transfer flow past a vertical porous plate in a porous medium. Reddy [9] have discussed Soret and Dufour effects on steady MHD free convection flow past a semi-infinite moving vertical plate in a porous medium with viscous dissipation. Dipak *et al.* [13] have studied Soret and Dufour effects on steady MHD convective flow past a continuously moving porous vertical plate. Influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects has studied by Postelnicu [7]. Ibrahim [8] has discussed analytic solution of heat and mass transfer over a permeable stretching plate affected by chemical reaction, internal heating, Dufour-Soret effect and Hall effect. Kesavaiah and Satyanarayana [15] have studied radiation absorption and Dufour effects to MHD flow in vertical surface.

In this paper we are analyzing Dufour effect on unsteady free convection MHD flow past a moving vertical plate through porous medium with variable temperature and constant mass diffusion in an inclined magnetic field.

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MATHEMATICAL ANALYSIS

In this paper we have considered the flow of unsteady viscous incompressible fluid. The *x*- axis is taken along the plate in the upward direction and *y*- axis is taken normal to plate. The plate considered is electrically non-conducting and its initial velocity is taken as u_0. A uniform inclined magnetic field B_0 is applied on the plate with angle α from vertical. Initially the fluid and plate are at the same temperature T_∞ and the concentration of the fluid is C_∞. At time $t > 0$, temperature of the plate is raised to T_w and the concentration of the fluid is raised to C_w.

The governing equations under the usual Boussinesq's approximations are as follows:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho}\sin^2(\alpha)u, \tag{1}$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2} + \frac{D_m K_T}{C_S C_P}\frac{\partial^2 C}{\partial y^2}, \tag{2}$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2}. \tag{3}$$

The initial and boundary conditions are given as:

$$\left.\begin{aligned} &t \le 0; u = 0, T = T_\infty, \mathrm{C} = \mathrm{C}_\infty \text{ for each value of } y,\\ &t > 0; u = u_0, T = T_\infty + (T_w - T_\infty)\frac{u_0^2 t}{\upsilon}, C = C_w \ at\ y = 0,\\ &u \to 0, T \to T_\infty, C \to C_\infty \text{ as } y \to \infty. \end{aligned}\right\} \tag{4}$$

Here u is the velocity of the fluid, g – the acceleration due to gravity, β – volumetric coefficient of thermal expansion, β^* – volumetric coefficient of concentration expansion, t – time, T – the temperature of the fluid, T_∞ – the temperature of the plate at $y \to \infty$, C – species concentration in the fluid, C_∞ – species concentration at $y \to \infty$, υ – the kinematic viscosity, ρ – the density, C_P – the specific heat at constant pressure, k – thermal conductivity of the fluid, K_T – thermal diffusion ratio, D – the mass diffusion constant, D_m – the effective mass diffusivity rate, T_w – the temperature of the plate at $y = 0$, C_w – species concentration at the plate at $y = 0$, C_S – Concentration susceptibility B_0 – the uniform magnetic field, σ – electrical conductivity and α – angle of inclination from vertical.

By using the following dimensionless quantities, the above equations (1), (2), and (3) can be transformed into dimensionless form.

$$\left.\begin{aligned} &\bar{y} = \frac{yu_0}{\upsilon}, \bar{t} = \frac{tu_0^2}{\upsilon}, \bar{u} = \frac{u}{u_0}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \bar{C} = \frac{C - C_\infty}{C_w - C_\infty}, Sc = \frac{\upsilon}{D}, Ha^2 = \frac{\sigma B_0^2 \upsilon}{\rho u_0^2}, \Pr = \frac{\mu C_p}{k}, \mu = \upsilon\rho,\\ &Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3}, A = Ha^2 * \sin^2(\alpha), Gm = \frac{g\beta^*\upsilon(C_w - C_\infty)}{u_0^3}, D_f = \frac{D_m K_T (C_w - C_\infty)}{\upsilon C_S C_P (T_w - T_\infty)}. \end{aligned}\right\} \tag{5}$$

Here $\bar{u}$ is dimensionless velocity, $\bar{t}$ – dimensionless time, *Pr*- Prandtl number, *Sc*- Schmidt number, *Gr*- thermal Grashof number, *Gm*- mass Grashof number, θ - dimensionless temperature, $\bar{C}$ - dimensionless concentration, *Ha*- the Hartmann number, μ - the coefficient of viscosity and D_f - Dufour number. Then model is transformed into the following non dimensional form of equations:

