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CERTAIN INTEGRAL TRANSFORMATIONS PERTAINING TO THE MULTIVARIABLE A- FUNCTION

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ABSTRACT

The object of the present paper is to obtain few general multiple integral transformations of the multivariable A-function (1981), as a kernel product with Fox's H-function [3, p. 408] and Laguerre polynomials respectively with the general class of polynomial ([4] and [7]). Several possible cases are also included.

Key Words And Phrases: *A-function, H-function, Laguerre polynomials, General class of polynomials.*

1. INTRODUCTION

Gautam and Goyal [1981] defined the multivariable A-function, which is a generalization of multivariable H-function of Srivastava and Panda [1976 b]. The definition of multivariable A-function runs as follows:

$$F[z_1,..,z_r]=A^{\mu,\lambda:\mu_1,\lambda_1;...;\mu_r,\lambda_r}_{\upsilon,C:\upsilon_1,C_1;...;\upsilon_r,C_r}\left[\begin{array}{c|l} z_1 & (a_j;A_j^{'};...;A_j^{(r)})_{1,\upsilon};(\tau_j^{'},C_j^{'})_{1,\upsilon_1};...;(\tau_j^{r},C_j^{r})_{1,\upsilon_r} \\ z_r & (b_j;B_j^{'};...;B_j^{(r)})_{1,C};(d_j^{'},D_j^{'})_{1,C_1};...;(d_j^{r},D_j^{r})_{1,C_r}\end{array}\right]$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}...\int_{L_r}\theta_1(s_1)...\theta_r(s_r)\,\Phi(s_1,...,s_r)\,z_1^{s_1}...z_r^{s_r}\,ds_1...ds_r, \tag{1.1}$$

$where, \omega=\sqrt{-1}$

$${}_1(s_1)=\frac{\prod_{j=1}^{\mu_1}\Gamma(d_j^{(i)}-D_j^{(i)}s_i)\prod_{j=1}^{\lambda_1}\Gamma(1-\tau_j^{(i)}+C_j^{(i)}s_i)}{\prod_{j=\mu_1+1}^{C_1}\Gamma(1-d_j^{(i)}+D_j^{(i)}s_i)\prod_{j=\lambda_1+1}^{D_1}\Gamma(\tau_j^{(i)}-C_j^{(i)}s_i)},\forall\, i=1,...,r, \tag{1.2}$$

$$\Phi(s_1,...,s_r)=\frac{\prod_{j=1}^{\lambda}\Gamma(1-a_j+\sum_{i=1}^{r}A_j^{(i)}s_i)\prod_{j=1}^{\mu}\Gamma(b_j^{(i)}-\sum_{i=1}^{r}B_j^{(i)}s_i)}{\prod_{j=\mu+1}^{C}\Gamma(1-b_j+\sum_{i=1}^{r}B_j^{(i)}s_i)\prod_{j=\lambda+1}^{D}\Gamma(a_j^{(i)}-\sum_{i=1}^{r}A_j^{(i)}s_i)}, \tag{1.3}$$

Here $\mu,\lambda,\upsilon,C,\mu_i,\lambda_i,\upsilon_i\ and\ c_i$ are non-negative integers and all $a_j's, b_j's, d_j^{(i)}'s, \tau_j^{(i)}'s, B_j^{(i)}'s$ complex numbers.

The multiple integral defining the A-function of r-variables converges absolutely if

$$\xi_i^*=0, \tag{1.4}$$

$$d_i>0, \tag{1.5}$$

and $$\left|\arg(\xi_i)z_k\right|<\frac{\pi}{2}\eta_i, \tag{1.6}$$

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where

$$\xi_i = \prod_{j=1}^{D}\{A_j^{(i)}\}^{A_j^{(i)}} \prod_{j=1}^{C}\{B_j^{(i)}\}^{-B_j^{(i)}} \prod_{j=1}^{c_i}\{D_j^{(i)}\}^{D_j^{(i)}} \prod_{j=1}^{\nu_i}\{C_j^{(i)}\}^{-C_j^{(i)}}\ , \tag{1.7}$$

$$\xi_i^* = img\left[\sum_{j=1}^{\upsilon} A_j^{(i)} - \sum_{j=1}^{C} B_j^{(i)} + \sum_{j=1}^{c_i} D_j^{(i)} - \sum_{j=1}^{\upsilon_i} C_j^{(i)}\right][0,1], \tag{1.8}$$

$$\eta_i = \mathrm{Re}\left[\sum_{j=1}^{\lambda} A_j^{(i)} - \sum_{j=1}^{\upsilon} A_j^{(i)} + \sum_{j=1}^{\mu} B_j^{(i)} - \sum_{j=1}^{C} B_j^{(i)} + \sum_{j=1}^{\mu_i} D_j^{(i)} - \sum_{j=1}^{c_i} D_j^{(i)} + \sum_{j=1}^{\lambda_i} C_j^{(i)} - \sum_{j=1}^{\upsilon_i} C_j^{(i)}\right],\ \forall i = 1,...,r. \tag{1.9}$$

If we take $A_j\text{'}s, B_j\text{'}s, C_j^{(i)}\text{'}s\ and\ D_j^{(i)}\text{'}s$ as real and $\mu = 0$, the A-function reduces to multivariable H-function of Srivastava and Panda [1976 b].

