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MHD AND HEAT TRANSFER EFFECTS ON AN OSCILLATORY FLOW OF JEFFREY FLUID IN A CIRCULAR TUBE

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ABSTRACT

We studied the effects of magnetic field and heat transfer on oscillatory flow of Jeffrey fluid in a circular tube. The expressions for the velocity field and temperature field are obtained analytically. It is observed that, the axial velocity increases with increasing $_1$, Gr and , while it decreases with increasing M, Pr, Re and . The temperature field decreases with increasing Pr.

1. INTRODUCTION

The study of oscillatory flow of a viscous fluid in cylindrical tubes has received the attention of many researchers as they play an important role in understanding the important physiological problems, namely the blood flow in arteriosclerotic blood vessel. Womersley [101] has investigated the oscillating flow of thin walled elastic tube. Detailed measurements of the oscillating velocity profiles were made by Linford and Ryan [54]. Unsteady and oscillatory flow of viscous fluids in locally constricted, rigid, axisymmetric tubes at low Reynolds number has been studied by Ramachandra Rao and Devanathan [71], Hall [41] and Schneck and Ostrach [83]. Haldar [40] have considered the oscillatory flow of a blood through an artery with a mild constriction. Several other workers, Misra and Singh [56], Ogulu and Alabraba [62], Tay and A Ogulu [97] and Elshahed [31], to mention but a few, have in one way or the other modeled and studied the flow of blood through a rigid tube under the influence of pulsatile pressure gradient.

Many researchers have studied blood flow in the artery by considering blood as either Newtonian or non-Newtonian fluids, since blood is a suspension of red cells in plasma; it behaves as a non-Newtonian fluid at low shear rate. Barnes *et al.* [13] have studied the behavior of no-Newtonian fluid flow through a straight rigid tube of circular cross section under the action of sinusoidally oscillating pressure gradient about a nonzero mean. Chaturani and Upadhya [21] have developed a method for the study of the pulsatile flow of couple stress fluid through circular tubes. The Poiseuille flow of couple stress fluid has been critically examined by Chaturani and Rathod [22]. Moreover, the Jeffrey model is relatively simpler linear model using time derivatives instead of convected derivatives for example the Oldroyd-B model does, it represents rheology different from the Newtonian [17]. None of these studies considered the effect of body temperature on the blood flow -prominent during deep heat muscle treatment.

The magnetohydrodynamic (MHD) flow between parallel plates is a classical problem that occurs in MHD power generators, MHD pumps, accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil and fluid droplets and sprays. Especially the flow of non-Newtonian fluids in channels is encountered in various engineering applications. For example, injection molding of plastic parts involves the flow of polymers inside channels. During the last few years the industrial importance of non-Newtonian fluids is widely known. Such fluids in the presence of a magnetic field have applications in the electromagnetic propulsion, the flow of nuclear fuel slurries and the flows of liquid state metals and alloys. Sarparkaya [83] have presented the first study for MHD Bingham plastic and power law fluids. Effect of magnetic field on pulsatile flow of blood in a porous channel was investigated by Bhuyan and Hazarika [16]. Hayat *et al.* [44] have studied the Hall effects on the unsteady hydromagnetic oscillatory flow of a second grade fluid in a channel. Couette and Poiseuille flows of an Oldroyd 6-constant fluid with magnetic field in a channel was investigated by Hayat *et al.* [45]. Hayat *et al.* [46] have studied the influence of heat transfer in an MHD second grade fluid film over- an unsteady stretching sheet. Vasudev *et al.* [99] have investigated the influence of magnetic field and heat transfer on peristaltic flow of Jeffrey fluid through a porous medium in an asymmetric channel. Vasudev *et al.* [100] have studied the MHD peristaltic flow of a Newtonian fluid through a porous medium in an asymmetric vertical channel with heat transfer. Unsteady flow of a Jeffrey fluid in an elastic tube with a stenosis have studied by Sreenadh Sridharamalle, Devaki Pulluri, Divakar Reddy and Krishnaiah *et al.* [29]. effect of jeffery fluid on heat and mass transfer past a vertical porous plate with soret and variable thermal conductivity *et al.* [29] have studied. Jeffrey Fluid Flow through Porous Medium in the Presence of Magnetic Field in Narrow Tubes santosh nallapu and G. Radhakrishnamacharya *et al.* [82] have studied.

