ON SOFT FUZZY ALMOST P-SPACES

A. HAYDAR EŞ*

Department of Mathematics Education, Baskent University, Baglica, 06490, Ankara, Turkey.

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ABSTRACT

In this paper the concepts of soft fuzzy almost P-spaces soft weak fuzzy P-spaces and soft fuzzy P-spaces are introduced and studied.

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1. INTRODUCTION

Zadeh introduced the concept of fuzzy sets and fuzzy set operations in [9]. Chang in [3] introduced and developed the concept of fuzzy topological spaces. The concept of P-Spaces in fuzzy setting was introduced by G. Balasubramanian in [2]. The concept of almost GP-spaces in classical topology was introduced by M.R. Ahmadi Zand [1]. The concept of almost P-spaces in fuzzy setting was introduced by the authors in [4, 5, 6]. The concept of soft fuzzy topological space is introduced by I.U. Tiryaki [7]. In this paper, the concepts of soft fuzzy almost P-spaces and soft fuzzy P-spaces are introduced and studied.

2. PRELIMINARIES

We introduce some basic notions and results that are used in the sequel.

Definition 2.1: [2] Let \( (X, \tau) \) be a fuzzy topological space. Let \( \lambda \) be any fuzzy set. Then \( \lambda \) is said to be fuzzy \( G_\delta \) set if 
\[ \lambda = \bigwedge_{i=1}^{\infty} \mu_i \]
where each \( \mu_i \) is fuzzy open set. The complement of a fuzzy \( G_\delta \) set is fuzzy \( F_\sigma \).

Definition 2.2: [7] Let \( X \) be a set, \( \mu \) be a fuzzy subset of \( X \) and \( M \subseteq X \). Then the pair \((\mu, M)\) will be called a soft fuzzy subset of \( X \). The set of all soft fuzzy subsets of \( X \) will be denoted by \( SF(X) \).

Proposition 2.3: [7] If \((\mu_j, M_j)\)\(\in SF(X)\), then the family \(\{(\mu_j, M_j)\} \in J \) has a meet, that is greatest lower bound, in \( SF(X) \), \( \subseteq \), denoted by \(\bigwedge\{\mu_j, M_j \} \) such that \(\bigwedge\{\mu_j, M_j \} = (\mu, M) \) where \(\mu(x) = \bigwedge_{\mu_j} \mu_j(x), \forall x, M = \bigwedge_{\mu_j} M_j, \forall x \subseteq X \).

Definition 2.4: [7] Let \( X \) be a non-empty set and the soft fuzzy sets \( A \) and \( B \) in the form,
\[ A = \{(\mu, M) | \mu(x) \in I^X, \forall x \in X, M \subseteq X \} \]
\[ B = \{(\lambda, N) | \lambda(x) \in I^X, \forall x \in X, N \subseteq X \} \]
Then,
(i) \( A \subseteq B \iff \mu(x) \leq \lambda(x), \forall x \in X, M \subseteq N \).
(ii) \( A = B \iff \mu(x) = \lambda(x), \forall x \in X, M = N \).
(iii) \( A \cup B \iff 1 - \mu(x), \forall x \in X, X \setminus M \).
(iv) \( A \cap B \iff \mu(x) \wedge \lambda(x), \forall x \in X \) and \( M \cap N \), for all \((\mu, M), (\lambda, N) \in SF(X) \).
(v) \( A \cup B \iff \mu(x) \vee \lambda(x), \forall x \in X \) and \( M \cup N \), for all \((\mu, M), (\lambda, N) \in SF(X) \).

Definition 2.5: [7]
\[ (0, \emptyset) = \{(\lambda, N) | \lambda = 0, N = \emptyset \} \]
\[ (1, X) = \{(\lambda, N) | \lambda = 1, N = X \} \]

*Corresponding Author: A. Haydar Eş*
Department of Mathematics Education, Baskent University, Baglica, 06490, Ankara, Turkey.
Definition 2.6: [7] For \((\mu, M) \in SF(X)\) the soft fuzzy set 
\((\mu, M) = (1 - \mu, X \setminus M)\) is called the complement of \((\mu, M)\).

Definition 2.7: [7] A subset \(\tau \subseteq SF(X)\) is called an SF-topology on \(X\) if
(i) \((0, \emptyset)\) and \((1, X)\) \(\in \tau\)
(ii) \((\mu_j, M_j), j = 1, 2, ..., n \Rightarrow \bigcap_{j=1}^{n}(\mu_j, M_j) \in \tau\)
(iii) \((\mu_j, M_j), j \in J \Rightarrow \bigcup_{j \in J}(\mu_j, M_j) \in \tau\). The elements of \(\tau\) are called soft fuzzy open, and those of 
\(\tau = \{(\mu, M) : (\mu, M)' \in \tau\}\) soft fuzzy closed.

