International Journal of Mathematical Archive-7(5), 2016, 151-160
Available online through www.ijma.info ISSN 2229 – 5046

# INTERVAL VALUED INTUITIONISTIC ANTI FUZZY PRIMARY IDEAL

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*(Received On: 09-05-16; Revised & Accepted On: 30-05-16)*

***ABSTRACT***

*The concept of interval valued intuitionistic anti fuzzy primary ideals and interval valued intuitionistic anti fuzzy semiprimary ideals is defined. Various properties of interval valued intuitionistic anti fuzzy primary ideals and interval valued intuitionistic anti fuzzy semiprimary ideals are discussed. Finally, interval valued intuitionistic anti fuzzy primary ideals and interval valued intuitionistic anti fuzzy semiprimary ideals is defined, some properties are established.*

***Keywords:*** *Intuitionistic fuzzy set, Intuitionistic anti fuzzy primary ideal, Intuitionistic anti fuzzy semi-primary ideal, Interval valued intuitionistic anti fuzzy primary ideals, Interval valued intuitionistic anti fuzzy semi primary ideals.*

## 1. INTRODUCTION

After on introduction of fuzzy sets by A.Zadeh[8], the fuzzy concept has introduced almost all branches of mathematics. The notion of intuitionistic fuzzy set and its operations were introduced by Atanassov [1], as a generalization of the notion of fuzzy set. Atanassov [2, 3] discussed the operators over interval valued intuitionistic fuzzy sets. Palanivelrajan and Nandakumar [6] introduced the definition and some properties of intuitionistic anti fuzzyprimary and semiprimary ideals and interval valued intuitionistic fuzzy primary ideals. In this paper interval valued intuitionistic anti fuzzy primary ideals and intuitionistic anti fuzzy semiprimary ideal are established.

## 2. PRELIMINARIES

In this section, some basic definitions that are essential for this paper are assembled.

**Definition 2.1:** Let $S$ be any nonempty set. A mapping $\mu : S \to [0,1]$ is called a fuzzy subset of $S$.

**Definition 2.2:** A fuzzy subset $\mu$ of a ring $R$ is called anti fuzzy ideal if for all $x, y \in R$ the following conditions are satisfied

$(i)\ \mu(x-y) \le \max(\mu(x), \mu(y))$

$(ii)\ \mu(xy) \le \min(\mu(x), \mu(y))$

**Definition 2.3:** A fuzzy ideal $\mu$ of a ring $R$ is called anti fuzzy primary ideal, if for all $a, b \in R$ either $\mu(ab) = \mu(a)$ or else $\mu(ab) \ge \mu(b^m)$ for some $m \in Z^+$.

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**Definition 2.4:** A fuzzy ideal $\mu$ of a ring $R$ is called anti fuzzy semiprimary ideal, if for all $a$ , $b \in R$ either $\mu(ab) \geq \mu(a^n)$, for some $n \in Z^+$ , or else $\mu(ab) \geq \mu(b^m)$ for some $m \in Z^+$.

**Definition 2.5:** An intuitionistic fuzzy set (IFS) $A$ in $X$ is defined as an object of the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle \mid x \in X\}$, where $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ are the degree of membership and the degree of non-membership of the element $x \in X$, respectively, for every $x \in X$, satisfying $0 \leq \mu_A(x) + \gamma_A(y) \leq 1$.

**Definition 2.6:** A fuzzy subset $\mu$ of a ring $R$ is called intuitionistic anti fuzzy ideal if for all $x, y \in R$ , the following conditions are satisfied,

$(i)$ $\mu_A(x-y) \leq max(\mu_A(x), \mu_A(y))$

$(ii)$ $\mu_A(xy) \leq min(\mu_A(x), \mu_A(y))$

$(iii)$ $\gamma_A(x-y) \geq min(\gamma_A(x), \gamma_A(y))$ $(iv)$ $\gamma_A(xy) \geq max(\gamma_A(x), \gamma_A(y))$

**Definition 2.7:** An intuitionistic anti fuzzy ideal $R$ of a ring $R$ is called Intuitionistic anti fuzzy primary ideal if for all $a, b \in R$ either $\mu_A(ab) = \mu_A(a)$ and $\gamma_A(ab) = \gamma_A(a)$, or $\mu_A(ab) \geq \mu_A(b^m)$ and $\gamma_A(ab) \leq \gamma_A(b^m)$, for some $m \in Z^+$.