In view of these, we studied the MHD and Heat transfer Effects on an oscillatory flow of Jeffrey fluid in a circular tube. The expressions for the velocity field and temperature field are obtained analytically. The effects of various pertinent parameters on the velocity field and temperature field studied in detail with the help of graphs.

2. MATHEMATICAL FORMULATION

We consider an oscillatory flow of a Jeffrey fluid through in a heated uniform cylindrical tube of constant radius R . A uniform magnetic field B_0 is applied in the transverse direction to the flow. The wall of the tube is maintained at a temperature T_w. We choose the cylindrical coordinates (r,θ,z) such that $r=0$ is the axis of symmetry. The flow is considered as axially symmetric and fully developed. The geometry of the flow is shown in Fig. 1.

The constitute equation of S for Jeffrey fluid is

$$\boldsymbol{S} = \frac{\mu}{1+\lambda_1}(\dot{\gamma} + \lambda_2 \ddot{\gamma}) \tag{2.1}$$

where μ is the dynamic viscosity, λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time, $\dot{\gamma}$ is the shear rate and dots over the quantities denote differentiation with time.

The equations governing the flow are given by

$$\rho\frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial\left(rS_{rz}\right)}{\partial r} - \sigma B_0^2 w + \rho g\beta\left(T - T_w\right) \tag{2.2}$$

$$\rho c_p \frac{\partial T}{\partial t} = k_0\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{2.3}$$

where ρ is the fluid density, μ is the fluid viscosity, p is the pressure, w is the velocity component in $z-$direction, g is the acceleration due to gravity, σ is the electrical conductivity of the fluid, β coefficient of thermal expansion, T is the temperature, k_0 is the thermal conductivity and c_p is the specific heat at constant pressure.

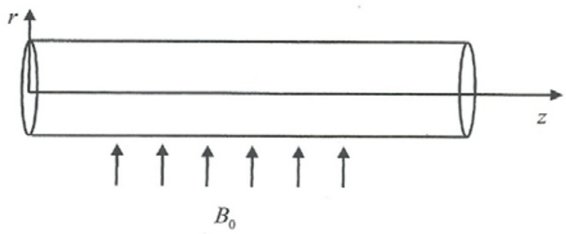

Fig. 1: The Physical Model

The appropriate boundary conditions are

$$w = 0,\ T = T_w \qquad at \qquad r = R$$

$$\frac{\partial w}{\partial r} = 0,\ T = T_\infty \qquad at \qquad r = 0 \tag{2.4}$$

Introducing the following non-dimensional variables

$$\bar{r} = \frac{r}{R}, \bar{z} = \frac{z}{R}, \bar{t} = \frac{w_0 t}{R}, \alpha^2 = \frac{\rho R^2}{\mu}, \lambda = \frac{R}{w_0}, \bar{w} = \frac{w}{w_0},$$

$$\bar{P} = \frac{p - p_w}{\mu}, \theta = \frac{T - T_w}{T_w - T_\infty} \Pr = \frac{\mu c_p}{k_0}, \mathrm{Re} = \frac{\rho w_0 R}{\mu},$$

Into the Eqs. (2.2) – (2.4), we get (after dropping bars)

$$Re\frac{\partial w}{\partial t} = -\lambda\frac{\partial p}{\partial z} + \frac{1}{1+\lambda_1}\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right] - M^2 w + \frac{Gr}{Re}\theta \tag{2.5}$$

$$\Pr\mathrm{Re}\frac{\partial\theta}{\partial t} = \frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} \tag{2.6}$$

where $\Pr$ is the Prandtl number, $M = RB_0\sqrt{\frac{\sigma}{\mu}}$ is Hartmann number, $Gr = \frac{\rho g\beta R^2\left(T_w - T_\infty\right)}{w_0\mu}$ is the Grashoff number and Re is the Reynolds number.

