EFFECT OF RADIATION AND ROTATION ON UNSTEADY MHD FLOW PAST AN IMPULSIVELY STARTED VERTICAL PLATE WITH VARIABLE MASS DIFFUSION IN POROUS MEDIUM IN THE PRESENCE OF HALL CURRENT

U. S. RAJPUT, MOHAMMAD SHAREEF*

Department of Mathematics and Astronomy, University of Lucknow, Lucknow-226007, India.

(Received On: 04-05-16; Revised & Accepted On: 24-05-16)

ABSTRACT

The present study is carried out to examine the effects of rotation, radiation and Hall current on unsteady free convection flow of a viscous, incompressible and electrically conducting fluid past an impulsively started vertical plate in a porous medium under the influence of transversely applied uniform magnetic field. Exact solution of energy equation is obtained in closed form by Laplace transform technique, and solution of momentum equation is obtained by defining a complex variable. The expression for the shear stress at the plate due to the primary and secondary flows is obtained. The results obtained are shown by graphs and table. Applications of the study arise in magnetic field control of materials processing systems, planetary and solar plasma fluid dynamics systems and rotating MHD induction machine energy generators etc.

Keywords: Radiation, Rotation effects, MHD, Concentration, Temperature, Porous medium, Hall Current, Heat Transfer.

1. INTRODUCTION

The impulsively started flow past a semi infinite flat plate is one of the classical problems in the fluid dynamics. Stokes [1] first studied the flow of a viscous incompressible fluid past an impulsively started infinite horizontal plate in its own plane. It is often called Rayleigh problem in the literature. Significant study to understand the behaviour of the fluids in unsteady boundary layer was done by Stewarton K. [10, 11]. His research was completely based within the context of boundary layer equations. As the influence of magnetic field on such flow within porous and nonporous media is of much significance in geothermal energy extraction, thermal insulation of buildings, sensible heat storage bed, enhanced recovery of petroleum products, plasma studies and on the performance of many engineering devices viz. MHD accelerators, MHD energy generators, MHD flow-meters, MHD pumps, Plasma jet engines etc. Due to the importance of the study, many researchers worked on impulsively started vertical plate with heat and mass transfer using different analytical and numerical methods. For instance, Soundalkar [12] analysed the effects of mass transfer and free-convection currents on the flow past an impulsively started vertical plate and his research shows that there is a rise in the velocity due to the presence of a foreign mass. But an increase in Sc (<1), Schmidt number, leads to a fall in the velocity. Further Soundhangar et al. [13] studied the heat and mass transfer effect on flow past impulsively started vertical plate. Radiation and mass transfer effects on two-dimensional flow past an impulsively started infinite vertical plate was studied by Prasad V. R et al. [14] and they solved the governing equations using an implicit finite-difference method of Crank–Nicolson type. And observed that, when the radiation parameter increases, the velocity and temperature decrease in the boundary layer. Cess [2] analysed the effect of radiation on free convection heat transfer from a vertical plate for an absorbing, emitting fluid in the optically thick region, using the singular perturbation technique. Hossain and Takhar [6] studied the radiation effects on mixed convection along a vertical plate with uniform surface temperature using Keller Box finite difference method. Raptis [5] has analyzed the thermal radiation and free convection flow through a porous medium by using perturbation technique. In all these papers the flow is considered to be steady. Further Raptis and Perdikis [8] studied the effects of thermal radiation and free convective flow past moving plate. Many other researchers [3, 4, 15, 16, 17, 18, 19] analysed the effect of heat and mass transfer on impulsively started vertical flat plate in porous or non porous medium under the influence of external transverse magnetic field.
However when the strength of magnetic field is very strong, one can not neglect the effect of Hall current. Also the rotating flows of viscous, incompressible and electrically conducting fluid have attracted attention of investigators due to their abundant geophysical and astrophysical applications. It is also important in the solar physics dealing with the sunspot development, the solar cycle and the structure of rotating magnetic stars. It is well known that a number of astronomical bodies possess fluid interiors and magnetic fields. Many scholars have studied such model for instance Mazumdar et al. [20] studied the hydrodynamic study flow with the effect of hall current. Agarwal et al. [21] analysed the combined influence of dissipation and Hall Effect on free convective flow in a rotating fluid and they analysed that the primary shear-stress increases and secondary shear-stress decreases with increase in magnetic and Hall parameters.