**Example 2.1:**

$$\mu_A(x) = \begin{cases} 0 & if \ x \in < 4 > \\ 0.3 & if \ x \in < 2 > \sim < 4 > \end{cases}$$

$$\gamma_A(x) = \begin{cases} 1 & if \ x \in < 8 > \\ 0.6 & if \ x \in < 2 > \sim < 8 > \end{cases}$$

**Definition 2.8:** An intuitionistic anti fuzzy ideal $A$ of a ring $R$ is called intuitionistic anti fuzzy semiprimary ideal if for all $a$ , $b \in R$ either $\mu_A(ab) \geq \mu_A(a^n)$ and $\gamma_A(ab) \leq \gamma_A(a^n)$ , for some $n \in Z^+$ or else $\mu_A(ab) \geq \mu_A(b^m)$ and $\gamma_A(ab) \leq \gamma_A(b^m)$ for some $m \in Z^+$.

**Definition 2.9:** An interval valued fuzzy set $A$ is specified by a function $M_A : E \to D[0,1]$, where $D[0,1]$ is the set of all intervals with in [0, 1], for all $x \in E, M_A(x)$ is an interval $[a,b]$, where $0 \leq a \leq b \leq 1$.

**Remark 2.1:** An interval valued intuitionistic fuzzy set. A over E is defined as an object of the form $A = \{x, M_A(x), N_A(x) \mid x \in E\}$ where $M_A(x) \subseteq [0,1]$ and $N_A(x) \subseteq [0,1]$ are interval and for all $x \in E$, $rmaxM_A(x) + rmaxN_A(x) \leq 1$.

**Definition 2.10:** Let A be an interval valued intuitionistic fuzzy sets. A fuzzy ideal A of a ring $R$ is said to be interval valued intuitionistic anti fuzzy primary ideal of $R$ if for all $a, b \in R$ then either

$\mu_A(ab) = rmaxM_A(ab) = rmaxM_A(a) = \mu_A(a)$ and $\gamma_A(ab) = rmaxN_A(ab) = rmaxN_A(a) = \gamma_A(a)$ or

$\gamma_A(ab)\ r\max M_A(ab) \geq r\max M_A(b^n) = \mu_A(b^n)$ and $\gamma_A(\mathrm{ab}) = rmaxN_A(ab) \leq rmaxN_A(b^n) = \gamma_A(b^n)$, for some $n \in Z^+$.

**Definition 2.11:** Let $A$ be an interval valued intuitionistic fuzzy sets. A fuzzy ideal $A$ of a ring $R$ is said to be interval valued intuitionistic anti fuzzy primary ideal of $R$ if for all $a,\ b \in R$ then either

$\mu_A(ab) = rmaxM_A(ab) \geq rmaxM_A(a^n) = \mu_A\left(a^n\right)$ and

$\gamma_A\left(ab\right) = rmax\ N_A\left(ab\right) \leq rmax\ N_A(a^n) = \gamma_A(a^n)$, for some $n \in Z^+$ or

$\mu_A(ab) = rmax\ M_A\left(ab\right) \geq rmax\ \mu_A(b^m) = \mu_A(b^m)$ and

$\gamma_A(ab) = rmax\ N_A(ab) \leq rmax N_A(b^m) = \gamma_A(b^m)$ for some $m \in Z^+$.

**Example 2.2:**

$$\mu_A(x) = rmaxM_A(x) = \begin{cases} [0,0] & if\ x = 0 \\ [0.2, 0.3] & \text{otherwise} \end{cases}$$

$$\gamma_A(x) = r\max N_A(x) = \begin{cases} [1,1] & if\ x = 0 \\ [0.4, 0.5] & \text{otherwise} \end{cases}$$

**Definition 2.12:** Let $A$ and $B$ are interval valued intuitionistic fuzzy set then A∩B is an interval valued intuitionistic fuzzy set E. If the following conditions are satisfied

$$A \cap B = \{\langle x, [min\left(rmin\ M_A\left(x\right), rmin\ M_B(x)\right), max(rminN_A(x), rminN_B(x)), \rangle / x \in E\},$$