If \(\tau\) is SF-topology on \(X\) we call the pair \((X, \tau)\) SF-topological space (in short SFTS).

Definition 2.8: [7] The closure of a soft fuzzy set \((\mu, M)\) will be denoted by \((\mu, M)'\). It is given by
\((\mu, M)' = \cup \{(y, N) : (\mu, M) \subseteq (y, N), (y, N) \in \tau\}\).

Likewise the interior is given by
\((\mu, M)^\prime = \cap \{(y, N) : (\mu, M) \subset (y, N), (y, N) \in \tau\}\).

Note: \((\mu, M)' = cl((\mu, M))\) and \((\mu, M)^\prime = int((\mu, M))\).

Definition 2.9: [8] Let \((X, \tau)\) be a soft fuzzy topological space. Let \((\lambda, N)\) be a soft fuzzy set in \((X, \tau)\). Then
(i) \((\lambda, N)\) is said to be soft fuzzy regular open if \((\lambda, N) = int(cl(\lambda, N)))
(ii) \((\lambda, N)\) is said to be soft fuzzy regular closed if \((\lambda, N) = cl(int(\lambda, N)))\).

Definition 2.10: [5] A fuzzy topological space \((X, \tau)\) is called a fuzzy P-space if countable intersection of fuzzy open sets in \((X, \tau)\) is fuzzy open. That is, every non-zero fuzzy \(G_\delta\) set in \((X, \tau)\), is fuzzy open in \((X, \tau)\).

Definition 2.11: [5] A fuzzy topological space \((X, \tau)\) is called a fuzzy almost P-space if for every non-zero fuzzy \(G_\delta\) set \(\lambda\) in \((X, \tau)\), \(int(\lambda) \neq 0\) in \((X, \tau)\).

Definition 2.12: [5] A fuzzy topological space \((X, \tau)\) is called a weak fuzzy P-space if the countable intersection fuzzy regular open sets in \((X, \tau)\) is a fuzzy regular open set in \((X, \tau)\).

3. ON SOFT FUZZY ALMOST P-SPACES

Definition 3.1: A soft fuzzy topological space \((X, \tau)\) is called a soft fuzzy P-space if countable intersection of soft fuzzy open sets in \((X, \tau)\) is soft fuzzy open. That is, every non-zero soft fuzzy \(G_\delta\) set in \((X, \tau)\), is soft fuzzy open in \((X, \tau)\).

Definition 3.2: A soft fuzzy topological space \((X, \tau)\) is called a soft fuzzy almost P-space if for every non-zero soft fuzzy \(G_\delta\) set \((\lambda, M)\) in \((X, \tau)\), \(int(\lambda, M) \neq 0\) in \((X, \tau)\). It is clear that in soft fuzzy topological spaces, we have the following implication:

Soft fuzzy P-space \(\Rightarrow\) Soft fuzzy almost P-space.

Proposition 3.3: If the soft fuzzy topological space \((X, \tau)\) is a soft P-space, then
\(int(\bigcap_{i=1}^{n}(\mu_i, M_i)) = \bigcap_{i=1}^{n}(\mu_i, M_i)\),
where \((\mu_i, M_i)\)'s are non-zero soft fuzzy open sets in \((X, \tau)\).

Proof: Let \((\mu_i, M_i)\)'s be non-zero soft fuzzy open sets in a soft fuzzy P-space \((X, \tau)\). Then \((\mu, M) = \bigcap_{i=1}^{n}(\mu_i, M_i)\) is a soft fuzzy \(G_\delta\) set in \((X, \tau)\). Since \((X, \tau)\) is a soft fuzzy P-space, the soft fuzzy \(G_\delta\) set \((\mu, M)\) is soft fuzzy open in \((X, \tau)\).

Hence, we have \(int(\mu, M) = (\mu, M)\). This implies that
\(int(\bigcap_{i=1}^{n}(\mu_i, M_i)) = \bigcap_{i=1}^{n}(\mu_i, M_i)\) = \(int(\bigcap_{i=1}^{n}(\mu_i, M_i))\), and hence
\(int(\bigcap_{i=1}^{n}(\mu_i, M_i)) = int(\bigcap_{i=1}^{n}(\mu_i, M_i))\),
where \((\mu_i, M_i)\)'s are non-zero soft fuzzy open sets in \((X, \tau)\).