This paper deals with an analysis of effects of Hall currents on unsteady free convective flow past an impulsively started vertical plate in the presence of transversely applied uniform magnetic field with heat and mass transfer in rotating system. The problem is solved analytically using the Laplace Transform technique. A selected set of graphical results illustrating the effects of various parameters involved in the problem are presented and discussed. The numerical values of skin-friction have been tabulated.

2. MATHEMATICAL ANALYSIS

Consider an unsteady MHD flow of a viscous, incompressible, electrically conducting fluid past an impulsively started vertical infinite flat plate in porous medium. Let \( \bar{x} \)- axis be chosen along the plate in the direction of flow, the \( \bar{z} \)- axis normal to the plate, the \( \bar{y} \)- axis normal to the \( \bar{x}-\bar{z} \) plane and the plate is assumed to coincide with \( \bar{z} = 0 \) plane. The fluid and the plate rotate as a rigid body with a uniform angular velocity \( \bar{\Omega} \) about \( \bar{z} \)-axis. A uniform magnetic field \( \bar{B} \) is applied along \( \bar{z} \)-axis and the plate is taken to be electrically non-conducting. The fluid motion is induced due to the impulsive movement of the plate as well as the free convection due to heating of the plate. As the plate occupying the plane \( \bar{z} = 0 \) is of infinite extent, all the physical quantities depend only on \( \bar{z} \) and \( \bar{t} \). Initially, at time \( \bar{t} \leq 0 \), the fluid and the plate are at rest and at a uniform concentration \( \bar{C}_\infty \) and temperature \( \bar{T}_\infty \). At time \( \bar{t} > 0 \), the plate starts moving with a velocity \( \bar{u} \) in its own plane and the concentration and temperature of the plate is raised to \( \bar{w}C \) and \( \bar{w}T \) respectively. Since the fluid is electrically conducting whose magnetic Reynolds number is very small, therefore the induced magnetic field produced by the fluid motion is negligible in comparison to the applied one. Also, due to the conservation of electric charge, current density along \( \bar{z} \)-direction \( \bar{J}_z \) is constant. Since the plate is assumed to be non-conducting therefore \( J_z \) can be assumed to be zero. So, under the above assumptions, the governing equations with Boussinesq’s approximation are as follows:

\[
\frac{\partial \bar{u}}{\partial \bar{t}} + 2\bar{\Omega} \bar{v} = \nu \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + g \beta (\bar{C} - \bar{C}_\infty) + g \beta' (\bar{T} - \bar{T}_\infty) + \frac{B}{\rho} \frac{\bar{J}_z}{K} \frac{\partial \bar{u}}{\partial \bar{z}},
\]

\[
\frac{\partial \bar{v}}{\partial \bar{t}} + 2\bar{\Omega} \bar{u} = \nu \frac{\partial^2 \bar{v}}{\partial \bar{z}^2} - \frac{B}{\rho} \frac{\bar{J}_z}{K} \frac{\partial \bar{v}}{\partial \bar{z}},
\]

\[
\frac{\partial \bar{C}}{\partial \bar{t}} = D \frac{\partial^2 \bar{C}}{\partial \bar{z}^2},
\]

\[

\rho C_p \frac{\partial \bar{T}}{\partial \bar{t}} = k \frac{\partial^2 \bar{T}}{\partial \bar{z}^2} - \frac{\partial q_r}{\partial \bar{z}},
\]

Where \( J_x = \frac{\sigma B_x (v + mu)}{1 + m^2} \), \( J_y = \frac{\sigma B_y (mv - u)}{1 + m^2} \)