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + Gr\theta + Gm\bar{C} - A\bar{u} \tag{6}$$

$$\frac{\partial \theta}{\partial \bar{t}} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial \bar{y}^2} + D_f \frac{\partial^2 \bar{C}}{\partial \bar{y}^2}, \tag{7}$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} = \frac{1}{Sc}\frac{\partial^2 \bar{C}}{\partial \bar{y}^2}. \tag{8}$$

The initial and boundary condition become:

$$\left.\begin{array}{l} \bar{t} \le 0; \bar{u} = 0, \theta = 0, \bar{C} = 0 \text{ for each value of } \bar{y}, \\ \bar{t} > 0; \bar{u} = 1, \theta = \bar{t}, \bar{C} = 1 \text{ at } \bar{y} = 0, \\ \bar{u} \to 0, \theta \to 0, \bar{C} \to 0 \text{ as } \bar{y} \to \infty. \end{array}\right\} \tag{9}$$

Dropping bars in the above equations, we get:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + GmC - Au, \tag{10}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + D_f \frac{\partial^2 C}{\partial y^2}, \tag{11}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}. \tag{12}$$

The initial and boundary condition become:

$$\left.\begin{array}{l} t \le 0; u = 0, \theta = 0, C = 0 \text{ for each value of } y, \\ t > 0; u = 1, \theta = t, C = 1 \text{ at } y = 0, \\ u \to 0, \theta \to 0, C \to 0 \text{ as } y \to \infty. \end{array}\right\} \tag{13}$$

Now the solution of equation (10), (11) and (12) under the boundary conditions (13) are obtained by the Laplace - transform technique. The exact solutions for species concentration *C*, fluid temperature θ and fluid velocity *u* are respectively:

$$C = Erfc\left[\frac{\sqrt{Sc}\,y}{2\sqrt{t}}\right] \tag{14}$$

$$\theta = -\frac{e^{\frac{-\Pr y^2}{4t}}\sqrt{\Pr t}\,y}{\sqrt{\pi}} - \frac{A_{25}}{2}\left(t + \frac{\Pr y^2}{2} + \frac{\Pr ScD_f}{Sc - \Pr}\right) + \frac{\Pr ScD_f}{2(Sc - \Pr)}A_{27} \tag{15}$$

$$u = \frac{1}{2}A_9B_{19} + \frac{1}{4A^2}Gr\left[2A_9B_{16}B_{13} + y\sqrt{A}A_9B_{17} + A_{13}B_{18}(1-\Pr) + \frac{A\Pr ScD_f}{Sc-\Pr}(-2A_9B_{19} + A_{13}B_{20})\right]$$
$$+\frac{(A_{18}B_{11} - A_9B_{12})}{2A}\left[\frac{Gr\Pr ScD_f}{Sc-\Pr} + Gm\right] - Gr\left[\begin{array}{l} \frac{1}{2A^2}\{-2A_{22}B_{13} + \frac{A\sqrt{\Pr}y}{\sqrt{\pi}}(2e^{\frac{-\Pr y^2}{4t}}\sqrt{t} + \sqrt{\pi\Pr}\,yA_{22}) \\ +\ \frac{1}{2}A_{13}B_{21}(\Pr-1)\} + \frac{\Pr Sc}{2A(Sc-\Pr)}(A_{25} + \frac{A_{13}}{2}(1 + A_{23} + A_{14}A_{26}))D_f \end{array}\right]$$
$$-\frac{Gr\Pr ScD_fB_{14}}{2A(Sc-\Pr)} - \frac{1}{2A}GmB_{15}$$

(16)

SKIN- FRICTION

We calculate the non-dimensional form of skin friction (τ) from the velocity field as:

$$\tau = \left(-\frac{\partial u}{\partial y}\right)_{y=0}$$

And numerical values of τ are given in table-1 for different parameters.

NUSSELT NUMBER

From temperature field, we study non dimensional form of rate of heat transfer (*Nu*) which is given as:

$$Nu = \left(-\frac{\partial \theta}{\partial y}\right)_{y=0}$$

Numerical values of *Nu* are given in table-2 for different parameters.