**Definition 2.13:** Let A and B are interval valued intuitionistic fuzzy set then $A \cup B$ is an interval valued intuitionistic fuzzy set E. If the following conditions are satisfied

$$A \cup B = \{\langle x, [max(rminM_A(x), rmin\ M_B(x)), min(rminN_A(x), rminN_B(x)), \rangle / x \in E\},$$

**Definition 2.14:** Let A and B are interval valuedintuitionistic fuzzy set then $A + B$ is an interval valued intuitionistic fuzzy set E. If the following conditions are satisfied

$$A + B = \{\langle x, rminM_A(x) + rminM_B(x) - r\min M_A(x).\ r\min M_B(x), r\min N_A(x).\ r\min N_B(x) \ \rangle \mid x \in E\},$$

**Definition 2.15:** Let A and B are interval valued intuitionistic fuzzy set then $A.B$ is an interval valued intuitionistic fuzzy set E. If the following conditions are satisfied

$$A.B = \{\langle x, rminM_A(x).rminM_B(x), rmin\ N_A(x) + rmin\ \ _B(x) - NrminM_A(x).\ rmin\ N_B(x), \rangle \mid xE\},$$

**Definition 2.16:** Let A and B are interval valued intuitionistic fuzzy set then $A @ B$ is an interval valued intuitionistic fuzzy set E. If the foll0owing conditions are satisfied

$$A @ B = \left\{\langle x, \frac{r\min M_A(x) + r\min M_B(x)}{2}, \frac{\text{rminN}_A(x) + \text{rminN}_B(x)}{2}, > / x \in E\right\}.$$

**Definition 2.17:** Let A and B are interval valued intuitionistic fuzzy set then $A\$B$ is an interval valued intuitionistic fuzzy set E. If the following conditions are satisfied

$$A\$B = \left\{\langle, \left[\sqrt{r\min M_A(x).r\min M_B(x)}, \sqrt{r\min N_A(x).r\min N_B(x)}\right]\rangle \mid x \in E\right\}.$$

**Definition 2.18:** Let A and B are interval valued intuitionistic fuzzy set then A#B is an interval valued intuitionistic fuzzy set E. If the following conditions are satisfied

$$A\#B = \{< x, \frac{2.\text{rminM}_A(x).\text{rminM}_B(x)}{\left(\text{rminM}_A(x) + \text{rminM}_B(x)\right)}, \frac{2.\text{rminN}_A(x).\text{rminN}_B(x)}{\left(\text{rminN}_A(x) + \text{rminN}_B(x)\right)} \rangle \mid x \in E\}$$

**Definition 2.19:** Let A and B are interval valued intuitionistic fuzzy set then A*B is an interval valued intuitionistic fuzzy set E. If the following conditions are satisfied

$$A*B = \left\{\langle x, \left[\frac{\text{rminM}_A(x) + \text{rminM}_B(x)}{2.\left(\text{rminM}_A(x).\text{rminM}_B(x) + 1\right)}, \frac{\text{rminN}_A(x) + \text{rminN}_B(x)}{2.\left(\text{rminN}_A(x).\ \text{rminN}_B(x) + 1\right)}\right], \rangle \mid x \in E.\right.$$

**3. INTERVAL VALUED INTUITIONISTIC ANTI FUZZY PRIMARY IDEALS AND INTERVAL VALUED INTUITIONISTIC ANTI FUZZY SEMIPRIMARY IDEALS**

**Theorem 3.1:** If A and B are interval valued intuitionistic anti fuzzy semiprimary ideal of *R*, then $A \cap B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of *R*.

**Proof:** Let A be an interval valued intuitionistic anti fuzzy semiprimary ideal of a ring R, then

$\mu_A(xy) = rmaxM_A(xy) \geq rmaxM_A(x^n) = \mu_A(\mathrm{xy})$ and

$\gamma_A(xy) = rmaxN_A(xy) \leq rmaxN_A(x^n) = \gamma_A(x^n)$ for some $n \in Z^+$ and $x, y \in R$.