The corresponding non-dimensional boundary conditions are

$$w=0,\ T=1 \quad at \quad r=1$$
$$\frac{\partial w}{\partial r}=0, T=0 \quad at \quad r=0 \tag{2.7}$$

3. SOLUTION

It is fairly unanimous that, the pumping action of the heart results in a pulsatile blood flow so that we can represent the pressure gradient (pressure in the left ventricle) as

$$-\frac{dp}{dz}=p_0 e^{i\omega t} \tag{3.1}$$

where p_0 is a real constant and ω is the frequency of the oscillation and flow variables expresses as

$$\theta(y,t)=\theta_0(r)e^{i\omega t} \tag{3.2}$$
$$w(y,t)=w_0(r)e^{i\omega t} \tag{3.3}$$

Substituting Eqs. (3.1) – (3.2) into Eqs. (2.5) and (2.6) and solving the resultant equations subject to the boundary conditions in (2.7), we obtain

$$\theta_0=\frac{I_0(\Omega r)}{I_0(\Omega)} \tag{3.4}$$
$$w_0=\frac{Gr}{\mathrm{Re}}\frac{(1+\lambda_1)}{(\beta_1^2+\Omega^2)}\left[\frac{I_0(\beta_1 r)}{I_0(\beta_1)}-\frac{I_0(\Omega r)}{I_0(h)}\right]+\frac{\lambda p_0}{\beta_1^2}(1+\lambda_1)\left[1-\frac{I_0(\beta_1 r)}{I_0(\beta_1)}\right] \tag{3.5}$$

Here $\Omega^2=i\omega \Pr \mathrm{Re}$ and $\beta_1^2=(M^2+i\omega \mathrm{Re})(1+\lambda_1)$

In Eqs. (3.4) and (3.5), $I_0(x)$ is the modified Bessel function of first kind of order zero.

Hence the temperature distribution and the axial velocity are given by

$$\theta=\frac{I_0(\Omega r)}{I_0(\Omega)}e^{i\omega t} \tag{3.6}$$
$$w=\left(\frac{Gr}{\mathrm{Re}}\frac{(1+\lambda_1)}{(\beta_1^2+\Omega^2)}\left[\frac{I_0(\beta_1 r)}{I_0(\beta_1)}-\frac{I_0(\Omega r)}{I_0(\Omega)}\right]+\frac{\lambda p_0}{\beta_1^2}(1+\lambda_1)\left[1-\frac{I_0(\beta_1 r)}{I_0(\beta_1)}\right]\right)e^{i\omega t} \tag{3.7}$$

in which $m_1=\sqrt{N^2-i\omega pe}$ and $m_2=\sqrt{M^2+i\omega \mathrm{Re}}$.

Solving equations (3.4) and (3.5) using the boundary conditions (3.6) and (3.7), we obtain

$$u_0(y)=-A\cosh m_2 y+C\frac{\sinh m_2 y}{\sinh m_2}+A+B\frac{\sin m_1 y}{\sin m_1} \tag{3.8}$$

and $$\theta_0(y)=\frac{\sin m_1 y}{\sin m} \tag{3.9}$$

where $A=\frac{\lambda(1+\lambda_1)}{m_2^2}, B=\frac{Gr(1+\lambda_1)}{(m_1^2+m_2^2)}$ and $C=(A\cosh m_2-A-B)$

Therefore, the fluid velocity and temperature are given as

$$u(y,t)=\left(-A\cosh m_2 y+C\frac{\sinh m_2 y}{\sinh m_2}+A+B\frac{\sin m_1 y}{\sin m_1}\right)e^{i\omega t} \tag{3.10}$$

and $$\theta(y,t)=\frac{\sin m_1 y}{\sin m}e^{i\omega t} \tag{3.11}$$

The rate of heat transfer coefficient in terms of Nusselt number Nu at the plate $y=0$ of the channel is given by

The rate of heat transfer coefficient in terms of Nusselt number on the boundary of the is

$$Nu = \left.\frac{\partial\theta}{\partial r}\right|_{r=1} = \frac{\Omega I_1(\Omega)}{I_0(\Omega)} e^{i\omega t} \tag{3.12}$$