Srivastava [4]introduce the general class of polynomials (see also Srivastava and Singh [7])

$$S_\alpha^\beta[z] = \sum_{k=0}^{[\beta/\alpha]} \frac{(-\beta)_{k\alpha}}{k!} A_{\beta,k} z^k,\ \ \beta = 0,1,2,..., \tag{1.10}$$

where α an arbitrary positive integer and coefficients $A_{\beta,k}(\beta, k \geq 0)$ are arbitrary constants, real or complex.

2. THE MAIN RESULTS

(i) $$\int_0^\infty ... \int_0^\infty x_1^{\sigma_1-1}...x_r^{\sigma_r-1}(k_1x_1^{\rho_1}+...+k_rx_r^{\rho_r})^\sigma S_\alpha^\beta[\eta(k_1x_1^{\rho_1}+...+k_rx_r^{\rho_r})^h]$$

$$H_{p,q}^{m,0}\left[\xi(k_1x_1^{\rho_1}+...+k_rx_r^{\rho_r})\left|\begin{matrix}(e_1,\varepsilon_1),...,(e_p,\varepsilon_p)\\(g_1,\gamma_1),...,(g_q,\gamma_q)\end{matrix}\right.\right].A_{\upsilon,C:\upsilon_1,c_1;...;\upsilon_r,c_r}^{\mu,\lambda:\mu_1,\lambda_1;...;\mu_r,\lambda_r}\begin{bmatrix}z_1X_1\\...\\z_rX_r\end{bmatrix}dx_1...dx_r$$

$$= \xi^{-s}\Psi(k_1,...,k_r)\sum_{k=0}^{[\beta/\alpha]}\frac{(-\beta)_{k\alpha}}{k!}A_{\beta,k}\eta^k\xi^{-hk}A_{\upsilon+r+q,C+p+:\upsilon_1,c_1;...;\upsilon_r,c_r}^{\mu,\lambda+r+m:\mu_1,\lambda_1;...;\mu_r,\lambda_r}$$

$$\left[1-\rho_j/\sigma_j : \xi'_j/\sigma_j,...,\xi_j^{(r)}/\sigma_j\right]_{1,r},\left[1-g_j-(S+hk)\gamma_j;N_1\gamma_j,...,N_r\gamma_j\right]_{1,q}:$$

$$\left[1-S+\sigma : N_1-n_1,...,N_r-n_r\right],\quad \left[1-e_j-(S+hk)\varepsilon_j;N_1\varepsilon_j,...,N_r\varepsilon_j\right]_{1,p}:$$

$$\begin{matrix}(a_j;A_j^{'};...;A_j^{(r)})_{1,\upsilon}(\tau_j^{'},C_j^{'})_{1,\upsilon_1};...;(\tau_j^{(r)},C_j^{(r)})_{1,\upsilon_r}\\(b_j;B_j^{'};...;B_j^{(r)})_{1,C}(d_j^{'},D_j^{'})_{1,c_1};...;(d_j^{(r)},D_j^{(r)})_{1,c_r}\end{matrix}\left|\begin{matrix}Z_1\\...\\Z_r\end{matrix}\right|, \tag{2.1}$$

where

$$X_i = x_1^{\xi_1^{(i)}}...x_r^{\xi_r^{(i)}}(k_1x_1^{\rho_1}+...+k_rx_r^{\rho_r})^{\eta_i}, \tag{2.2}$$

$$S = \sigma + \frac{\sigma_1}{\rho_1} + ... + \frac{\sigma_r}{\rho_r}, \tag{2.3}$$

$$\Psi(k_1,...,k_r) = (\sigma_1...\sigma_r)^{-1}k_1^{-\sigma_1/\rho_1},...,k_r^{-\sigma_r/\rho_r}, \tag{2.4}$$

$$N_i = n_i + \frac{\xi_1^{(i)}}{\rho_1} + ... + \frac{\xi_r^{(i)}}{\rho_r}, \tag{2.5}$$

and

$$Z_i = z_i\xi_1^{-N_1}k_1^{-\xi_1^{(i)}/\rho_1}...k_r^{-\xi_r^{(i)}/\rho_r}.\qquad \text{S.} \tag{2.6}$$

The above integral formula (2.1) is valid under the following sufficient conditions:

(a) $k_i > 0, \rho_i > 0, n_i \geq 0, \xi_i^{(i)} > 0, \ \forall i, \ j \in \{1,...,r\}$, (2.7)

(b) $\mathrm{Re}(\sigma_i) > 0, i = 1,...,r$ and

$$\mathrm{Re}(S) > -\sum_{i=1}^{r} N_i \delta_i - \min_{1 \leq i \leq m} \left\{ \mathrm{Re}\left(\frac{g_j}{\gamma_j} \right) \right\}, \tag{2.8}$$

where $\delta_i = \min \left\{ \mathrm{Re}\left(\frac{d_j^{(i)}}{D_j^{(i)}} \right) \right\}, \ j = 1,...,\mu_i$, (2.9)