SHERWOOD NUMBER

From concentration field, we study non-dimensional form of rate of mass transfer (Sh) which is given as:

$$Sh = \left(-\frac{\partial C}{\partial y} \right)_{y=0}$$

Numerical values of *Sh* are given in table-3 for different parameters.

RESULTS AND DISCUSSION

The numerical values of velocity, skin-friction, Nusselt number and Sherwood number are computed for different parameters like thermal Grashof number *Gr*, mass Grashof number *Gm*, Hartmann number *Ha*, Prandtl number *Pr*, Schmidt number *Sc*, inclination α , Dufour number D_f and time *t*. The values of the parameters considered are *Gr* = 5, 10, 15, *Ha* = 2, 4, 6, *Gm* = 50, 60, 70, $\alpha = 15°, 30°, 60°$, *Pr* = 7, 10, *Sc* = 2.01, 2.10, 2.20, D_f = 0.15, 0.23, 0.50 and *t* = 0.15, 0.18, 0.20. Figures 3, 6 and 8 show that velocity increases when *Gm*, *Pr* and *t* are increased. Figures 1, 2, 4, 5 and 7 show that velocity decreases when D_f , α, *Gr*, *Ha*, and *Sc* are increased.

Numerical value of Skin- friction is given in table-1. The value of skin-friction increases with increasing the values of *Ha, Sc, Gr,* α and D_f and decreases with increasing the values of *Gm, t , and Pr*.

Numerical value of Nusselt number is given in table-2. The value of *Nu* increases with increasing the values of *t* and decreases with increasing the values of *Pr*, D_f and *Sc*.

In table-3 the numerical value of Sherwood number is given, the value of *Sh* increases with *Sc* and decreases with increasing the values of *t.*