When $M \to 0,$ the Eq. (3.7) becomes

$$w = \left(\frac{Gr}{\mathrm{Re}} \frac{(1+\lambda_1)}{\left(\beta_2^2+\Omega^2\right)} \left[\frac{I_0(\beta_2 r)}{I_0(\beta_2)} - \frac{I_0(\Omega r)}{I_0(\Omega)} \right] + \frac{\lambda p_0}{\beta_2^2}(1+\lambda_1)\left[1-\frac{I_0(\beta_2 r)}{I_0(\beta_2)}\right] \right) e^{i\omega t} \tag{3.13}$$

Here $\beta_2^2 = i\omega\,\mathrm{Re}(1+\lambda_1)$

4. DISCUSSION OF THE RESULTS

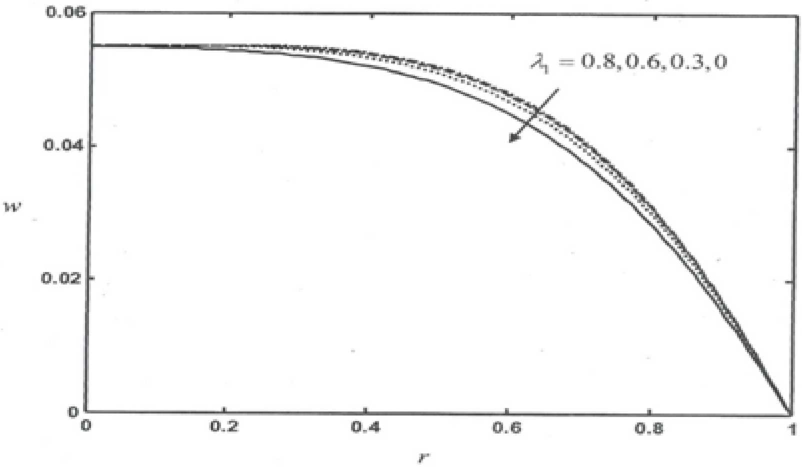

Fig. 2: Effects of material parameter λ_1 on w for M = 0.1, p = 1, ω = 10, λ = 0.5, Pr = 2, Gr = 1, Re =1 and t = 0.1

Above Fig. 2 depicts the effects of material parameter λ_1 on w for $Da = 0.1,\ p = 1,\ \omega = 10,\ \lambda = 0.5, \mathrm{Pr} = 2, Gr = 1, \mathrm{Re} = 1$ and $t = 0.1$. It is observed that, the axial velocity w increases at the axis of tube with increasing material parameter λ_1.

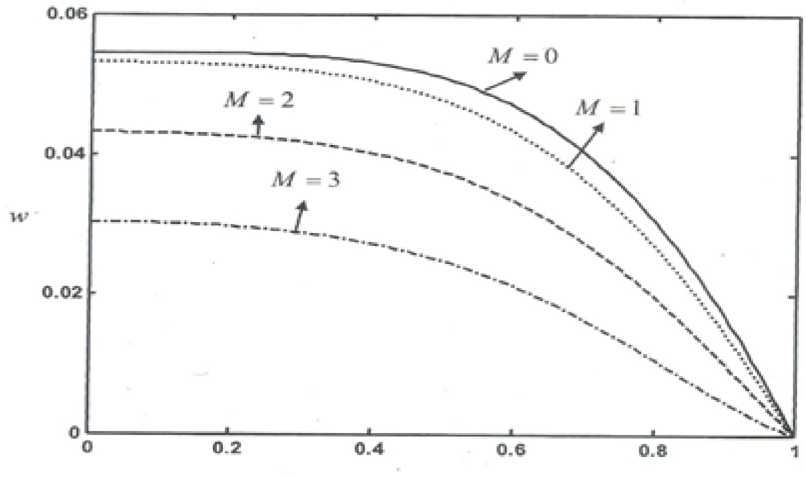

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Fig. 3: Effects of Hartman number M on w for λ_1 = 0.3, p = 1, ω = 10, λ = 0.5, Pr = 2, Gr = 1, Re = 1 and t = 0.1

In order to see the effects of Hartmann number M on w for $\lambda_1 = 0.3, p = 1, w = 10,\ \lambda = 0.5, \mathrm{Pr} = 2, Gr = 1, \mathrm{Re} = 1$ and $t = 0.1$ we plotted Above Fig. 3. It is found that, the axial velocity w decreases with an increase in Hartmann number M. Further, it is found that the velocity is more for non-conducting (magnetic) (i.e., $M \to 0$) Jeffrey fluid than that of conducting Jeffery fluid.

