ACTIVITY ORIENTED LEARNING OF THE AREA OF A CIRCLE

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ABSTRACT

Learning mathematics from the early stage plays a tremendous role in developing learners’ rationality which ultimately enrich the human wealth of a country by creating scientifically oriented skilled persons, and which, in the present process of globalization, has a direct impact towards the economic growth and hence the overall development of a nation.

But it is a hard fact that the teaching–learning process of mathematics is not satisfactory in our country. Actually, the learners face troubles to understand the mathematical concepts because of the inherent abstract nature of the subject. In this regard, educational research demands that it is definitely possible to ease the process of abstraction of the subject through the activity oriented learning. The activity based learning process creates the joyful learning environment in the classroom which boosts the will force of the learners and which may cause a significant positive change in their attitude towards the subject. But it is matter of pity that there is no such stress on mathematical activity in the prescribed text books of mathematics at secondary level of different Boards in our country. Good books containing sufficient number of need based activities (both illustrations & exercises) are not available to the teachers and learners. The researcher feels that this issue is very important in the context of school education of mathematics and the need for developing class wise standard activity oriented books for the secondary level is an urgent issue in the context of mathematical education.

In this paper, five activity based illustrations for finding the area of a circle under the branch of mensuration have been presented.

Keywords: secondary level, learning mathematics, mensuration, circle, area, activity.

MSC: AMS Subject Classification 2010: 97C70, 97D40, 97U60.

1. INTRODUCTION

Mathematics is such a crucial subject which plays an important role for the development of society, development of civilization and searches the answers of many mysteries in the world as well as universe. For the complete logical development of the subject, the learners of the subject undergo good exercise of their intellect leading to the development of rational mind-set, which is an index of modernized human being. Considering this, the subject has been introduced from very early stage at school level in almost all the countries. But unfortunately, a large number of students cannot have the taste of the subject properly. In this regard, National Curriculum Framework (NCF)-2005 has mentioned some major problems of teaching mathematics only in India [29]. Undoubtedly, it is said that the subject is being presented to the learners in a difficult way for its abstraction. It is found that although the learners recall the mathematical formulae but almost all of them can not explain how the formula logically been framed or how these are established through the concrete activities. They can not solve the real life problems (including the problems of text books) involving formula for their lack of collaboration between real direct experiences and the so-called formulae. Although, somebody solve the problems of text books but they only follow the mechanical process. Generally, they can not form formula related to real life problems. Hence, here the objective of the teaching of mathematics fails. Therefore it is an important thing that the content of mathematics has to be presented carefully so that the learners could learn mathematical concept and have a glimpse of its abstraction through direct activities and real experience. In this regard,
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Edger Dale (1964) established his ‘cone of experience’ model for communication in education. He stressed from real direct experience to verbal symbols in his cone of experience for the concretization of the subject [28]. On the teaching aids for communication in education, S. Roy (1972) quoted the comments of the psychologist Barnard, “Because of their variety and vividness, instructional aids speed and strengthen learning [33].” Educational research says that children are very sensitive and their learning process starts only when a comfortable learning environment is presented before them in which they can participate actively. This process of learning is none other than ‘learning through activities’. Activity based teaching-learning paradigm is not properly followed in our usual class room teaching-learning process. The children can understand the mathematical concept through the proper activity based teaching-learning paradigm to make concretization of the content. In this context, Donna H. Henry (2007) said, “...that my students will create their own math knowledge through activities that involve them in explorations and inquiry. It is also my intent that the knowledge gained through the manipulative activities can be applied in solving problems. Math that arises from real-life situations is more relevant to the students’ lives. I believe that a concept should grow out of a student’s need to know, not just because it is the next chapter in the book [8],” Research has documented that children in early grades learn mathematics more effectively when they use physical objects in their lessons (6), [7], [8], [9], [15], [17], [32], [34], [38]). The use of both manipulative materials and pictorial representations is highly effective whereas symbolic treatments alone are less effective. So that lessons using manipulative materials have a higher probability of producing greater mathematical achievement than that do the non–manipulative lessons. Learners can observe and hold the physical objects which are particularly important in the early stages of learning the mathematical concepts because these objects help the learners to understand by visualization. The type or design of the objects can be blocks, marbles, poker chips, cardboard, cutouts –almost anything. These materials should be inexpensive which may be made at home or at math lab. The cognitive development of the children and their ability to understand ordinarily move from the concrete to the abstract. Learning from real objects takes benefits for this. It gives a firm foundation for the later development of skills and concepts. In this context, National Curriculum Framework-2000 said, ‘Mathematics learning should be imparted through activities from the very beginning of school education, i.e. from the primary stage itself. These activities may involve the use of concrete materials, models, patterns, charts, pictures, posters, games, puzzles and experiments. The importance of using learning aids needs to be stressed [29].’ These activities may be implemented/developed in the class room as well as mathematics laboratory. Again, National Curriculum Framework-2005 opines on mathematics education at the secondary stage, ‘At this stage students integrate many concepts and skills that they have learnt into a problem-solving ability. Mathematical modeling, data analysis and interpretation taught at this stage can consolidate a high level of mathematical literacy. Individual and group exploration of connections and patterns, visualization and generalization, and making and proving conjectures are important at this stage, and can be encouraged through the use of appropriate tools that include concrete models as in mathematics laboratories and computers [30].’ After all according to position paper National Focus Group on Teaching of Mathematics of National Council of Educational Research & Training (N.C.E.R.T.)-2006, “At the secondary stage, a special emphasis on experimentation and exploration may be worthwhile. Mathematics laboratories are a recent phenomenon, which hopefully will expand considerably in future. Activities in practical mathematics help learner immensely in visualization. Indeed Singh, Avtar and Singh offer excellent suggestions for activities at all stages [31].” The status of implementation of the activities based mathematics learning in India is poor. In this regard, S. Anandalakshmy & Bala Mandir Team (2007) said in ‘A Report on an Innovative Method in Tamil Nadu’ on Activity Based Learning that innovative methods which engage the children and enable them to achieve mastery over school-related competencies and skills can be located here and there [16]. However, they are small in scale and number in India.