(c) m, n, q are integers such that $1 \leq m \leq q \ and \ p \geq 0, \varepsilon_j > 0 \, (j = 1,..., p),$

$$\gamma_j > 0 (j = 1,...,q), \ \Omega_1 \equiv \sum_{j=1}^{p} \varepsilon_j - \sum_{j=1}^{q} \gamma_j < 0, \ \Omega_1 \equiv \sum_{j=1}^{m} \gamma_j - \sum_{j=m+1}^{q} \gamma_j - \sum_{j=1}^{p} \varepsilon_j > 0 \text{ and } \left|\arg(\xi)\right| < \frac{1}{2}\pi\,\Omega_2 \tag{2.9}$$

(d) $A_{\beta,k}$ are arbitrary constants, real or complex and $\beta, k \geq 0$.

(e) Conditions corresponding appropriately (1.4) through (1.6) are satisfied by each of the multivariable A-function occurring in (2.1). Here $H_{p,q}^{m,0}\left[z\right]$ denotes the familier H-function of C. Fox ([3], p. 408, see also [5], p. 310).

(ii) $$\int_0^\infty ... \int_0^\infty x_1^{\sigma_1 - 1} ... x_r^{\sigma_r - 1} (k_1 x_1^{\rho_1} + ... + k_r x_r^{\rho_r})^{\sigma} S_\alpha^\beta [\eta (k_1 x_1^{\rho_1} + ... + k_r x_r^{\rho_r})^h]$$

$$\exp\left[-\gamma(k_1 x_1^{\rho_1} + ... + k_r x_r^{\rho_r})\right].L_w^{(u)}\left[\gamma(k_1 x_1^{\rho_1} + ... + k_r x_r^{\rho_r})\right] A_{\upsilon,C:\upsilon_1,c_1;...;\upsilon_r,c_r}^{\mu,\lambda:\mu_1,\lambda_1;...;\mu_r,\lambda_r} \begin{bmatrix} z_1 x_1^{\xi_1} \\ ... \\ z_r x_r^{\xi_r} \end{bmatrix} dx_1 ... dx_r$$

$$= \frac{(-1)^w \gamma^{-S}}{(w)!} \Psi(k_1,...,k_r) \sum_{k=0}^{\left[\beta/\alpha\right]} \frac{(-\beta)_{k\alpha}}{k!} A_{\beta,k} \eta^k \gamma^{-hk} . A \begin{matrix} \mu, \lambda+2: \mu_1, \lambda_1+1; ...; \mu_r, \lambda_r+1 \\ \upsilon+2, C+1: \upsilon_1+1, c_1; ...; \upsilon_r+1, c_r \end{matrix}$$

$$\left[\begin{matrix} \left[1-S-hk; \xi_1/\rho_1,...,\xi_r/\rho_r\right], \left[1-S-hk+u; \xi_1/\rho_1,...,\xi_r/\rho_r\right], (a_j; A_j^{'};...;A_j^{(r)})_{1,\upsilon}; \\ \left[1-S-hk+u+w; \xi_1/\rho_1,...,\xi_r/\rho_r\right], \end{matrix} \right.$$

$$\left. \begin{matrix} (1-\sigma_1/\rho_1; \xi_1/\rho_1), (\tau_j^{'}, C_j^{'})_{1,\upsilon_1}; ...; (1-\sigma_r/\rho_r; \xi_r/\rho_r); (\tau_j^{(r)}, C_j^{(r)})_{1,\upsilon_r} \\ (b_j; B_j^{'};...;B_j^{(r)})_{1,C} (d_j^{'}, D_j^{'})_{1,c_1}; ...; (d_j^{(r)}, D_j^{(r)})_{1,c_r} \end{matrix} \middle| \begin{matrix} \zeta_1 \\ ... \\ \zeta_r \end{matrix} \right], \tag{2.10}$$

where $L_w^{(u)}(z)$ be the Laguerre polynomial of order u and degree w in z,

$$w \geq 0, k_i > 0, \rho_i > 0, \xi_i > 0, \mathrm{Re}(\sigma_i) > 0, \forall i = 1,...,r, \mathrm{Re}(S) > -\sum_{i=1}^{r} \left(\frac{\xi_i \delta_i}{\rho_i} \right), \mathrm{Re}(\gamma) > 0, \tag{2.11}$$

$\Psi(k_1,...,k_r)$, S and δ_i being given by (2.4), (2.3) and (2.8), respectively, $\zeta_i = z_i \left(\gamma k_i\right)^{-\xi_i/\rho_i}$, i=1,…,r and conditions given by (1.4) through (1.6) are assumed to hold the multivariable A-function.