The boundary conditions taken are as under:

\[
\bar{t} \leq 0: \bar{u} = 0, \bar{v} = 0, \bar{C} = \bar{C}_\infty, \bar{T} = \bar{T}_\infty (\forall \bar{z}),
\]

\[
\bar{t} > 0: \bar{u} = \bar{u}_0, \bar{v} = 0, \bar{T}_{\bar{u}_0} = \bar{C}_{\bar{u}_0}, \bar{C} = \bar{C}_{\bar{u}_0} + (\bar{C}_\infty - \bar{C}_{\bar{u}_0}) \frac{\bar{u}_0^2}{v} \bar{t} (a\bar{z} = 0),
\]

\[
\bar{u} \to 0, \bar{C} \to \bar{C}_{\bar{u}_0}, \bar{T} \to \bar{T}_{\bar{u}_0} (a\bar{z} = 0),
\]

By using Rosseland approximation (Brewster [9]), the radiative heat flux \( q_r \) is given by

\[
q_r = \frac{4\sigma}{3k_c} \frac{\partial \bar{T}^4}{\partial \bar{z}}
\]

© 2016, IJMA. All Rights Reserved

167
where the symbols are: $C$ – concentration of the fluid, $T$ – temperature of the fluid, $\bar{C}_\infty$ – concentration of the fluid far away from the plate, $\bar{T}_\infty$ – temperature of the fluid far away from the plate, $\bar{C}_w$ – concentration at the wall, $\bar{T}_w$ – temperature at the wall, $\mathbf{B}_n$ – external magnetic field, $\bar{u}$ – primary velocity of the fluid, $\bar{v}$ – secondary velocity of the fluid, $\bar{u}_o$ – velocity of the Plate, $\mathbf{K}$ – permeability parameter, $z$ – spatial coordinate normal to the plate, $t$ – time, $\beta$ – volumetric coefficient of concentration expansion, $\gamma$ – volumetric coefficient of thermal expansion, $q_r$ – radiative heat flux, $\alpha$ – thermal diffusivity, $k_c$ – mean absorption coefficient, $g$ – acceleration due to gravity, $\rho$ – density, $\nu$ – kinematic viscosity, $\sigma$ – Stefan-Boltzmann constant, $m$ – hall parameter, $x_J$ – current density along $x$–axis and $y_J$ – current density along $y$–axis.

If temperature differences within the flow are sufficiently small, then expanding $\bar{T}^3$ by using Taylor series about $\bar{T}_\infty$ and neglecting higher order terms, we get,

$$\bar{T}^3 \approx 4 \bar{T}_\infty^3 \bar{T}^3$$ (7)

By using (6) and (7), (4) reduces to

$$\frac{\partial \bar{T}}{\partial t} = \alpha \frac{\partial^2 \bar{T}}{\partial z^2} + \frac{16 \sigma \bar{K}}{\nu \bar{K}} \frac{\partial^2 \bar{T}}{\partial z^2}$$ (8)

To obtain the equations in dimensionless form, the following non-dimensional quantities are introduced:

$$u = \frac{\bar{u}}{\bar{u}_o}, \quad v = \frac{\bar{v}}{\bar{u}_o}, \quad t = \frac{\bar{u}_o}{\nu} \bar{z}, \quad K = \frac{\bar{u}_o}{\bar{K}}, \quad z = \frac{\bar{u}_o}{\nu} \bar{z}, \theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_\infty - \bar{T}_\infty},$$

$$G_m = \frac{\bar{g} \beta \nu (\bar{C}_w - \bar{C}_\infty)}{\bar{u}_o^3}, \quad S_c = \frac{\nu}{D}, \quad P_r = \frac{\nu}{\alpha \sigma}, \quad N = \frac{k_c \alpha c_p}{4 \sigma \bar{T}_\infty},$$

$$c = \frac{(\bar{C}_w - \bar{C}_\infty)}{(\bar{C}_w - \bar{C}_\infty)}, \quad M = \frac{\bar{g} \beta \nu (\bar{T}_\infty - \bar{T}_\infty)}{\bar{u}_o^3}, \quad \Omega = \frac{\nu}{\bar{u}_o}$$ (9)

where $u$ – dimensionless primary velocity of the fluid, $v$ – dimensionless secondary velocity of the fluid, $z$ – dimensionless spatial coordinate normal to the plate, $c$ – dimensionless concentration, $\theta$ – dimensionless temperature, $S_c$ – Schmidt number, $P_r$ – Prandlt number, $G_m$ – Mass Grashof number, $G_i$ – Thermal Grashof number, $t$ – dimensionless time, $N$ – Radiation parameter, $\Omega$ – dimensionless rotation parameter and $M$ – magnetic field parameter.