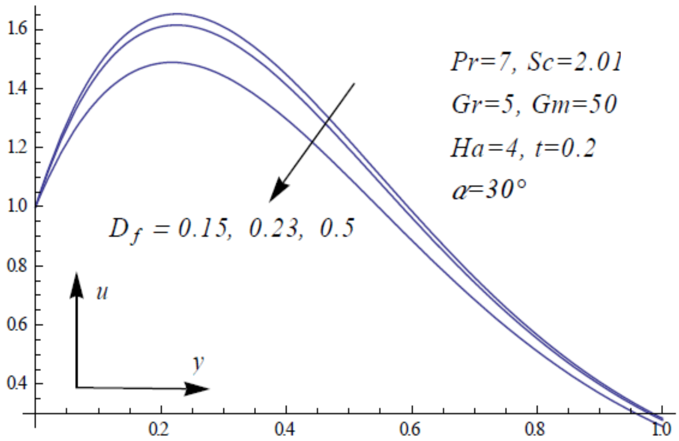

Figure-1: Velocity profile for different values of D_f

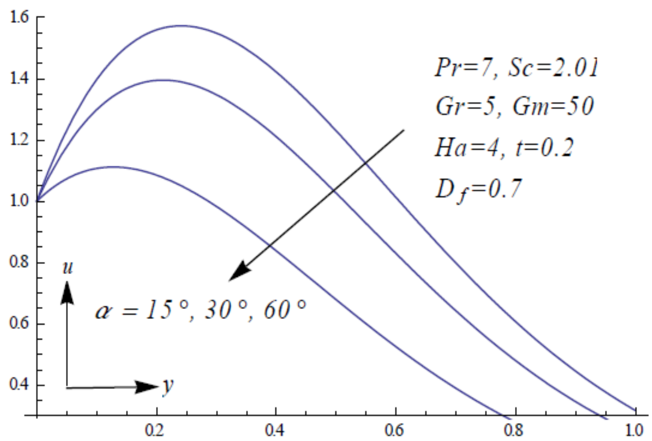

Figure-2: Velocity profile for different values of α

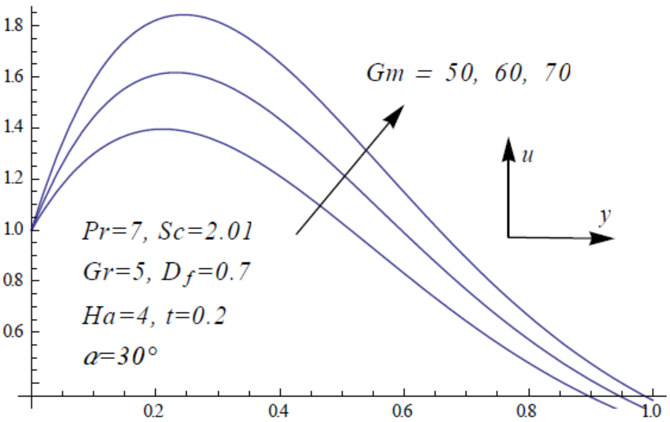

Figure-3: Velocity profile for different values of *Gm*

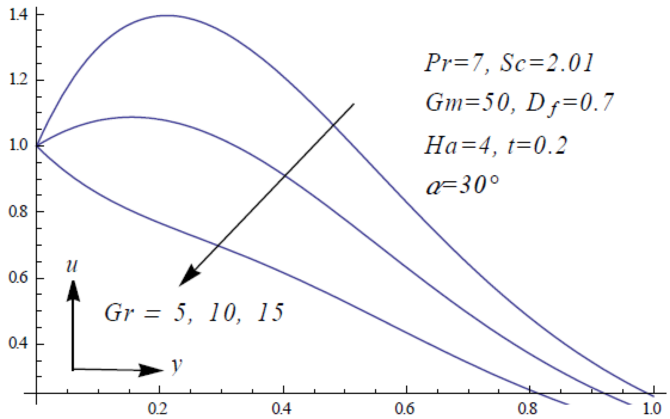

Figure-4: Velocity profile for different values of *Gr*

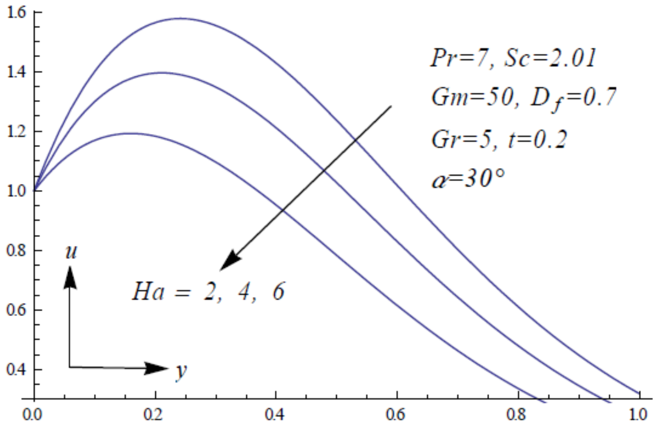

Figure-5: Velocity profile for different values of *Ha*

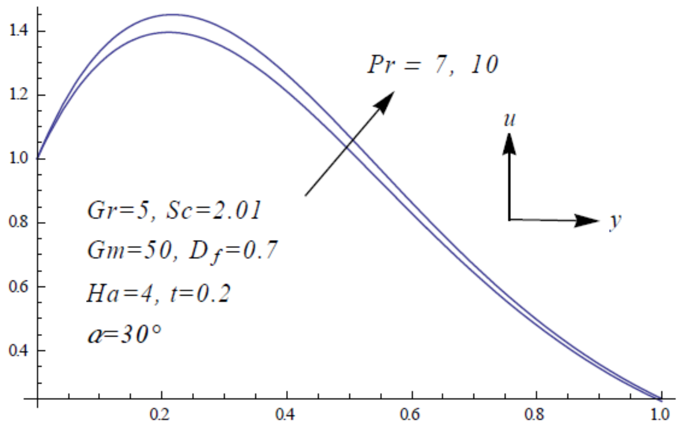

Figure-6: Velocity profile for different values of *Pr*

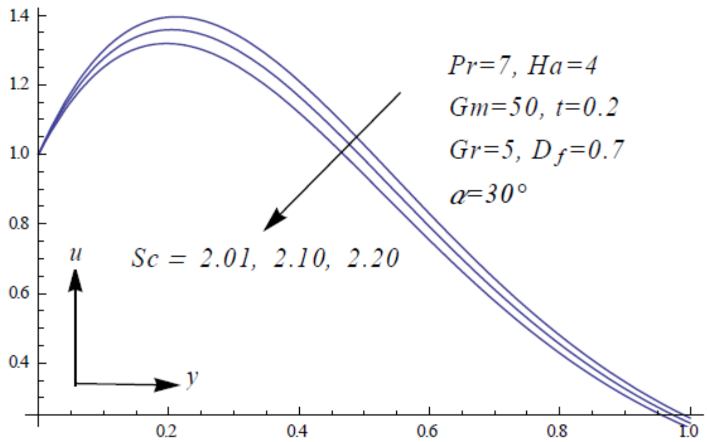

Figure-7: Velocity profile for different values of *Sc*

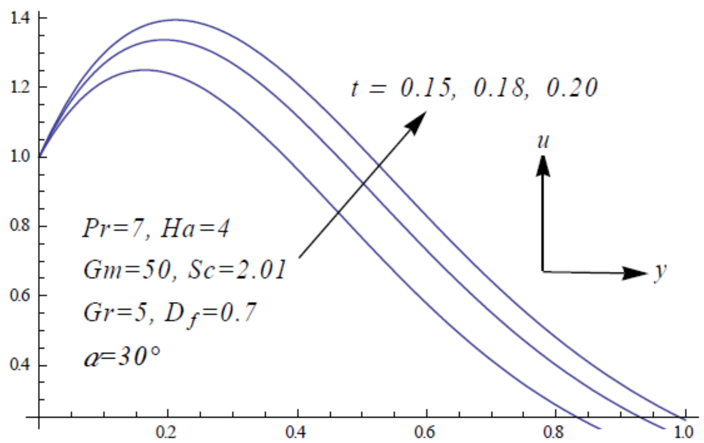

Figure-8: Velocity profile for different values of *t*

Table-1: Skin-Friction for different values of parameters

α (In degree)	Ha	Pr	D_f	t	Gm	Gr	Sc	τ
15	4	07	0.70	0.20	50	05	2.01	-5.3663
30	4	07	0.70	0.20	50	05	2.01	-4.2274
60	4	07	0.70	0.20	50	05	2.01	-1.9156
30	2	07	0.70	0.20	50	05	2.01	-5.3970
30	6	07	0.70	0.20	50	05	2.01	-2.6773
30	4	03	0.70	0.20	50	05	2.01	+0.2277
30	4	10	0.70	0.20	50	05	2.01	-4.6882
30	4	07	0.15	0.20	50	05	2.01	-6.6936
30	4	07	0.23	0.20	50	05	2.01	-6.3348
30	4	07	0.50	0.20	50	05	2.01	-5.1242
30	4	07	0.70	0.15	50	05	2.01	-3.3951
30	4	07	0.70	0.18	50	05	2.01	-3.9195
30	4	07	0.70	0.20	60	05	2.01	-6.1139