3. PROOFS

To prove the main results, we take some assumptions for convenience $\sum n_i s_i$ and $\sum \xi_j^{(i)} s_i$ denotes the r-terms sums

$\sum_{i=1}^{r} n_i s_i$ and $\sum_{i=1}^{r} \xi_i^{(i)} s_i$, respectively $\forall \ j = 1,...,r.$ (3.1)

Also, let

$$\Delta = \int_0^\infty ... \int_0^\infty x_1^{\sigma_1 - 1} ... x_r^{\sigma_r - 1} \ f(k_1 x_1^{\rho_1} + ... + k_r x_r^{\rho_r}) A_{\upsilon,C:\upsilon_1,c_1;...;\upsilon_r,c_r}^{\mu,\lambda:\mu_1,\lambda_1;...;\mu_r,\lambda_r} \begin{bmatrix} z_1 x_1^{\xi_1} \\ ... \\ z_r x_r^{\xi_r} \end{bmatrix} dx_1 ... dx_r, \tag{3.2}$$

where the X_i are defined by (2.2) and the function f is such that the multiple integral converges. On replacing the multivariable A-function occurring in (3.2) by contour integral given by (1.1), under the various conditions stated with (2.1), we find that

$$\Delta = \frac{1}{(2\pi\omega)^r} \int_{L_1} ... \int_{L_r} \theta_1(s_1)...\theta_r(s_r)\,\Phi(s_1,...,s_r)\, z_1^{s_1}...z_r^{s_r}$$

$$\left\{ \int_0^\infty ... \int_0^\infty x_1^{\sigma_1+\sum \xi_1^{(i)} s_i - 1} ... x_r^{\sigma_r + \sum \xi_{r1}^{(i)} s_i - 1}\, (k_1 x_1^{\rho_1} + ... + k_r x_r^{\rho_r})^{\sum n_i s_i} \right.$$

$$\left. f(k_1 x_1^{\rho_1} + ... + k_r x_r^{\rho_r}) dx_1 ... dx_r \right\} ds_1 ... ds_r. \tag{3.3}$$

Now we integrate the innermost $(x_1,...,x_r)$ -integral by using the following form of a known result [1, p. 173].

$$\int_0^\infty ... \int_0^\infty x_1^{\sigma_1 - 1} ... x_r^{\sigma_r - 1}\, (k_1 x_1^{\rho_1} + ... + k_r x_r^{\rho_r})^{\sigma} f(k_1 x_1^{\rho_1} + ... + k_r x_r^{\rho_r}) dx_1 ... dx_r$$

$$\Psi(k_1,...,k_r) \frac{\Gamma(\sigma_1/\rho_1)...\Gamma(\sigma_r/\rho_r)}{\Gamma(\sigma_1/\rho_1 + ... + \sigma_r/\rho_r)} \int_0^\infty z^{\sigma_1/\rho_1 + ... + \sigma_r/\rho_r + \sigma - 1} f(z) dz, \tag{3.4}$$

where $\Psi(k_1,...,k_r)$ is given by (2.4) and $\min\limits_{1 \le i \le r} \{k_i, \rho_i, \mathrm{Re}(\sigma_j)\} > 0$ then (3.3) reduces in the following form

$$\Delta = \frac{\Psi(k_1,...,k_r)}{(2\pi\omega)^r} \int_{L_1} ... \int_{L_r} \theta_1(s_1)...\theta_r(s_r)\,\Phi(s_1,...,s_r)\, Y_1^{s_1} ... Y_r^{s_r}\, \frac{\Gamma(\sigma_1^*/\rho_1)...\Gamma(\sigma_r^*/\rho_r)}{\Gamma(\sigma_1^*/\rho_1 + ... + \sigma_r^*/\rho_r)}$$

$$\left\{ \int_0^\infty z^{\sigma_1/\rho_1 + ... + \sigma_r/\rho_r + \sigma - 1} f(z) dz \right\} ds_1 ... ds_r, \tag{3.5}$$

where $\Psi(k_1,...,k_r),\ N_i$ and S are given by (2.4), (2.5) and (2.3) respectively, and $Y_i = z k_i^{\sum \xi_j^{(i)} / \rho_j}$,

(3.6)

$$\sigma_i^* = \sigma_j + \sum_{i=1}^{r} \xi_j^{(i)} s_j,\ \forall\ j = 1,...,r. \tag{3.7}$$

Now in the integral (3.5), we set

$$f(z) = z^{\sigma} H_{p,q}^{m,0} \left[z\xi \left| \begin{matrix} (e_1,\varepsilon_1),...,(e_p,\varepsilon_p) \\ (g_1,\gamma_1),...,(g_q,\gamma_q) \end{matrix} \right. \right] S_\beta^\alpha \left[z^h \gamma \right], \tag{3.8}$$

and evaluate the z-integral by following familiar formula (when n=0), expressing the Mellin transform of Fox's H-function [5, p.311, eq (3.3)]

$$M\left\{ H_{p,q}^{m,n}(zx) : s \right\} = \frac{\prod_{j=1}^{m} \Gamma(\beta_j + B_j s) \prod_{j=1}^{n} \Gamma(1 - \alpha_j - A_j s)}{\prod_{j=m+1}^{q} \Gamma(1 - \beta_j - B_j s) \prod_{j=n+1}^{p} (\alpha_j + A_j s)} z^{-s}. \tag{3.9}$$

Interpret the resulting $(s_1,...,s_r)$ - integral as an A-function of r-variable, we will obtain the required result given in (2.1).