Now, text books of mathematics of West Bengal Board of Primary Education (W.B.B.P.E.), West Bengal Board of Secondary Education (W.B.B.S.E.) and N.C.E.R.T. up to secondary level has been considered the activities. But W.B.B.S.E. & N.C.E.R.T. have not considered the activities sufficiently in their prescribed text books at secondary level. On the other hand, generally, teacher as well as learner does not get readily available all proper activities for introducing any new mathematical concept. So, the author as a mathematics teacher at secondary level under W.B.B.S.E. feels the need to collect the different activities and develop them sequentially considering the learners’ ability level, target group etc. He has already done some works on activity based learning ([35], [36], [37]).

In this paper, as an example, author has presented below five activities for finding the area of a circle which will encourage the learners to consolidate their knowledge and to practice mathematics joyfully and surely, they will relish the simplicity & the logical beauty of the subject.

2. OBJECTIVE OF THE STUDY

The aim of this study is to develop various learning activities of the area of a circle.

3. MATERIALS AND METHODS

Materials and methods have been discussed below:
3.1 Collection of text books:
The prescribed text books of mathematics from class-I to class-X standard of WBBPE, WBBSE and NCERT and other available books from the market were collected at first (from [1] to [5], [10] to [14], [18] to [27], [39]).

3.2 Analysis of Text books/text book scanning:
The books were analysed to identify the activities of the ‘area of circle’ in geometry as well as in mensuration.

3.3 Developing the activities:
The activities for verifying the ‘area of circle’ were developed considering the psychological order of learners and the learners’ ability level.

3.4 Sequencing of activities:
The developed activities were sequenced keeping in view the logical order of the subject matter and the psychological order of learners. The gaps in activities, if any, detected by the researcher were filled in by him at the initial stage.

3.5 Experts’ opinions:
Experts’ opinions were taken on the developed activities.

Finally, the sequential form of the activities ‘area of circle’ incorporating the experts’ opinion was developed.

4. TOTAL FRAME WORK

Five activities for finding the area of a circle are presented below. Learners will acquire the knowledge about how to find the area of a circle through their active participation.

Activity-I: Verifying the area of a circle by approximating the circle as a polygon.

Requirements: Paper, pencil, instruments of geometrical box, scissors.