Let B be an interval valued intuitionistic anti fuzzy semiprimary ideal of the ring R, then

$\mu_B(xy) = rmax\mu_B(xy) \geq rmaxM_B(x^n) = \mu_B(x^n)$ *and* $\gamma_B(xy) = rmaxN_B(xy) \leq rmaxN_B(x^n) = \gamma_B(x^n)$, for some $n \in Z^+$ and $x, y \in R$.

Consider $x, y \in R$, then $x, y \in A \cap B$ implies $x, y \in A$ *and* $x, y \in B$

Consider

$$
\begin{aligned}
\mu_{A\cap B}(xy) &= r\max M_{A\cap B}(xy) \\
&= \max(r\max M_A(xy), r\max M_B(xy)) \\
&\geq \max(r\max M_A(x^n), r\max M_B(x^n)) \\
&= r\max M_{A\cap B}(x^n) \\
&= \mu_{A\cap B}(x^n)
\end{aligned}
$$

Therefore, $\mu_{A\cap B}(xy) \geq \gamma_{A\cap B}(x^n)$.

Consider

$$
\begin{aligned}
\gamma_{A\cap B}(xy) &= r\max N_{A\cap B}(xy) \\
&= \min(r\max N_A(xy), r\max N_B(xy)) \\
&\leq \min(r\max N_A(x^n), r\max N_B(x^n)) \\
&= r\max N_{A\cap B}(x^n) \\
&= \gamma_{A\cap B}(x^n)
\end{aligned}
$$

Therefore, $\gamma_{A\cap B}(xy) \leq \gamma_{A\cap B}(x^n)$.

Therefore $A \cap B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

**Theorem 3.2:** If A and B are interval valued intuitionistic anti fuzzy semiprimary ideal of R, then $A \cup B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

**Proof:** Let A be an interval valued intuitionistic anti fuzzy semiprimary ideal of a ring R, then

$\mu_A(xy) = rmaxM_A(xy) \geq rmaxM_A(x^n) = \mu_A(x^n)$ and

$\gamma_A(xy) = rmaxN_A(xy) \leq rmaxN_A(x^n) = \gamma_A(x^n)$ for some $n \in Z^+$ and $x, y \in R$.

Let B be an interval valued intuitionistic anti fuzzy semiprimary ideal of the ring R, then

$\mu_B(xy) = rmax\mathrm{M}_B(xy) \geq rmaxM_B(x^n) = \mu_B(x^n)$ and

$\gamma_B(xy) = rmaxN_B(xy) \leq rmaxN_B(x^n) = \gamma_B(x^n)$, for some $n \in Z^+$ and $x, y \in R$.

Consider $x, y \in R$, then $x, y \in A \cup B$ implies $x, y \in A$ *and* $x, y \in B$

 

Consider

$$\begin{aligned}\mu_{A\cup B}(xy) &= r\max M_{A\cup B}(xy)\\ &= \min(r\max M_A(xy), r\max M_B(xy))\\ &\geq \min(r\max M_A(x^n), r\max M_B(x^n))\\ &= r\max M_{A\cap B}(x^n)\\ &= \mu_{A\cup B}(x^n)\end{aligned}$$

Therefore, $\mu_{A\cup B}(xy) \geq \mu_{A\cup B}(x^n)$.

Consider

$$\begin{aligned}\gamma_{A\cup B}(xy) &= r\max N_{A\cup B}(xy)\\ &= \max(r\max N_A(xy), r\max N_B(xy))\\ &\leq \max(r\max N_A(x^n), r\max N_B(x^n))\\ &= r\max N_{A\cup B}(x^n)\\ &= \gamma_{A\cup B}(x^n)\end{aligned}$$

Therefore, $\gamma_{A\cup B}(xy) \leq \gamma_{A\cup B}(x^n)$.