Moreover to establish the other main integral (2.10) we can find relationship (3.5) in similar way and then we set

$$f(z) = z^{\sigma} \exp(-\gamma z) S_\beta^\alpha \left[z^h \eta \right] \tag{3.10}$$

Evaluate the innermost z-integral by using to a slightly modified version of following well-known integral [2, p. 292, eq. (1)]

$$M\left\{e^{-\gamma x}L_m^{(\alpha)}(\gamma x);s\right\}=\frac{\Gamma(\alpha-s+m+1)\Gamma(s)}{m!\Gamma(\alpha-s+1)}\gamma^{-s}. \tag{3.11}$$

If we interpret the resulting multiple contour integral as an A-function of r-variable, we will get desired result (2.10).

4. SPECIAL CASES

(1) For the general class of polynomials, we take the case of Hermite polynomials ([8, p. 106, eq. (5.54)] and [7, p. 158]) by setting $S_\beta^2[z]=z^{\beta/2}H_\beta\left[\frac{1}{2\sqrt{z}}\right]$ in which case $\alpha=2, A_{\beta,k}=(-1)^k$.

(i) **Integral 1(a):** The result (2.1) reduces in following form

$$\int_0^\infty\cdots\int_0^\infty x_1^{\sigma_1-1}\ldots x_r^{\sigma_r-1}(k_1x_1^{\rho_1}+\ldots+k_rx_r^{\rho_r})^{\sigma+\beta h/2}\eta^{\beta/2}H_\beta\left[\frac{1}{2\sqrt{\eta(k_1x_1^{\rho_1}+\ldots+k_rx_r^{\rho_r})^h}}\right]$$

$$H_{p,q}^{m,0}\left[\xi(k_1x_1^{\rho_1}+\ldots+k_rx_r^{\rho_r})\left|\begin{matrix}(e_1,\varepsilon_1),\ldots,(e_p,\varepsilon_p)\\(g_1,\gamma_1),\ldots,(g_q,\gamma_q)\end{matrix}\right.\right].A_{\upsilon,C:\upsilon_1,c_1;\ldots;\upsilon_r,c_r}^{\mu,\lambda:\mu_1,\lambda_1;\ldots;\mu_r,\lambda_r}\begin{bmatrix}z_1X_1\\\ldots\\z_rX_r\end{bmatrix}dx_1\ldots dx_r$$

$$=\xi^{-s}\Psi(k_1,\ldots,k_r)\sum_{k=0}^{[\beta/2]}\frac{(\beta)!(-1)^k}{(\beta-2k)!(k)!}\eta^k\xi^{-hk}.A_{\upsilon+r+q,C+p+1:\upsilon_1,c_1;\ldots;\upsilon_r,c_r}^{\mu,\lambda+r+m:\mu_1,\lambda_1;\ldots;\mu_r,\lambda_r}$$

$$\left[\begin{matrix}\left[1-\rho_j/\sigma_j:\xi'_j/\sigma_j,\ldots,\xi_j^{(r)}/\sigma_j\right]_{1,r},\left[1-g_j-(S+hk)\gamma_j;N_1\gamma_j,\ldots,N_r\gamma_j\right]_{1,q}:\\\left[1-S+\sigma:N_1-n_1,\ldots,N_r-n_r\right],\quad\left[1-e_j-(S+hk)\varepsilon_j;N_1\varepsilon_j,\ldots,N_r\varepsilon_j\right]_{1,p}:\end{matrix}\right.$$

$$\left.\begin{matrix}(a_j;A_j^{'};\ldots;A_j^{(r)})_{1,\upsilon}(\tau_j^{'},C_j^{'})_{1,\upsilon_1};\ldots;(\tau_j^{(r)},C_j^{(r)})_{1,\upsilon_r}\\(b_j;B_j^{'};\ldots;B_j^{(r)})_{1,C}(d_j^{'},D_j^{'})_{1,c_1};\ldots;(d_j^{(r)},D_j^{(r)})_{1,c_r}\end{matrix}\right|\begin{matrix}Z_1\\\ldots\\Z_r\end{matrix}\right] \tag{4.1}$$

Valid under the same conditions as obtainable from (2.1).