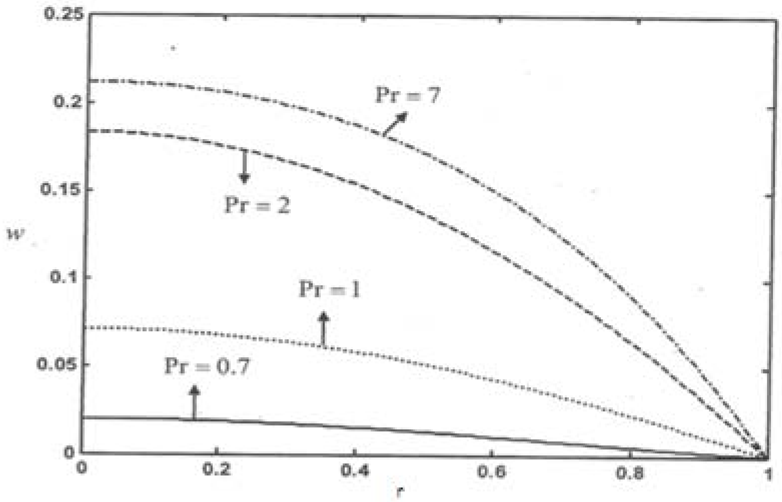

Fig. 4: Effects of Hartman number Pr on w for $\lambda_1 = 0.3$, $p = 1$, $\omega = 10$, $\lambda = 0.5$, $M = 1$, Gr = 1, Re = 1 and $t = 0.1$

Above Fig. 4 shows the effects of Prandtl number Pr on w for $\lambda_1 = 0.3, p = 1, \omega = 10, \lambda = 0.5$, $Da = 0.1, Gr = 1, \mathrm{Re} = 1$ and $t = 0.1$. It is noted that, an increase in the Prandtl number Pr decreases the axial velocity w.

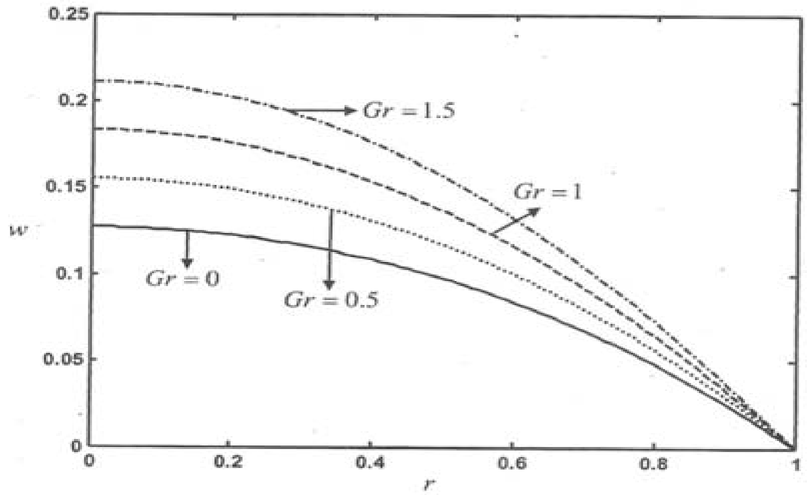

Fig. 5: Effects of Grashof number Gr on w for $\lambda_1 = 0.3$, $p = 1$, $\omega = 10$, $\lambda = 0.5$, Pr = 2, $M = 1$, Re = 1 and $t = 0.1$

Above Fig. 5 illustates the effects of Grashof number Gr on w for $\lambda_1 = 0.3, p = 1, \omega = 10, \lambda = 0.5, \mathrm{Pr} = 2, Da = 0.1, Gr = 1, \mathrm{Re} = 1$ and $t = 0.1$. It is observed that, the axial velocity w increases with an increase in Grashof number Gr.