Therefore $A\cup B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

**Theorem 3.3:** If A and B are interval valued intuitionistic anti fuzzy semiprimary ideal of R, then $A+B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

**Proof:** Let A be an interval valued intuitionistic anti fuzzy semiprimary ideal of a ring R, then

$$\mu_A(xy) = rmaxM_A(xy) \geq rmaxM_A(x^n) = \mu_A(x^n) \text{ and}$$

$$\gamma_A(xy) = rmaxN_A(xy) \leq rmaxN_A(x^n) = \gamma_A(x^n) \text{ for some } n\in Z^+ \text{ and } x, y\in R.$$

Let B be an interval valued intuitionistic anti fuzzy semiprimary ideal of the ring R, then

$$\mu_B(xy) = rmax\mathrm{M}_B(xy) \geq rmaxM_B(x^n) = \mu_B(x^n) \; and$$

$$\gamma_B(xy) = rmax\; N_B(xy) \leq rmaxN_B(x^n) = \gamma_B(x^n), \text{ for some } n\in Z^+ \text{ and } x, y\in R.$$

Consider $x, y\in R$, then $x, y\in A+B$ implies $x, y\in A \; and \; x,\; y\in B$

consider

$$\begin{aligned}\mu_{A+B}(xy) &= r\max M_{A+B}(xy)\\ &= (r\max M_A(xy) + r\max M_B(xy) - r\max M_A(xy).r\max M_B(xy))\\ &\geq (r\max M_A(x^n) + r\max M_B(x^n) - r\max M_A(x^n).r\max M_B(x^n))\\ &= \mu_A(x^n) + \mu_B(x^n) - \mu_A(x^n).\mu_B(x^n)\\ &= \mu_{A+B}(x^n).\end{aligned}$$

;

Therefore, $\mu_{A+B}(xy) \geq \mu_{A+B}(x^n)$

Consider

$$\begin{aligned}\gamma_{A+B}(xy) &= r\max N_{A+B}(xy)\\ &= r\max N_A(xy).r\max N_B(xy)\\ &\geq r\max N_A(x^n).r\max N_B(x^n)\\ &= \gamma_A(x^n).\gamma_B(x^n)\\ &= \gamma_{A+B}(x^n).\end{aligned}$$

Therefore, $\gamma_{A+B}(xy) \leq \gamma_{A+B}(x^n)$

Therefore, $A+B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

**Theorem 3.4:** If A and B are interval valued intuitionistic anti fuzzy semiprimary ideal of R, then A.B is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

**Proof:** Let A be an interval valued intuitionistic anti fuzzy semiprimary ideal of a ring R, then

$$\mu_A(xy) = rmaxM_A(xy) \geq rmax\ M_A(x^n) = \mu_A(x^n)\ and$$

$$\gamma_A(xy) = rmaxN_A(xy) \leq rmaxN_A(x^n) = \gamma_A(x^n) \text{ for some } n \in Z^+ \text{ and } x, y \in R.$$

Let B be an interval valued intuitionistic anti fuzzy semiprimary ideal of the ring R, then

$$\mu_B(xy) = rmax\mathrm{M}_B(xy) \geq rmaxM_B(x^n) = \mu_B(x^n)\ and$$

$$\gamma_B(xy) = rmaxN_B(xy) \leq rmaxN_B(x^n) = \gamma_B(x^n), \text{ for some } n \in Z^+ \text{ and } x, y \in R.$$

Consider $x, y \in R,$ then $x, y \in A.B$ implies $x, y \in A\ and\ x,\ y \in B$

Consider

$$\begin{aligned}\mu_{A.B}(xy) &= r\max M_{A.B}(xy)\\ &= r\max M_A(xy).r\max M_B(xy))\\ &\geq r\max M_A(x^n).r\max M_B(x^n))\\ &= \mu_A(x^n).\mu_B(x^n)\\ &= \mu_{A.B}(x^n).\end{aligned}$$

Therefore, $\mu_{A.B}(xy) \geq \mu_{A.B}(x^n)$

Consider

$$\begin{aligned}\gamma_{A.B}(xy) &= r\max N_{A.B}(xy)\\ &= (r\max N_A(xy) + r\max N_B(xy) - r\max N_A(xy).r\max N_B(xy))\\ &\leq (r\max N_A(x^n) + r\max N_B(x^n) - r\max N_A(x^n).r\max N_B(x^n))\\ &= \gamma_A(x^n).\gamma_B(x^n) - \gamma_A(x^n).\gamma_B(x^n)\\ &= \gamma_{A.B}(x^n).\end{aligned}$$

Therefore, $\gamma_{A.B}(xy) \leq \gamma_{A.B}(x^n)$

Therefore, $A.B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

**Theorem 3.5:** If A and B are interval valued intuitionistic anti fuzzy semiprimary ideal of R, then A@B is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