(ii) **Integral 1 (b):** The result (2.10) reduces in following form

$$\int_0^\infty\cdots\int_0^\infty x_1^{\sigma_1-1}\ldots x_r^{\sigma_r-1}(k_1x_1^{\rho_1}+\ldots+k_rx_r^{\rho_r})^{\sigma+\beta h/2}\eta^{\beta/2}H_\beta\left[\frac{1}{2\sqrt{\eta(k_1x_1^{\rho_1}+\ldots+k_rx_r^{\rho_r})^h}}\right]$$

$$\exp\left[-\gamma(k_1x_1^{\rho_1}+\ldots+k_rx_r^{\rho_r})\right].L_w^{(u)}\left[\gamma(k_1x_1^{\rho_1}+\ldots+k_rx_r^{\rho_r})\right]A_{\upsilon,C:\upsilon_1,c_1;\ldots;\upsilon_r,c_r}^{\mu,\lambda:\mu_1,\lambda_1;\ldots;\mu_r,\lambda_r}\begin{bmatrix}z_1x_1^{\xi_1}\\\ldots\\z_rx_r^{\xi_r}\end{bmatrix}dx_1\ldots dx_r$$

$$=\frac{(-1)^w\gamma^{-S}}{(w)!}\Psi(k_1,\ldots,k_r)\sum_{k=0}^{[\beta/2]}\frac{(\beta)!(-1)^k}{(\beta-2k)!(k)!}\eta^k\gamma^{-hk}.A_{\upsilon+2,C+1:\upsilon_1+1,c_1;\ldots;\upsilon_r+1,c_r}^{\mu,\lambda+2:\mu_1,\lambda_1+1;\ldots;\mu_r,\lambda_r+1}$$

$$\left[\begin{matrix}\left[1-S-hk;\xi_1/\rho_1,\ldots,\xi_r/\rho_r\right],\left[1-S-hk+u;\xi_1/\rho_1,\ldots,\xi_r/\rho_r\right],(a_j;A_j^{'};\ldots;A_j^{(r)})_{1,\upsilon};\\\left[1-S-hk+u+w;\xi_1/\rho_1,\ldots,\xi_r/\rho_r\right],\end{matrix}\right.$$

$$\left.\begin{matrix}(1-\sigma_1/\rho_1;\xi_1/\rho_1),(\tau_j^{'},C_j^{'})_{1,\upsilon_1};\ldots;(1-\sigma_r/\rho_r;\xi_r/\rho_r);(\tau_j^{(r)},C_j^{(r)})_{1,\upsilon_r}\\(b_j;B_j^{'};\ldots;B_j^{(r)})_{1,C}(d_j^{'},D_j^{'})_{1,c_1};\ldots;(d_j^{(r)},D_j^{(r)})_{1,c_r}\end{matrix}\right|\begin{matrix}\zeta_1\\\ldots\\\zeta_r\end{matrix}\right] \tag{4.2}$$

Valid under the same conditions as obtainable from (2.10).

(2) If we set

$\alpha = 1 \, and \; A_{\beta,k} = \binom{\beta+\nu}{\beta}\frac{1}{(\nu+1)_k}$, the general class of polynomial reduces in Laguerre polynomials ([8, p. 106, eq. (15, 16)] and [7, p. 159]) where Laguerre polynomials is given by

$$L_\beta^{(\nu)}[z] = \sum_{k=0}^{\beta}\binom{\beta+\nu}{\beta-k}\frac{(-z)^k}{(k)!}.$$

(i) **Integral 2(a):** The result (2.1) reduces in following form

$$\int_0^\infty \dots \int_0^\infty x_1^{\sigma_1-1}\dots x_r^{\sigma_r-1}(k_1x_1^{\rho_1}+\dots+k_rx_r^{\rho_r})^\sigma L_\beta^{(\nu)}[\eta(k_1x_1^{\rho_1}+\dots+k_rx_r^{\rho_r})^h]$$

$$H_{p,q}^{m,0}\left[\xi(k_1x_1^{\rho_1}+\dots+k_rx_r^{\rho_r})\left|\begin{matrix}(e_1,\varepsilon_1),\dots,(e_p,\varepsilon_p)\\(g_1,\gamma_1),\dots,(g_q,\gamma_q)\end{matrix}\right.\right].A_{\upsilon,C:\upsilon_1,c_1;\dots;\upsilon_r,c_r}^{\mu,\lambda:\mu_1,\lambda_1;\dots;\mu_r,\lambda_r}\begin{bmatrix}z_1X_1\\ \dots\\ z_rX_r\end{bmatrix}dx_1\dots dx_r$$

$$= \xi^{-s}\Psi(k_1,\dots,k_r)\sum_{k=0}^{\beta}\binom{\beta+\nu}{\beta-k}\frac{(-\eta)^k}{(k)!}\xi^{-hk}.A\begin{matrix}\mu,\lambda+r+m:\mu_1,\lambda_1;\dots;\mu_r,\lambda_r\\ \upsilon+r+q,C+p+1:\upsilon_1,c_1;\dots;\upsilon_r,c_r\end{matrix}$$

$$\left[\begin{matrix}\left[1-\rho_j/\sigma_j:\xi'_j/\sigma_j,\dots,\xi_j^{(r)}/\sigma_j\right]_{1,r},\left[1-g_j-(S+hk)\gamma_j;N_1\gamma_j,\dots,N_r\gamma_j\right]_{1,q}:\\ \left[1-S+\sigma:N_1-n_1,\dots,N_r-n_r\right],\quad\left[1-e_j-(S+hk)\varepsilon_j;N_1\varepsilon_j,\dots,N_r\varepsilon_j\right]_{1,p}:\end{matrix}\right.$$

$$\left.\begin{matrix}(a_j;A_j^{'};\dots;A_j^{(r)})_{1,\upsilon}(\tau_j^{'},C_j^{'})_{1,\upsilon_1};\dots;(\tau_j^{(r)},C_j^{(r)})_{1,\upsilon_r}\\(b_j;B_j^{'};\dots;B_j^{(r)})_{1,C}(d_j^{'},D_j^{'})_{1,c_1};\dots;(d_j^{(r)},D_j^{(r)})_{1,c_r}\end{matrix}\right|\begin{matrix}Z_1\\ \dots\\ Z_r\end{matrix}\right] \tag{4.3}$$

Valid under the same conditions as obtainable from (2.1).