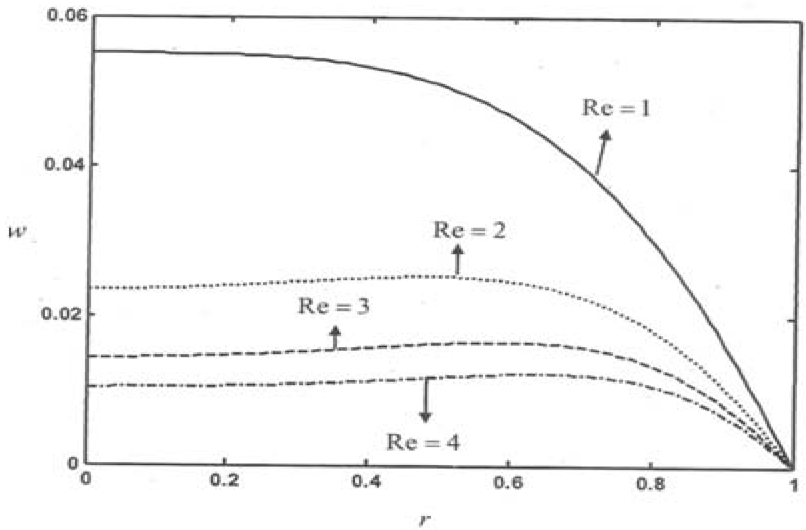

Fig. 6: Effects of Reynolds number Re on w for $\lambda_1 = 0.3$, $p = 1$, $\omega = 10$, $\lambda = 0.5$, Pr = 2, $M = 1$, $Gr = 1$ and $t = 0.1$

Above Fig. 6 shows the effects of Reynolds number Re on w for $\lambda_1 = 0.3, p = 1, \omega = 10, \lambda = 0.5, \Pr = 2, Da = 0.1, Gr = 1, \mathrm{Re} = 1$ and $t = 0.1$. It is found that, the axial velocity w decreases on increasing Reynolds number Re

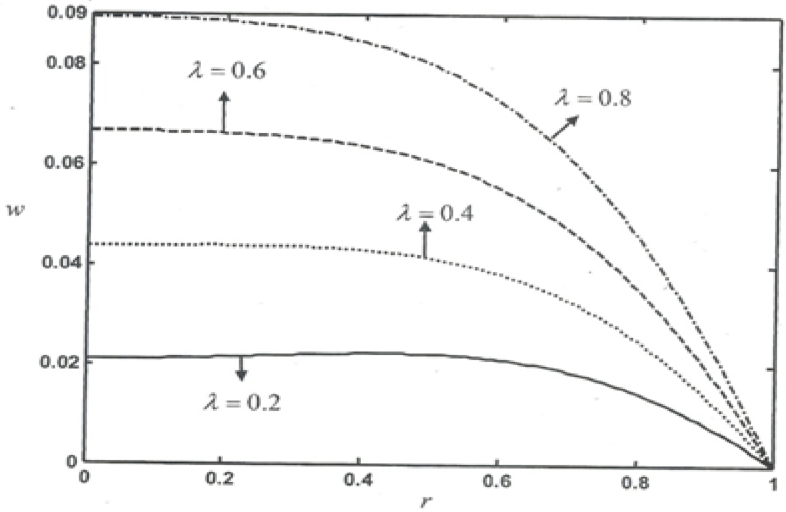

Fig. 7: Effects of λ on w for λ_1= 0.3, M = 1, Pr = 2, Gr = 1, P = 1 ω = 10, Re = 1 and t = 0.1.

In order to see the effects of λ on w for λ1 = 0.3, Da = 0.1, Pr-2, Gr-1, p=1, ω=10, Re-1 and t=0.1 we plotted Fig. 7. It is observed that, the axial velocity increases with increasing λ.