30	4	07	0.70	0.20	70	05	2.01	-8.0004
30	4	07	0.70	0.20	50	10	2.01	-1.1771
30	4	07	0.70	0.20	50	15	2.01	+1.8731
30	4	07	0.70	0.20	50	05	2.10	-3.9405
30	4	07	0.70	0.20	50	05	2.20	-3.6205

Table-2: Nusselt number for different values of parameters

Pr	t	D_f	Sc	Nu
03	0.20	0.70	2.01	-0.8165
07	0.20	0.70	2.01	-1.7226
10	0.20	0.70	2.01	-2.2798
07	0.15	0.70	2.01	-2.3745
07	0.18	0.70	2.01	-1.7226
07	0.20	0.15	2.01	0.6798
07	0.20	0.23	2.01	0.3304
07	0.20	0.50	2.01	-0.8490
07	0.20	0.70	2.10	-1.8350
07	0.20	0.70	2.20	-1.9586

Table-3: Sherwood number for different values of parameters

Sc	t	Sh
2.01	0.20	1.78858
2.10	0.20	1.82818
2.20	0.20	1.87121
2.01	0.15	2.06527
2.01	0.18	1.88533

CONCLUSION

Some conclusions of study are as below:

- The velocity of the fluid increases with increasing the values of Gm, t and Pr.
- The velocity of the fluid decreases with increasing the values of Ha, Gr, Sc, α and D_f.
- The skin-Friction of the fluid increases with increasing the values of Ha, Sc, Gr, α and D_f.
- The skin-Friction of the fluid decreases with increasing the values of Gm, t and Pr .
- Nusselt number increases with increasing the values of t.
- Nusselt number decreases with increasing the values of Pr, Sc, D_f.
- Sherwood number increases with increasing the values of Sc.
- Sherwood number decreases with increasing the values of t.