(ii) **Integral 2 (b):** The result (2.10) reduces in following form

$$\int_0^\infty \dots \int_0^\infty x_1^{\sigma_1-1}\dots x_r^{\sigma_r-1}(k_1x_1^{\rho_1}+\dots+k_rx_r^{\rho_r})^\sigma L_\beta^{(\nu)}[\eta(k_1x_1^{\rho_1}+\dots+k_rx_r^{\rho_r})^h]$$

$$\exp\left[-\gamma(k_1x_1^{\rho_1}+\dots+k_rx_r^{\rho_r})\right].L_w^{(u)}\left[\gamma(k_1x_1^{\rho_1}+\dots+k_rx_r^{\rho_r})\right]A_{\upsilon,C:\upsilon_1,c_1;\dots;\upsilon_r,c_r}^{\mu,\lambda:\mu_1,\lambda_1;\dots;\mu_r,\lambda_r}\begin{bmatrix}z_1x_1^{\xi_1}\\ \dots\\ z_rx_r^{\xi_r}\end{bmatrix}dx_1\dots dx_r$$

$$\Psi(k_1,\dots,k_r)\sum_{k=0}^{\beta}\binom{\beta+\nu}{\beta-k}\frac{(-\eta)^k}{(k)!}\gamma^{-hk}.A\begin{matrix}\mu,\lambda+2:\mu_1,\lambda_1+1;\dots;\mu_r,\lambda_r+1\\ \upsilon+2,C+1:\upsilon_1+1,c_1;\dots;\upsilon_r+1,c_r\end{matrix}$$

$$\left[\begin{matrix}\left[1-S-hk;\xi_1/\rho_1,\dots,\xi_r/\rho_r\right],\left[1-S-hk+u;\xi_1/\rho_1,\dots,\xi_r/\rho_r\right],(a_j;A_j^{'};\dots;A_j^{(r)})_{1,\upsilon};\\ \left[1-S-hk+u+w;\xi_1/\rho_1,\dots,\xi_r/\rho_r\right],\end{matrix}\right.$$

$$\left.\begin{matrix}(1-\sigma_1/\rho_1;\xi_1/\rho_1),(\tau_j^{'},C_j^{'})_{1,\upsilon_1};\dots;(1-\sigma_r/\rho_r;\xi_r/\rho_r);(\tau_j^{(r)},C_j^{(r)})_{1,\upsilon_r}\\(b_j;B_j^{'};\dots;B_j^{(r)})_{1,C}(d_j^{'},D_j^{'})_{1,c_1};\dots;(d_j^{(r)},D_j^{(r)})_{1,c_r}\end{matrix}\right|\begin{matrix}\zeta_1\\ \dots\\ \zeta_r\end{matrix}\right] \tag{4.4}$$

Valid under the same conditions as obtainable from (2.10).

(2) For the Jacobi polynomials [8, p. 68, eq. (15, 16)] and [7, p. 159] by setting

$S_\beta^1[z] = P_\beta^{(s,t)}[1-2z]$ in which case $\alpha = 1$ and $A_{\beta,k} = \binom{\beta+s}{\beta}\frac{(s+t+\beta+1)_k}{(s+1)_k}$

(i) **Integral 3(a):** The result (2.1) reduces in following form

$$\int_0^\infty \cdots \int_0^\infty x_1^{\sigma_1-1} \cdots x_r^{\sigma_r-1}(k_1x_1^{\rho_1}+\ldots+k_rx_r^{\rho_r})^{\sigma} P_\beta^{(s,t)}[1-2\eta(k_1x_1^{\rho_1}+\ldots+k_rx_r^{\rho_r})^h]$$

$$H_{p,q}^{m,0}\left[\xi(k_1x_1^{\rho_1}+\ldots+k_rx_r^{\rho_r})\left|\begin{matrix}(e_1,\varepsilon_1),\ldots,(e_p,\varepsilon_p)\\(g_1,\gamma_1),\ldots,(g_q,\gamma_q)\end{matrix}\right.\right].A_{\upsilon,C:\upsilon_1,c_1;\ldots;\upsilon_r,c_r}^{\mu,\lambda:\mu_1,\lambda_1;\ldots;\mu_r,\lambda_r}\begin{bmatrix}z_1X_1\\ \ldots\\ z_rX_r\end{bmatrix}dx_1\ldots dx_r$$