Proposition 3.4: If \((\lambda_i, M_i)\)'s are soft fuzzy regular closed sets in a soft fuzzy P-space \((X, \tau)\), then
\(cl(\bigcup_{i=1}^{n}(\lambda_i, M_i)) = \bigcup_{i=1}^{n}(\lambda_i, M_i)\).

Proof: Let \((\lambda_i, M_i)\)'s be soft fuzzy regular closed sets in a soft fuzzy P-space \((X, \tau)\). Then \((\lambda_i, M_i)\)'s are soft fuzzy closed sets in \((X, \tau)\), which implies that \((1, X) - (\lambda_i, M_i)\)'s are soft fuzzy open sets in \((X, \tau)\). Then \(\bigcap_{i=1}^{n}((1, X) - (\lambda_i, M_i))\) is a non-zero soft fuzzy \(G_\delta\) set in \((X, \tau)\).
Proposition 3.11: Hence

\[ \text{int}((\prod_{i=1}^{\infty} [1, X] - (\lambda_i, M_i))] = \prod_{i=1}^{\infty} [1, X] - (\lambda_i, M_i)). \]

Therefore \((1, X) - \text{cl}(\bigcup_{i=1}^{\infty} (\lambda_i, M_i)) = (1, X) - \bigcup_{i=1}^{\infty} (\lambda_i, M_i). \) Hence we have \(\text{cl}(\bigcup_{i=1}^{\infty} (\lambda_i, M_i)) = \bigcup_{i=1}^{\infty} (\lambda_i, M_i). \)

Definition 3.5: A soft fuzzy set \((\lambda, M)\) in a soft fuzzy topological space \((X, \tau)\) is called a soft fuzzy nowhere dense if there exists no non-zero soft fuzzy open set \((\mu, N)\) in \((X, \tau)\) such that \((\mu, N) \subset \text{cl}(\lambda, M). \)

That is, \(\text{int}(\text{cl}(\lambda, M)) = (0, \emptyset). \)

Definition 3.6: A soft fuzzy set \((\lambda, M)\) in a soft fuzzy topological space \((X, \tau)\) is called a soft fuzzy dense if there exists no soft fuzzy closed set \((\mu, N)\) in \((X, \tau)\) such that \((\mu, N) \subset \text{cl}(\lambda, M)\).

That is, \(\text{cl}(\lambda, M) = (1, X)\).

Definition 3.7: A soft fuzzy topological space \((X, \tau)\) is called a soft fuzzy submaximal space if for each soft fuzzy set \((\lambda, M)\) in \((X, \tau)\) such that \(\text{cl}(\lambda, M) = (1, X)\), then \((\lambda, M)\) in \((X, \tau)\).

Proposition 3.8: If each soft fuzzy \(G_\delta\) set is a soft fuzzy dense set in a soft fuzzy submaximal space \((X, \tau)\), then \((X, \tau)\) is a soft fuzzy P-space.

Proof: Let \((\lambda, M)\) be a soft fuzzy \(G_\delta\) set in a soft fuzzy submaximal space \((X, \tau)\). By hypothesis, \((\lambda, M)\) is a soft fuzzy dense set in \((X, \tau)\). Then \((\lambda, M)\) is a soft fuzzy open set in \((X, \tau)\). That is, every soft fuzzy \(G_\delta\) set in \((X, \tau)\) is a soft fuzzy open set in \((X, \tau)\). Hence \((X, \tau)\) is a soft fuzzy P-space.

Proposition 3.9: If \(\text{cl}(\text{int}(\lambda, M)) = (1, X)\), for each soft fuzzy \(G_\delta\) set \((\lambda, M)\) in a soft fuzzy submaximal space \((X, \tau)\), then \((X, \tau)\) is a soft fuzzy P-space.

Proof: Let \((\lambda, M)\) be a soft fuzzy \(F_\sigma\) set in a soft fuzzy submaximal space \((X, \tau)\). Then \((\lambda, M)\) is a soft fuzzy \(G_\delta\) set in \((X, \tau)\). By hypothesis, \(\text{cl}(\text{int}(\lambda, M)) = (1, X)\). Then \((1, X) - \text{cl}(\text{int}(\lambda, M)) = (0, \emptyset). \)

This implies that \((1, X) - [(1, X) - \text{int}(\text{cl}(\lambda, M)) = (0, \emptyset). \)

That is, \(\text{int}(\text{cl}(\lambda, M)) = (0, \emptyset)\) and hence \((\lambda, M)\) is a soft fuzzy nowhere dense set in \((X, \tau)\). Thus the soft fuzzy \(F_\sigma\) set \((\lambda, M)\) is a soft fuzzy nowhere dense set in a soft fuzzy submaximal space \((X, \tau)\). Since each soft fuzzy \(F_\sigma\) set is a soft fuzzy nowhere dense set in a soft fuzzy submaximal space \((X, \tau)\), then \((X, \tau)\) is a soft fuzzy P-space.