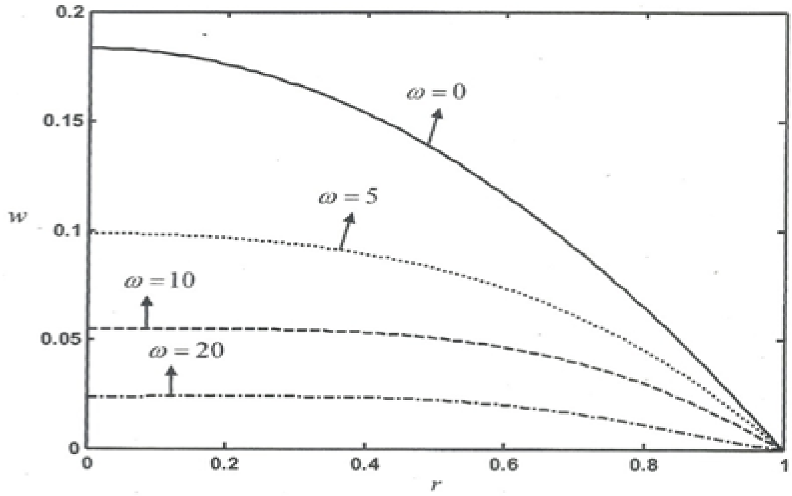

Fig. 8: Effects of ω on w for λ_1 = 0.3, M = 1, Pr = 2, Gr = 1, P = 1, λ = 0.5, Re = 1 and t = 0.1.

Fig. 8 shows the effects of ω on w for λ_1 = 0.3, Da = 0.1, Pr=2, Gr = 1, p = 1, λ = 0.5, Re = 1 and t = 0.1 is shown in

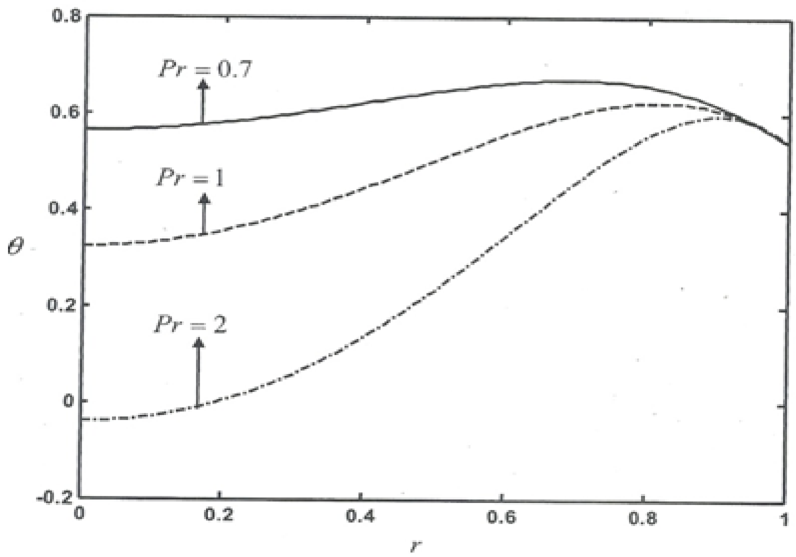

Fig. 9: Effects of Pradtl number Pr on θ for ω = 10, Re = 1 and t = 0.1.

Above Fig. 9. It is observed that, the axial velocity decreases on increasing ω.

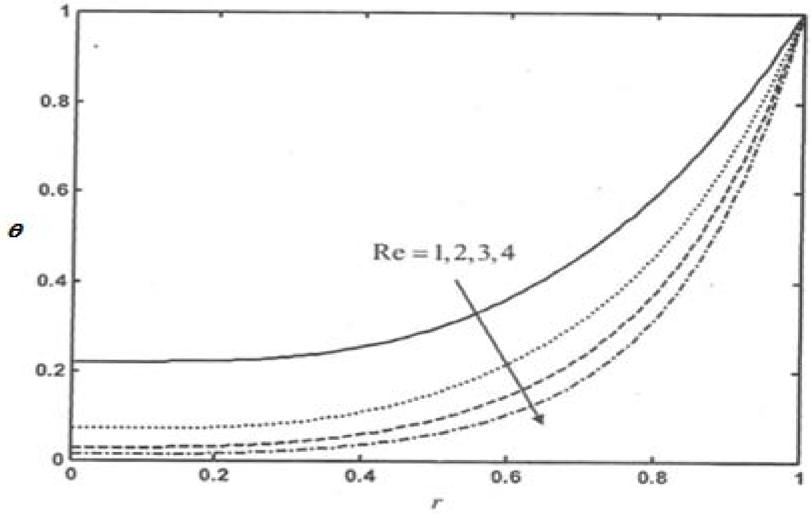

Fig. 10: Effects of Reynolds number *Re* on θ for ω = 10, Pr = 2 and *t* = 0.1.