The equations (1), (2), (3), (5) and (8) become:

$$\frac{\partial^2 u}{\partial t^2} - 2 \Omega \nu \frac{\partial^2 u}{\partial z^2} = \frac{M}{(1 + m^2)}(mv - u) + G_m c + G_i \theta - \frac{u}{K},$$ (10)

$$\frac{\partial^2 v}{\partial t^2} + 2 \Omega u \frac{\partial^2 v}{\partial z^2} = \frac{M}{(1 + m^2)}(v + mu) - \frac{v}{K},$$ (11)

$$\frac{\partial^2 c}{\partial t^2} = \frac{1}{S_c} \frac{\partial^2 c}{\partial z^2},$$ (12)

$$\frac{\partial \theta}{\partial t} = \frac{R_e \partial \theta}{P_r \partial z^2}. $$ (13)

$$t \leq 0: u = 0, v = 0, c = 0, \theta = 0(\forall z),$$

$$t > 0: u = 1, v = 0, c = t, \theta = 1(atz = 0),$$ (14)

$$u \rightarrow 0, c \rightarrow 0, \theta \rightarrow 0(asz \rightarrow \infty).$$

To solve above system, take $V = u + iv$. Then using equations (10) and (11), we get,

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial z^2} - bV + G_m c + G_i \theta,$$ (15)
The boundary conditions (14) are reduced to:

\[ t \leq 0 : V = 0, c = 0, \theta = 0(\forall z), \]
\[ t > 0 : V = 1, c = t, \theta = 1(atz = 0), \]
\[ V \to 0, c \to 0, \theta \to 0(asz \to \infty). \]

The governing non-dimensional partial differential equations (12), (13) and (15) subject to the above boundary conditions prescribed in equation (16) are solved using the Laplace Transform technique. The solution is as under:

\[
V(z, t) = (\phi(t) + F_i) \left\{ 2 \cosh(z\sqrt{b}) + e^{-z\sqrt{b}} \operatorname{Erfc}\left(\sqrt{bt} - \frac{z}{2\sqrt{t}}\right) - e^{z\sqrt{b}} \operatorname{Erfc}\left(\sqrt{bt} + \frac{z}{2\sqrt{t}}\right) \right\}
- F_2 e^{-z\sqrt{b}} \left( z \cosh(z\sqrt{b}) - E_i \operatorname{Erfc}\left(\frac{z}{2\sqrt{t}}\right) - e^{-z\sqrt{b}} \operatorname{Erfc}\left(\frac{z}{2\sqrt{t}}\right) - iE_i \operatorname{Erfc}\left(\frac{z}{2\sqrt{t}} + i\sqrt{B_1}\right) \right)
+ F_3 e^{-z\sqrt{b}} \left( 2 \sinh(z\sqrt{b}) + e^{-z\sqrt{b}} \operatorname{Erfc}\left(\sqrt{bt} - \frac{z}{2\sqrt{t}}\right) - e^{z\sqrt{b}} \operatorname{Erfc}\left(\sqrt{bt} + \frac{z}{2\sqrt{t}}\right) \right)
+ F_4 z \left( 2 \sqrt{t} e^{-z\sqrt{b}} - E_i \operatorname{Erfc}\left(\frac{z}{2\sqrt{t}}\right) - F_i \operatorname{Erfc}\left(\frac{z}{2\sqrt{t}}\right) + \nu(t) \operatorname{Erf}\left(\frac{z}{2\sqrt{t}}\right) \right)
- F_5 e^{-z\sqrt{b}} \left( 2 \cos(zE_i) - e^{-z\sqrt{b}} \operatorname{Erfc}\left(\sqrt{bt} + \frac{z}{2\sqrt{t}}\right) - e^{z\sqrt{b}} \operatorname{Erfc}\left(\sqrt{bt} - \frac{z}{2\sqrt{t}}\right) \right),
\]
\[
\theta(z, t) = \operatorname{Erfc}\left(\frac{z}{\sqrt{t}}\right), \quad c(z, t) = \frac{1}{2} \operatorname{Erfc}\left(\frac{z}{2\sqrt{t}}\right) (2t + z^2 S_c) - ze^{-z\sqrt{b}} \sqrt{\frac{S_c}{\pi}}.
\]