Definition 3.10: A soft fuzzy topological space \((X, \tau)\) is called a soft fuzzy weak P-space if the countable intersection of soft fuzzy regular open sets in \((X, \tau)\) is a soft fuzzy regular open sets in \((X, \tau)\). That is, \(\prod_{i=1}^{\infty} (\lambda_i, M_i)\) is a soft fuzzy regular open in \((X, \tau)\), where \((\lambda_i, M_i)\)’s are soft fuzzy regular open sets in \((X, \tau)\). It is clear that in soft fuzzy topological spaces, we have the following implication:

Soft fuzzy P-space ⇒ Soft fuzzy weak P-space.

Proposition 3.11: A soft fuzzy topological space \((X, \tau)\) is a soft fuzzy weak P-space if \(\prod_{i=1}^{\infty} (\lambda_i, M_i)\), where \((\lambda_i, M_i)\)’s are soft fuzzy regular closed sets in \((X, \tau)\), is a soft fuzzy regular closed in \((X, \tau)\).

Proof: Let \((X, \tau)\) be a soft fuzzy weak P-space. Then \(\text{int}(\text{cl}(\prod_{i=1}^{\infty} (\lambda_i, M_i))) = \prod_{i=1}^{\infty} (\lambda_i, M_i)\), where \((\lambda_i, M_i)\)’s are soft fuzzy regular open sets in \((X, \tau)\). Now

\[ (1, X) - \text{int}(\text{cl}(\prod_{i=1}^{\infty} (\lambda_i, M_i))) = (1, X) - \prod_{i=1}^{\infty} (\lambda_i, M_i), \]

implies

\[ \text{cl}(\text{int}(\prod_{i=1}^{\infty} (1, X) - (\lambda_i, M_i))) = \prod_{i=1}^{\infty} (1, X) - (\lambda_i, M_i)). \]

Since \([1, X) - (\lambda_i, M_i)]\) is a soft fuzzy regular closed set in \((X, \tau)\). Then we have

\[ \text{cl}(\text{int}(\prod_{i=1}^{\infty} (1, X) - (\lambda_i, M_i))) = \prod_{i=1}^{\infty} [(1, X) - (\lambda_i, M_i)]. \]

Hence \(\prod_{i=1}^{\infty} [(1, X) - (\lambda_i, M_i)]\) is a soft fuzzy regular closed in \((X, \tau)\).

Conversely, suppose that \(\text{cl}(\text{int}(\prod_{i=1}^{\infty} (1, X) - (\lambda_i, M_i))) = \prod_{i=1}^{\infty} (1, X) - (\lambda_i, M_i)). \)

where \([1, X) - \prod_{i=1}^{\infty} (\lambda_i, M_i)]\) are soft fuzzy regular closed sets in \((X, \tau)\). Then

\[ (1, X) - \text{cl}(\text{int}(\prod_{i=1}^{\infty} (1, X) - (\lambda_i, M_i))) = (1, X) - \prod_{i=1}^{\infty} (1, X) - (\lambda_i, M_i)]. \]
which implies that 

\[
\text{int}(cl(\prod_{i=1}^{\infty}(1, X) - ((1, X) - (\lambda, M_1)))) = \prod_{i=1}^{\infty}[((1, X) - ((1, X) - (\lambda, M)))] = \prod_{i=2}^{\infty}(\lambda, M_i).
\]

Hence \((X, \tau)\) is a soft fuzzy weak P-space.

**Proposition 3.12**: If a soft fuzzy topological space \((X, \tau)\) is a soft fuzzy weak P-space, then 

\[
cl(\prod_{i=1}^{\infty}(\lambda, M)) = \prod_{i=1}^{\infty} cl(\lambda, M_i),
\]

where \((\lambda, M_i)'s\) are non-zero soft fuzzy open sets in \((X, \tau)\).

**Proof**: Proof is similar to the Proposition 3.4.

**Definition 3.13**: A soft fuzzy topological space \((X, \tau)\) is called a soft fuzzy almost Lindelöf space if every soft fuzzy open cover \((\lambda, M_a)_{a\in\mathcal{A}}\) of \((X, \tau)\) there exists a countable subcover \((\lambda, M_{n})_{n\in\mathbb{N}}\) such that \(\bigcup_{n\in\mathbb{N}} cl(\lambda, M_n) = (1, X)\).