**Proof:** Let A be an interval valued intuitionistic anti fuzzy semiprimary ideal of a ring R, then

$$\mu_A(xy) = rmaxM_A(xy) \geq \ rmaxM_A(x^n) = \mu_A(x^n) \ and$$

$$\gamma_A(xy) = rmaxN_A(xy) \leq rmaxN_A(x^n) = \gamma_A(x^n) \text{ for some } n \in Z^+ \text{ and } x, y \in R.$$

Let B be an interval valued intuitionistic anti fuzzy semiprimary ideal of the ring R, then

$$\mu_B(xy) = rmaxM_B(xy) \geq rmaxM_B(x^n) = \mu_B(x^n) \ and$$

$$\gamma_B(xy) = rmax\ N_B(xy) \leq rmaxN_B(x^n) = \gamma_B(x^n), \text{ for some } n \in Z^+ \text{ and } x, y \in R.$$

Consider $x, y \in R,$ then $x, y \in A@B$ implies $x, y \in A \ and \ x, \ y \in B$

Consider

$$\begin{aligned}\mu_{A@B}(xy) &= r\max M_{A@B}(xy)\\ &= \frac{r\max M_A(xy) + r\max M_B(xy)}{2}\\ &\geq \frac{r\max M_A(x^n) + r\max M_B(x^n)}{2}\\ &= \frac{\mu_A(x^n) + \mu_A(x^n)}{2}\\ &= \mu_{A@B}(x^n)\end{aligned}$$

Therefore, $\mu_{A@B}(xy) \geq \mu_{A@B}(x^n)$

Consider

$$\begin{aligned}\gamma_{A@B}(xy) &= r\max N_{A@B}(xy)\\ &= \frac{r\max N_A(xy) + r\max N_B(xy)}{2}\\ &\leq \frac{r\max N_A(x^n) + r\max N_B(x^n)}{2}\\ &= \frac{\gamma_A(x^n) + \gamma_A(x^n)}{2}\\ &= \mu_{A@B}(x^n)\end{aligned}$$

Therefore, $\gamma_{A@B}(xy) \leq \gamma_{A@B}(x^n)$

Therefore, $A@B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

**Theorem 3.6:** If A and B are interval valued intuitionistic anti fuzzy semiprimary ideal of R, then A$B is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

**Proof:** Let A be an interval valued intuitionistic anti fuzzy semiprimary ideal of a ring R, then

$$\mu_A(xy) = rmaxM_A(xy) \geq \ rmax\ M_A(x^n) = \mu_A(x^n) \ and$$

$$\gamma_A(xy) = rmaxN_A(xy) \leq rmax\ N_A(x^n) = \ \gamma_A(x^n) \text{ for some } n \in Z^+ \text{ and } x, y \in R.$$

Let B be an interval valued intuitionistic anti fuzzy semiprimary ideal of the ring R, then

$$\mu_B(xy) = rmaxM_B(xy) \geq rmaxM_B(x^n) = \mu_B(x^n) \ and$$

$$\gamma_B(xy) = rmaxN_B(xy) \leq rmaxN_B(x^n) = \gamma_B(x^n), \text{ for some } n \in Z^+ \text{ and } x, y \in R.$$

Consider $x, y \in R,$ then $x, y \in A\$B$ implies $x, y \in A \; and \; x, \; y \in B$

Consider

$$\begin{aligned}
\mu_{A\$B}(xy) &= r\max M_{A\$B}(xy) \\
&= \sqrt{r\max M_A(xy) r\max M_B(xy)} \\
&\geq \sqrt{r\max M_A(x^n) r\max M_B(x^n)} \\
&= \sqrt{\mu_A(x^n)\mu_B(x^n)} \\
&= \mu_{A\$B}(x^n)
\end{aligned}$$

Therefore, $\mu_{A\$B}(xy) \geq \mu_{A\$B}(x^n)$ .