Mode: Pair group.

Strategy: Learning through activities.

Objective of the development: Cognitive development.

Activity follows:

Stage-I: Teacher will do the following activities involving the learners.
1. The teacher will ask the learners to tell the formula of the area of a triangle.
2. In order to demonstrate limiting behavior of arc and chord with respect to their lengths, the teacher shall first draw a curve on a paper and then to make several pieces and finally consider each small piece as a part of a straight line.

Stage-II: Learners will do the following activities with help of the teacher if needed.
Each pair group:
1. Draws a circle of radius r on a paper whose centre is O. (Pl. see figure-1.1)
2. Takes 12 points namely A, B, C, ………, L at equal distance (using protractor) on the circle. (Pl. see figure-1.1)
3. Joins A, B, C, ………., L with O. (Pl. see figure-1.1)
4. Takes midpoints P1, P2, P3, ………., P12 of AB, BC, CD,… …LA (considered as straight lines) respectively. (Pl. see figure-1.1)
5. Again Joins P1, P2, P3, ………, P12 with O by dotted line. (Pl. see figure-1.1)
6. Finds the relation between area of the circle and areas of the sectors from the figure. (Pl. see figure-1.1)
7. Joins AP1, P, B, BP2, P2C, ……, P12A.
8. Joins AB, BC, ………., LA.
9. Names points of intersection namely Q1, Q2, ….Q12.
10. Calculates the area of the circle.
Here, the circumference of the circle is $2\pi r$.

Now, the calculated area of the circle is approximately $\pi r^2$. (since, an approximation is due to arc AB, arc BC, ...are approximated to chord AB, chord BC, ...)

**Remarks:** Area of $\Delta OAB <$ Area of quadrilateral $OAP_1B <$ Area of sector $OAB$.

Also as the number of points $n \rightarrow \infty$, chord $AB +$ chord $BC +$ ... $\rightarrow$ arc $AB +$ arc $BC +$ ....... = Circumference of the circle.

Area of $\Delta OAB +$ Area of $\Delta OBC +$ ...... $\rightarrow$ Area of sector $OAB +$ Area of sector $OBC +$ ...... $= $ Area of the circle

So when $n$ is sufficiently large, perimeter of the polygon $AB$...$LA \approx (AB + BC +$......$LA) = $ Circumference of the circle, and area of the polygon $AB$...$LA \approx $ Area of the circle.

**Activity-2: Verifying the area of a circle when like a rectangle is made by the sectors of a circular hard paper/thermocal.**

**Requirements:** Thermocal/ pitch board/ hard paper, pencil, instruments of geometrical box, scissors, colour.

**Mode:** Pair group.

**Strategy:** Learning through activities.

**Objective of the development:** Cognitive development.
Activity follows:

Stage-I: Teacher will do the following activity involving the learners.

Showing a piece of rectangular paper, the teacher will ask the learners the formula of the area of a rectangle.

Stage-II: Learners will do the following activities with help of the teacher if needed.

Each pair group:
1. Cuts a circular area of a radius r from a thermocal and mark the centre. (Pl. see figure-2.1)
2. Colours half of the circular area of thermocal. (Pl. see figure-2.2)
3. Draws 12 equal sectors (6 in the coloured portion and 6 in the uncoloured portion).
4. Cuts the circular area into 12 equal sectors. (Pl. see figure-2.3)
5. Now, joins 12 sectors side by side: one coloured sector and one uncoloured sector with their top and bottom in the reverse order. The shape thus formed will look like a parallelogram (Pl. see figure-2.4)
6. Further, to make the shape more like a rectangle, cut half portion from top to bottom of the 1st sector and place the portion to the side of the last one. (Pl. see figure-2.4 & 2.5)

It is mentioned that we cut the circular thermocal into 24 equal sectors (Pl. see figure-2.6), 48 equal sectors (Pl. see figure-2.8) or more equal sectors then follow by the same method and then we get like rectangle. (Pl. see figure-2.7 & 2.9)

The work is illustrated below:
Here, the figure-2.5/figure2.6 obtained by placing the sectors of the circular area side by side almost coincides with rectangle.