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APPENDIX

$$A_9 = e^{-\sqrt{A}y},\ A_{10} = Erfc\left[\frac{2\sqrt{A}t + y}{2\sqrt{t}}\right],\ A_{11} = Erf\left[\frac{2\sqrt{A}t - y}{2\sqrt{t}}\right],\ A_{12} = Erf\left[\frac{2\sqrt{A}t + y}{2\sqrt{t}}\right]$$

$$A_{13} = 2e^{\frac{At}{\Pr-1} - \sqrt{\frac{A\Pr}{\Pr-1}}y},\ A_{14} = e^{2\sqrt{\frac{A\Pr}{\Pr-1}}y}\quad A_{15} = Erf\left[\frac{2\sqrt{\frac{A\Pr}{\Pr-1}}t - y}{2\sqrt{t}}\right],\ A_{16} = Erf\left[\frac{2\sqrt{\frac{A\Pr}{\Pr-1}}t + y}{2\sqrt{t}}\right]$$

$$A_{17} = Erfc\left[\frac{2\sqrt{\frac{A\Pr}{\Pr-1}}t + y}{2\sqrt{t}}\right],\ A_{18} = e^{\frac{At}{Sc-1} - \sqrt{\frac{ASc}{Sc-1}}y},\ A_{19} = e^{2\sqrt{\frac{ASc}{Sc-1}}y},\ A_{20} = Erf\left[\frac{2\sqrt{\frac{ASc}{Sc-1}}t - y}{2\sqrt{t}}\right]$$

$$A_{21} = Erf\left[\frac{2\sqrt{\frac{ASc}{Sc-1}}t + y}{2\sqrt{t}}\right],\ A_{22} = -1 + Erf\left[\frac{\sqrt{\Pr}y}{2\sqrt{t}}\right],\ A_{23} = Erf\left[\frac{2\sqrt{\frac{A}{\Pr-1}}t - \sqrt{\Pr}y}{2\sqrt{t}}\right]$$

$$A_{24} = Erf\left[\frac{2\sqrt{\frac{A}{\Pr-1}}t + \sqrt{\Pr}y}{2\sqrt{t}}\right],\ A_{25} = -2Erfc\left[\frac{\sqrt{\Pr}y}{2\sqrt{t}}\right],\ A_{26} = Erfc\left[\frac{2\sqrt{\frac{A}{\Pr-1}}t + \sqrt{\Pr}y}{2\sqrt{t}}\right]$$

$$A_{27} = -2Erfc\left[\frac{\sqrt{Sc}y}{2\sqrt{t}}\right],\ A_{28} = Erf\left[\frac{2\sqrt{\frac{A}{Sc-1}}t - \sqrt{Sc}y}{2\sqrt{t}}\right],\ A_{29} = Erfc\left[\frac{2\sqrt{\frac{A}{Sc-1}}t + \sqrt{Sc}y}{2\sqrt{t}}\right]$$

$$A_{30} = Erf\left[\frac{2\sqrt{\frac{A}{Sc-1}}t + \sqrt{Sc}y}{2\sqrt{t}}\right],\ B_{11} = 1 + A_{19} + A_{20} - A_{19}A_{21},\ B_{12} = 1 + A_{11} + A_9^{-2}A_{10}$$

$$B_{13} = 1 - \Pr - At,\ B_{14} = A_{27} + A_{18}(1 + A_{28} + A_{19}A_{29}),\ B_{15} = A_{18}(1 + A_{19} + A_{28} - A_{19}A_{30}) + A_{27}$$

$$B_{16} = 1 + A_9^{-2} + A_{11} - A_9^{-2}A_{12},\ B_{17} = 1 - A_9^{-2} + A_{11} + A_9^{-2}A_{12},\ B_{18} = -1 - A_{14} - A_{15} + A_{14}A_{17}$$

$$B_{19} = 1 + A_{11} + A_9^{-2}A_{10},\ B_{20} = 1 + A_{15} + A_{14}A_{17},\ B_{21} = 1 + A_{14} + A_{23} - A_{14}A_{24}$$

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