Consider

$$\begin{aligned}
\gamma_{A\$B}(xy) &= r\max N_{A\$B}(xy) \\
&= \sqrt{r\max N_A(xy) r\max N_B(xy)} \\
&\leq \sqrt{r\max N_A(x^n) r\max N_B(x^n)} \\
&= \sqrt{\gamma_A(x^n)\gamma_B(x^n)} \\
&= \gamma_{A\$B}(x^n)
\end{aligned}$$

Therefore, $\gamma_{A\$B}(xy) \leq N_{A\$B}(x^n)$

Therefore, $A\$B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

**Theorem 3.7:** If A and B are interval valued intuitionistic anti fuzzy semiprimary ideal of R, then A#B is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

**Proof:** Let A be an interval valued intuitionistic anti fuzzy semiprimary ideal of a ring R, then

$$\mu_A(xy) = rmaxM_A(xy) \geq \; rmaxM_A(x^n) = \mu_A(x^n) \; and$$

$$\gamma_A(xy) = rmaxN_A(xy) \leq rmaxN_A(x^n) = \; \gamma_A(x^n) \text{ for some } n \in Z^+ \text{ and } x, y \in R.$$

Let B be an interval valued intuitionistic anti fuzzy semiprimary ideal of the ring R, then

$$\mu_B(xy) = rmax\mathrm{M}_B(xy) \geq rmaxM_B(x^n) = \mu_B(x^n) \; and$$

$$\gamma_B(xy) \; = rmaxN_B(xy) \leq rmaxN_B(x^n) = \gamma_B(x^n), \text{ for some } n \in Z^+ \text{ and } x, y \in R.$$

Consider $x, y \in R,$, then $x, y \in A\#B$ implies $x, y \in A \; and \; x, \; y \in B$

$$\begin{aligned}
\mu_{A\#B}(xy) &= r\max M_{A\#B}(xy) \\
&= \frac{2r\max M_A(xy) r\max M_B(xy)}{r\max M_A(xy) + r\max M_B(xy)} \\
&\geq \frac{2r\max M_A(x^n) r\max M_B(x^n)}{r\max M_A(x^n) + r\max M_B(x^n)} \\
&= \frac{2\mu_A(x^n)\mu_B(x^n)}{\mu_A(x^n) + \mu_B(x^n)} \\
&= \mu_{A\#B}(x^n).
\end{aligned}$$

 

Therefore, $\mu_{A\#B}(xy) \geq \mu_{A\#B}(x^n)$

Consider

$$\begin{aligned}\gamma_{A\#B}(xy) &= r\max N_{A\#B}(xy)\\ &= \frac{2r\max N_A(xy)r\max N_B(xy)}{r\max N_A(xy)+r\max N_B(xy)}\\ &\leq \frac{2r\max N_A(x^n)r\max N_B(x^n)}{r\max N_A(x^n)+r\max N_B(x^n)}\\ &= \frac{2\gamma_A(x^n)\gamma_B(x^n)}{\gamma_A(x^n)+\gamma_B(x^n)}\\ &= \gamma_{A\#B}(x^n).\end{aligned}$$

Therefore, $\gamma_{A\#B}(xy) \leq \gamma_{A\#B}(x^n)$

Therefore, $A\#B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

**Theorem 3.8:** If A and B are interval valued intuitionistic anti fuzzy semiprimary ideal of R, then A*B is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

**Proof:** Let A be an interval valued intuitionistic anti fuzzy semiprimary ideal of a ring R, then

$$\mu_A(xy) = rmaxM_A(xy) \geq \ rmaxM_A(x^n) = \mu_A(x^n) \ and$$

$$\gamma_A(xy) = rmaxN_A(xy) \leq r\max N_A(x^n) = \gamma_A(x^n) \text{ for some } n \in Z^+ \text{ and } x, y \in R.$$

Let B be an interval valued intuitionistic anti fuzzy semiprimary ideal of the ring R, then

$$\mu_B(xy) = rmax\mathrm{M}_B(xy) \geq rmaxM_B(x^n) = \mu_B(x^n) \ and$$

$$\gamma_B(xy) \ = rmaxN_B(xy) \leq rmaxN_B(x^n) = \gamma_B(x^n), \text{ for some } n \in Z^+ \text{ and } x, y \in R.$$

Consider $x, y \in R,$ then $x, y \in A*B$ implies $x, y \in A \ and \ x, \ y \in B$