Now, the area of the rectangle = length × width

\[= \frac{1}{2} \text{ of the circumference} \times \text{radius}\]

\[= \frac{1}{2} \times 2\pi r \times r \quad \text{[Since, the circumference of the circular thermocal =}2\pi r]\]

\[= \pi r \times r\]

\[= \pi r^2\]

So, the area of the circle is \(\pi r^2\). (However, this is an approximate relation since the small parts of circular arcs are approximated to line segments)

**Activity-3: Verifying the area of a circle using the area of a triangle whose height is \(r\) and base is \(2\pi r\).**

**Requirements:** Circular coloured hard paper, plain paper, thread, instruments of scale box, tape, scissors, gum, pitch board, sketch pen.

**Mode:** Pair group.

**Strategy:** Learning through activities.

**Objective of the development:** Cognitive development.

**Activity follows:**

**Stage-I:** Teacher will do the following activity involving the learners.

1. Showing a piece of triangular shaped paper, the teacher will ask the learners the formula of the area of a triangle.

**Stage-II:** Learners will do the following activities with help of the teacher if needed.

Each pair group:

1. Takes a circular and coloured hard paper of radius \(r\). (Pl. see figure 3.1)
2. Measures the circumference of the circular hard paper using a thin thread.
3. Measures the length of the thread. (Pl. see figure 3.2)
4. Takes a line segment on a plain paper of the same length as that of the thread. (Pl. see figure 3.3)
5. Takes a point outside the straight line at a distance \(r\) from that straight line. (Pl. see figure 3.4)
6. Joins the end points of the straight line with the outside point. (Pl. see figure 3.5)
7. Now, finds out the area of the triangle.

The work is illustrated below:
Now, area of the triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 2\pi r \times r$ [Here, base of the triangle = the circumference of the circular coloured hard paper = $2\pi r$]

$= \pi r \times r$

$= \pi r^2$

Now, is there any relationship between the area of circular hard paper and that of the above framed triangle?

**To find the relationship between the area of the coloured circular hard paper and of the above mentioned triangle, do another activity.**

**Activity follows:**

**Stage-III:** Teacher will do the following activities involving the learners.
1. Drawing figures of a parallelogram and a triangle with a common base and between two parallel straight lines, the teacher will ask the learners about the relation between their areas.

**Stage-IV:** Learners will do the following activities with help of the teacher if needed.

Each pair group:
1. Takes the previous coloured circular hard paper of radius $r$. (Pl. see figure 3.6)
2. Draws 12 equal sectors on the circular paper. (Pl. see figure 3.7)
3. Cuts the circular area into 12 equal sectors. (Pl. see figure 3.7)
4. Joins the sectors side by side and paste them on a pitch board forming a base AB. (Pl. see figure 3.8)
5. Draws a line segment BC at the last end point B for the last sector parallel in direction and equal in length with one of the side of the last triangular sector. (Pl. see figure 3.9)
6. Joins the top vertices of each sector and extend it up to the parallel line by sketch pen forming a line segment CD. (Pl. see figure 3.9 & 3.10)
7. Completes the parallelogram ABCD. (Pl. see figure 3.10)
8. Takes a point O on the side DC which is opposite to the base of parallelogram. (Pl. see figure 3.11)
9. Joins the point with the end points of the base line of the parallelogram by a dotted line using sketch pen. (Pl. see figure 3.11)
10. Finds out the area of the parallelogram.
11. Finds out the area of the circle.
12. Finds out the area of the triangle.
13. Finds out the relationship between the area of the circle and that of the triangle.

The work is illustrated below:
Now, area of the parallelogram = base × height
= \(2\pi r \times r\)
= \(2\pi r^2\) \(\cdots \cdots \) (i)

Figure 3.10 reveals that the parallelogram is a sum of 12 sectors of coloured hard paper and 12 sectors of uncoloured triangular portions.

Now, according to construction, each coloured triangular sector is congruent with each uncoloured sector, the proof which can be given from geometrical arguments.