The effects of Reynolds number Re on θ for ω = 10, Pr = 2 and *t* = 0.1 is deoicted in Above Fig. 10. It is found that, the temperature θ decreases with increasing Reynolds number Re.

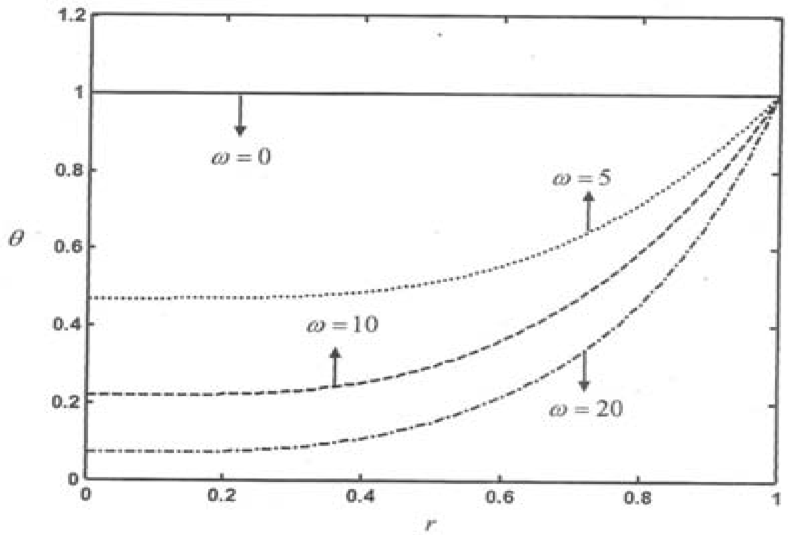

Fig. 11: Effects of ω on θ for *Re* = 1, *Pr* = 2 and *t* = 0.1.

Above Fig. 11 shows the effects of ω on θ for Pr = 2, Re = 1 and *t* = 0.1. It is found that, the temperature θ decreases with increasing Prandtl number ω.

Table – 1: Effect of Prandtl number *Pr* on *Nu* for ω = 10, Re = 1 and t = 0.1

Pr	*Nu*
0.7	0.3219
1	0.3823
2	0.4289

Above Table-1 depicts the effect of Prandtl number *Pr* on *Nu* for ω = 10, Re = 1 and *t*= 0.1**.** It is observed that, the *Nu* increases with increasing *Pr* .

Table – 2: Effect of Reynolds number Re on *Nu* for ω = 10, Pr = 2 and *t* = 0.1

Re	*Nu*
1	0.4289
2	0.4602
3	0.4755
4	0.4844

Above Table-2 illustrates the effect of Reynolds number Re on Nu for ω = 10, Pr = 2 and *t* = 0.1. It is noted that, the Nu increases with increasing Re.

Table – 3: Effect of ω on *Nu* for Pr = 1 and *t* = 0.1

Re	*Nu*
0	0
5	0.7384
10	0.4289
20	-0.4472

Above Table-3shows the effect of ω on Nu for Pr - 2, Re = 1 and t = 0.1. It is found that, the Nu oscillates with increasing ω.

Table – 4: Effect of *t* on *Nu* for ω = 10, Re = 1 and Pr = 2

Re	*Nu*
0	0.9225
0.5	0.3408
1	-0.7291
2	0.3011

Above Table-4 depicts the effect of t on Nu for ω = 10, Re = 1 and Pr = 2 . It is observed that, the Nu oscillates with time t.

5. CONCLUSIONS

In this chapter, we studied the effects of magnetic field and heat transfer on oscillatory flow of Jeffrey fluid in a circular tube. The expressions for the velocity field and temperature field are obtained analytically. It is observed that, the axial velocity increases with increasing λ_1, Gr and λ, while it decreases with increasing M, Pr, Re and λ. The temperature field decreases with increasing Pr.

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