**Definition 3.14**: A soft fuzzy topological space \((X, \tau)\) is said to be soft fuzzy weakly Lindelöf space if every soft fuzzy open cover \((\lambda, M_a)_{a\in\mathcal{A}}\) of \((X, \tau)\) there exists a countable subcover \((\lambda, M_{n})_{n\in\mathbb{N}}\) such that 

\[
cl(\bigcup_{n\in\mathbb{N}} (\lambda, M_n)) = (1, X).
\]

Obviously every soft fuzzy almost Lindelöf space is a soft fuzzy weakly Lindelöf space.

**Proposition 3.15**: If the soft fuzzy topological space \((X, \tau)\) is a soft fuzzy weak P-space, then every soft fuzzy weakly Lindelöf space is a soft fuzzy almost Lindelöf space.

**Proof**: Let \((X, \tau)\) be a soft fuzzy weakly Lindelöf space and \((\lambda, M_a)_{a\in\mathcal{A}}\) be a soft fuzzy open cover of \((X, \tau)\). Then there exists a countable subcover \((\lambda, M_{n})_{n\in\mathbb{N}}\) such that 

\[
cl(\bigcup_{n\in\mathbb{N}} (\lambda, M_n)) = (1, X).
\]

Since \((X, \tau)\) is a soft fuzzy weak P-space, 

\[
cl(\bigcup_{n\in\mathbb{N}} (\lambda, M_n)) = \bigcup_{n\in\mathbb{N}} cl(\lambda, M_n)
\]

where \((\lambda, M_{n})\)'s are non-zero soft fuzzy open sets in \((X, \tau)\). Hence for the soft fuzzy open cover \((\lambda, M_a)_{a\in\mathcal{A}}\) of \((X, \tau)\), there exists a countable subcover 

\[
(\lambda, M_{n})_{n\in\mathbb{N}}\text{ such that }\bigcup_{n\in\mathbb{N}} cl(\lambda, M_n) = (1, X).
\]

Hence \((X, \tau)\) is a soft fuzzy almost Lindelöf space.

**Proposition 3.16**: If a soft fuzzy topological space \((X, \tau)\) is a soft fuzzy P-space, then \((X, \tau)\) is a soft fuzzy weak P-space.

**Proof**: Let \((\lambda, M)\)'s be soft fuzzy regular closed sets in \((X, \tau)\). Since \((X, \tau)\) is a soft fuzzy P-space, we have 

\[
cl(\bigcup_{i=1}^{\infty}(\lambda, M_i)) = \bigcup_{i=1}^{\infty} cl(\lambda, M_i).
\]

Now \(cl(int(\bigcup_{i=1}^{\infty}(\lambda, M_i))) \subseteq cl(\bigcup_{i=1}^{\infty}(\lambda, M_i)) = \bigcup_{i=1}^{\infty}(\lambda, M_i)\).

Since \(cl(int(\lambda, M_i)) = (\lambda, M_i)\), then 

\[
\bigcup_{i=1}^{\infty} cl(int(\lambda, M_i)) = \bigcup_{i=1}^{\infty}(\lambda, M_i),
\]

which implies that 

\[
\bigcup_{i=1}^{\infty}(\lambda, M_i) \subseteq cl(int(\bigcup_{i=1}^{\infty}(\lambda, M_i))).
\]

Hence \(cl(int(\bigcup_{i=1}^{\infty}(\lambda, M_i))) = \bigcup_{i=1}^{\infty}(\lambda, M_i)\). From the Proposition 3.11, \((X, \tau)\) is a soft fuzzy weak P-space.

**Proposition 3.17**: If \((\lambda, M)\) is a non-zero soft fuzzy nowhere dense and soft fuzzy \(G_\delta\) set in a soft fuzzy topological space \((X, \tau)\), then \((X, \tau)\) is not a soft fuzzy almost P-space.

**Proof**: Let \((\lambda, M)\) be a non-zero soft fuzzy nowhere dense soft fuzzy \(G_\delta\) set \((\lambda, M)\) in \((X, \tau)\). Then 

\[
int(\lambda, M) \subseteq int(cl(\lambda, M)) \text{ and } cl(int(\lambda, M)) = (0, \emptyset),
\]

implies that \(int(\lambda, M) = (0, \emptyset)\). Hence for the non-zero soft fuzzy \(G_\delta\) set \((\lambda, M)\) in \((X, \tau)\), \(int(\lambda, M) = (0, \emptyset)\) in \((X, \tau)\). Therefore \((X, \tau)\) is not a soft fuzzy almost P-space.

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