Consider

$$\begin{aligned}\mu_{A*B}(xy) &= r\max M_{A*B}(xy)\\ &= \frac{r\max M_A(xy)+r\max M_B(xy)}{2(r\max M_A(xy)r\max M_B(xy)+1)}\\ &\geq \frac{r\max M_A(x^n)+r\max M_B(x^n)}{2(r\max M_A(x^n)r\max M_B(x^n)+1)}\\ &= \frac{\mu_A(x^n)+\mu_B(x^n)}{2(\mu_A(x^n)\mu_B(x^n)+1)}\\ &= \mu_{A*B}(x^n)\end{aligned}$$

Therefore, $\mu_{A*B}(xy) \geq \mu_{A*B}(x^n)$

Consider

$$\begin{aligned}\gamma_{A*B}(xy) &\geq r\max N_{A*B}(xy)\\ &= \frac{r\max N_A(xy)+r\max N_B(xy)}{2(r\max N_A(x^n)r\max N_B(x^n)+1)}\end{aligned}$$

$$\leq \frac{r \max N_A(x^n) + r \max N_B(x^n)}{2(r \max N_A(x^n) r \max N_B(x^n) + 1)}$$
$$= \frac{\gamma_A(x^n) + \gamma_B(x^n)}{2\ \gamma_A((x^n)\gamma_B(x^n) + 1)}$$
$$= \gamma_{A*B}(x^n).$$

Therefore, $\gamma_{A*B}(xy) \leq \gamma_{A*B}(x^n)$

Therefore, $A*B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

**Theorem 3.9:** If A and B are interval valued intuitionistic anti fuzzy primary ideal of R then $A \cap B$ is an interval valued intuitionistic anti fuzzy primary ideal of R.

**Theorem 3.10:** If A and B are interval valued intuitionistic anti fuzzy primary ideal of R then $A \cup B$ is an interval valued intuitionistic anti fuzzy primary ideal of R.

**Theorem 3.11:** If A and B are interval valued intuitionistic anti fuzzy primary ideal of R then $A + B$ is an interval valued intuitionistic anti fuzzy primary ideal of R

**Theorem 3.12:** If A and B are interval valued intuitionistic anti fuzzy primary ideal of R then $A.B$ is an interval valued intuitionistic anti fuzzy primary ideal of R.

**Theorem 3.13:** If A and B are interval valued intuitionistic anti fuzzy primary ideal of R then $A@B$ is an interval valued intuitionistic anti fuzzy primary ideal of R.

**Theorem 3.14:** If A and B are interval valued intuitionistic anti fuzzy primary ideal of R then A$B is an interval valued intuitionistic anti fuzzy primary ideal of R.

**Theorem 3.15:** If A and B are interval valued intuitionistic anti fuzzy primary ideal of R then A#B is an interval valued intuitionistic anti fuzzy primary ideal of R.

**Theorem 3.16:** If A and B are interval valued intuitionistic anti fuzzy primary ideal of R then A*B is an interval valued intuitionistic anti fuzzy primary ideal of R.

**REFERENCES**

1. Atanassov, K. T., Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, Vol. 20, 1986, No. 1, 87–96.
2. Atanassov, K., Operators over interval valued intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, Vol. 64, 1994, No. 2, 159–174.
3. Atanassov, K., Intuitionistic Fuzzy Sets, Springer, Heidelberg, 1999.
4. Atanassov, K. T, G. Gargov. Interval valued intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, Vol. 31, 1989, 343–349.
5. Chakrabarty, K., R. Biswas, S. Nanda, A note on union and intersection of intuitionistic fuzzy sets, *Notes on Intuitionistic Fuzzy Sets*, 3(4), (1997).
6. Palanivelrajan.M, S.Nandakumar some properties of intuitionistic fuzzy primary and semiprimary ideals, *Notes on Intuitionistic Fuzzy Sets*, Vol.18, 2012, No.3, 68-74.
7. Rajesh, K. Fuzzy semiprimary ideals of rings, *Fuzzy Sets and Systems*, Vol. 42, 1991, 263–272.
8. Zadeh, L. A., Fuzzy sets, *Information and Control*, Vol. 8, 1965, 338–353.

**Source of support: Nil, Conflict of interest: None Declared**