### 3. SKIN FRICTION

The skin-friction components \( \tau_x \) and \( \tau_y \) are obtained as:

\[
\tau_x + i\tau_y = -\frac{\partial F}{\partial z}|_{z=0}.
\]

### 4. RESULT AND DISCUSSION

In order to get a physical insight of the problem, a representative set of numerical results is shown graphically in Figures 1–25. It is noticed from figures 1 to 25 that primary velocity \( u \) and secondary velocity \( v \) attain a distinctive maximum value near the surface of plate and then decrease properly on increasing boundary layer coordinate \( z \) to approach free stream value. From figures 5 and 6, it is observed that increasing the permeability parameter \( K \) of the porous medium, the primary velocity and secondary velocities increases. This is because an increase in \( K \) implies that there is a decrease in the resistance of the porous medium which tends to accelerate primary velocity as well as secondary velocity in the boundary layer region. Figures 9 and 10 shows the influence of Hall current on primary and secondary velocities respectively. Primary velocity \( u \) increases rapidly near the surface of the plate whereas secondary velocity \( v \) increases throughout the boundary layer region on increasing Hall current parameter \( m \). This shows that Hall current tends to accelerate primary velocity in the region near the surface of the plate whereas it tends to accelerate secondary velocity throughout the boundary layer region. Effect of rotation on flow behavior is shown by figures 15 and 16 and it is observed that the an increase in rotation parameter \( \Omega \) primary velocity \( u \) decreases throughout the boundary layer region whereas secondary velocity \( v \) increases near the surface of the plate. This implies that rotation tends to accelerate secondary velocity whereas retards primary velocity in the boundary layer region. As time \( t \) increases both the primary and secondary velocities increases (figure [17, 18]). Effect of magnetic parameter \( M \) on velocity profile is shown by figures 7 and 8 and it is observed that, effect is almost similar as that of rotation parameter. Figures 1 to 4 shows the buoyancy effect and it is observed that both the primary and secondary velocities increases on increasing thermal Grashof number \( G_t \) and mass Grashof number \( G_m \). Therefore it concludes that buoyancy force tends to accelerate primary and secondary velocities. Figures 11 to 14 shows the effect of Prandtl number and Schmidt number on velocity profile and it is observed that primary and secondary velocities goes on decreasing with increase in
This is due to the fact with large values of $P_r$ and $S_c$ the viscous diffusivity dominates the behavior. Figure 23 and 24 shows the effect of radiation parameter on the both component of velocity and it is observed that it retards the flow. Concentration and temperature profiles are illustrated in figure – 19 to 22 and 25 for different values of $P_r$, $S_c$, $N$, and time. In figure 21, it can be seen that the concentration of the fluid is inversely proportional to the value of Schmidt number $S_c$. Thus, the increase in $S_c$ reduces the concentration in the system. This is due to the fact that there would be a decrease of concentration boundary layer thickness with the increase of Schmidt number $S_c$. Similar effect can be seen for temperature profile with Prandlt number $P_r$ (figure-19). Also concentration and temperature boundary layer increases with time (figure – 20, 22). While the radiation parameter decreases the temperature boundary layer.

The effects of various parameters on the skin-friction are shown in tables -1. It is found from table -1, that the value of $\tau_x$ increases when the values of $\Omega$ and N are increased (keeping other parameters fixed) but if values of $m$ and $K$ are increased, gets decreased. Also, it is observed that $\tau_y$ decreases with $\Omega$ and $K$ and it is increased $m$ and N are increased.