$$=\xi^{-s}\Psi(k_1,\ldots,k_r)\sum_{k=0}^{\beta}\binom{\beta+s}{\beta-k}\binom{\beta+t+k+s}{k}\xi^{-hk}.A\begin{matrix}\mu,\lambda+r+m:\mu_1,\lambda_1;\ldots;\mu_r,\lambda_r\\ \upsilon+r+q,C+p+1:\upsilon_1,c_1;\ldots;\upsilon_r,c_r\end{matrix}$$

$$\left[\begin{matrix}\left[1-\rho_j/\sigma_j:\xi'_j/\sigma_j,\ldots,\xi_j^{(r)}/\sigma_j\right]_{1,r},\left[1-g_j-(S+hk)\gamma_j;N_1\gamma_j,\ldots,N_r\gamma_j\right]_{1,q}:\\ \left[1-S+\sigma:N_1-n_1,\ldots,N_r-n_r\right],\quad\left[1-e_j-(S+hk)\varepsilon_j;N_1\varepsilon_j,\ldots,N_r\varepsilon_j\right]_{1,p}:\end{matrix}\right.$$

$$\left.\begin{matrix}(a_j;A_j';\ldots;A_j^{(r)})_{1,\upsilon}(\tau_j',C_j')_{1,\upsilon_1};\ldots;(\tau_j^{(r)},C_j^{(r)})_{1,\upsilon_r}\\(b_j;B_j';\ldots;B_j^{(r)})_{1,C}(d_j',D_j')_{1,c_1};\ldots;(d_j^{(r)},D_j^{(r)})_{1,c_r}\end{matrix}\right|\begin{matrix}Z_1\\ \ldots\\ Z_r\end{matrix}\right] \tag{4.5}$$

Valid under the same conditions as obtainable from (2.1).

(ii) **Integral 3 (b):** The result (2.10) reduces in following form

$$\int_0^\infty \cdots \int_0^\infty x_1^{\sigma_1-1} \cdots x_r^{\sigma_r-1}(k_1x_1^{\rho_1}+\ldots+k_rx_r^{\rho_r})^{\sigma} P_\beta^{(s,t)}[1-2\eta(k_1x_1^{\rho_1}+\ldots+k_rx_r^{\rho_r})^h]$$

$$\exp\left[-\gamma(k_1x_1^{\rho_1}+\ldots+k_rx_r^{\rho_r})\right].L_w^{(u)}\left[\gamma(k_1x_1^{\rho_1}+\ldots+k_rx_r^{\rho_r})\right]A_{\upsilon,C:\upsilon_1,c_1;\ldots;\upsilon_r,c_r}^{\mu,\lambda:\mu_1,\lambda_1;\ldots;\mu_r,\lambda_r}\begin{bmatrix}z_1x_1^{\xi_1}\\ \ldots\\ z_rx_r^{\xi_r}\end{bmatrix}dx_1\ldots dx_r$$

$$=\frac{(-1)^w\gamma^{-S}}{(w)!}\Psi(k_1,\ldots,k_r)\sum_{k=0}^{\beta}\binom{\beta+s}{\beta-k}\binom{\beta+t+k+s}{k}\gamma^{-hk}.A\begin{matrix}\mu,\lambda+2:\mu_1,\lambda_1+1;\ldots;\mu_r,\lambda_r+1\\ \upsilon+2,C+1:\upsilon_1+1,c_1;\ldots;\upsilon_r+1,c_r\end{matrix}$$

$$\left[\begin{matrix}\left[1-S-hk;\xi_1/\rho_1,\ldots,\xi_r/\rho_r\right],\left[1-S-hk+u;\xi_1/\rho_1,\ldots,\xi_r/\rho_r\right],(a_j;A_j';\ldots;A_j^{(r)})_{1,\upsilon};\\ \left[1-S-hk+u+w;\xi_1/\rho_1,\ldots,\xi_r/\rho_r\right],\end{matrix}\right.$$

$$\left.\begin{matrix}(1-\sigma_1/\rho_1;\xi_1/\rho_1),(\tau_j',C_j')_{1,\upsilon_1};\ldots;(1-\sigma_r/\rho_r;\xi_r/\rho_r);(\tau_j^{(r)},C_j^{(r)})_{1,\upsilon_r}\\(b_j;B_j';\ldots;B_j^{(r)})_{1,C}(d_j',D_j')_{1,c_1};\ldots;(d_j^{(r)},D_j^{(r)})_{1,c_r}\end{matrix}\right|\begin{matrix}\zeta_1\\ \ldots\\ \zeta_r\end{matrix}\right] \tag{4.6}$$

Valid under the same conditions as obtainable from (2.10).

(2) If we take $\beta \to 0$ *and* $\mu = 0$ in result (2.1), we obtain a known result obtained by Srivastava and Panda [6, p. 354, eq. (1.8)].

(3) If we take $\beta \to 0$ *and* $\mu = 0$ in result (2.10), we obtain a known result obtained by Srivastava and Panda [6, p. 354, eq. (1.14)].

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