5. CONCLUSION

It is found that hall current has tendency to accelerate the fluid flow in both the primary and secondary flow directions. When the radiation parameter increases, the velocity and temperature decrease in the boundary layer. Whereas the skin-friction increases with the increase in radiation parameter. Rotation retards primary flow whereas it accelerates secondary flow. Also the flow in both the directions accelerated by the porosity of the medium. As the value of Schmidt number and Prandlt number increases the fluid velocity in both the directions gets decreases but the velocity in both the directions increases with time.

- Temperature increases with time but decreases with Prandlt number.
- Concentration increases with time but decreases with Schmidt number.
- Skin friction-
  - $\tau_x$ increases when $m$, $G_m$, $G_r$, $K$, $R$ and $t$ are increased but it decreases with $M$, $S_c$, $P_r$ and $\Omega$.
  - $\tau_y$ increases when $m$, $G_m$, $G_r$, $K$, $M$, $\Omega$, $N$ and $t$ are increased but it decreases if $P_r$ & $S_c$ are increased.
Table-1: Skin friction for different parameters
(at particular values of $M$, $S_m$, $P_r$, $G_m$, $G_r$ and $t$)

<table>
<thead>
<tr>
<th>$K$</th>
<th>$\Omega$</th>
<th>$m$</th>
<th>$N$</th>
<th>$r_m$</th>
<th>$r_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.5</td>
<td>1.5</td>
<td>3.0</td>
<td>0.7519</td>
<td>0.4653</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>1.5</td>
<td>3.0</td>
<td>0.4189</td>
<td>0.5038</td>
</tr>
<tr>
<td>0.7</td>
<td>0.5</td>
<td>1.5</td>
<td>3.0</td>
<td>0.2678</td>
<td>0.5221</td>
</tr>
<tr>
<td>0.5</td>
<td>0.3</td>
<td>1.5</td>
<td>3.0</td>
<td>0.4078</td>
<td>0.3996</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>1.5</td>
<td>3.0</td>
<td>0.4189</td>
<td>0.5038</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>2.0</td>
<td>3.0</td>
<td>0.3588</td>
<td>0.4181</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>4.0</td>
<td>3.0</td>
<td>0.2745</td>
<td>0.3981</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>6.0</td>
<td>3.0</td>
<td>0.2538</td>
<td>0.3601</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>1.5</td>
<td>2.0</td>
<td>0.3067</td>
<td>0.5553</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>1.5</td>
<td>3.0</td>
<td>0.4189</td>
<td>0.5038</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>4.0</td>
<td>3.0</td>
<td>0.4853</td>
<td>0.4733</td>
</tr>
</tbody>
</table>

6. REFERENCES


7. APPENDIX

\[
\begin{align*}
    b &= \frac{Mt}{m+i} + \frac{1}{K} + 2\Omega, \quad R_s = 1 + \frac{4}{3N}, \quad A_1 = \frac{G}{P - R_s}, \quad A_2 = \frac{G}{S - 1}, \quad B_1 = \frac{-bR_s}{P - R_s}, \quad B_2 = \frac{-b}{S - 1}, \quad F_1 = \frac{A_1}{B_1}, \\
    \phi(t) &= \frac{1}{2} A_1 + \frac{A_1 t}{B_2}, \quad \psi(t) = \frac{A_1}{B_2} (1 - tB_2), \quad F_2 = \frac{A_1}{B_2}, \quad E_1 = \sqrt{b - B_2}, \quad E_2 = \frac{P}{R_s}, \quad E_3 = \sqrt{\frac{P}{R_s}}, \quad E_4 = \frac{\pi}{2} B_2, \quad E_5 = \sqrt{\frac{\pi}{2} S}, \quad E_6 = \sqrt{\frac{\pi}{2} S}.
\end{align*}
\]

Source of support: Nil, Conflict of interest: None Declared

[Copy right © 2016. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]