Therefore, the area of each coloured triangle = the area of each uncoloured triangle

Then, sum of the areas of coloured triangles (sectors) = sum of the areas of uncoloured triangles

Now, the area of the circle = sum of the areas of coloured triangles (sectors)
= \(\frac{1}{2} \times \text{Area of the parallelogram}\)
= \(\frac{1}{2} \times 2\pi r^2\) (Since, area of the parallelogram = \(2\pi r^2\))
= \(\pi r^2\) \(\cdots \cdots \) (ii)

Again, area of the triangle OAB = \(\frac{1}{2} \times \text{Area of the parallelogram [since, the triangle and the parallelogram are on the same base and between the same parallels]}\)
= \(\frac{1}{2} \times 2\pi r^2\) (Since, area of the parallelogram = \(2\pi r^2\))
= \(\pi r^2\) \(\cdots \cdots \) (iii)
Now, from the relations (ii) and (iii), we get:

The area of the circle = The area of the triangle OAB

The learners may extend this study as follows:

Cut half portion from top to bottom of the 1st sector and place the portion to the side of the last one as in figure- 3.12 & 3.13. Then the shape will be like a rectangle.

Then, the areas of the rectangle, circle and triangle (Pl. see figure 3.14) can be found.

Activity-4: Verifying the area of a circle when like a triangle is made by cord.

Requirements: Cord, scissors, sketch pen, instruments of geometrical box, gum, plain paper.

Mode: Pair group.

Strategy: Learning through activities.

Objective of the development: Cognitive development.

Activity follows:

Stage-I: Teacher will do the following activity involving the learners.

1. The teacher will ask the formula of area of a triangle/a right angled triangle to the learners showing a right angled triangular shaped paper.

Stage-II: Learners will do the following activities with help of the teacher if needed.

Each pair group:

1. Takes a piece of cord.
2. Winds the cord round and round in the spiral form on a plane starting at the centre until a suitable circle is obtained. (Pl. see figure-4.1)
3. Draws a coloured line between the starting point and the end point on the circular area of the cord. (Pl. see figure-4.1)
4. Cuts carefully along the coloured line. (Pl. see figure-4.1)
5. Re-arranges the pieces one by one in order of their lengths to make a right angled triangle by gum on a plain paper. (Pl. see figure-4.2)
The work is illustrated below:

Here, the area of the triangle $\approx \frac{1}{2} \times (r \times 2\pi r) = \pi r^2$

So, the area of the circle $\approx \pi r^2$. (However, this is an approximate relation since the hypotenuse of the right angled triangle is approximated to a line segment and the lengths of the cut-piece cords are approximated to $2\pi \times$ their radii)

**Alternative form of Activity-4: Verifying the area of a circle when like a triangle is made by cord.**

**Requirements:** Cord, scissors, sketch pen, instruments of geometrical box, gum, plain paper.

**Mode:** Pair group.

**Strategy:** Learning through activities.

**Objective of the development:** Cognitive development.

**Activity follows:**

**Stage-I:** Teacher will do the following activity involving the learners.
1. The teacher will ask the formula of area of a triangle/a right angled triangle to the learners showing a right angled triangular shaped paper.

**Stage-II:** Learners will do the following activities with help of the teacher if needed.

Each pair group:
1. Takes a piece of cord.
2. Winds the cord round and round in the spiral form on a plane starting at the centre until a suitable circle is obtained. (Pl. see figure-4.3)
3. Draws a coloured line along diameter through the starting point and end point on the circular area of the cord. (Pl. see figure-4.3)
4. Cuts carefully along the coloured line i.e. diametric line (Pl. see figure-4.3)
5. Re-arranges the pieces one by one in order of their lengths to make a right angled triangle by gum on a plain paper. (Pl. see figure-4.4)
The work is illustrated below:

Here, the area of the triangle \( \approx \frac{1}{2} \times (2r \times \pi r) \)
\[ = \pi r^2 \]
So, the area of the circle \( \pi r^2 \). {However, this is an approximate relation since the hypotenuse of the right angled triangle is approximated to a line segment and the lengths of the cut-piece cords are approximated to \( \pi \times \) their radii and the right angled triangular figure formed with height \( 2r \) and base approximately equal to \( \pi r \) (approximation is due to the thickness of the cord)}.

5. IMPLEMENTATION

This study may be implemented in text books of mathematics at school level. It could also be used in the teaching-learning process.

6. CONCLUSION

1. Five illustrations of activities for finding the area of a circle have been presented in this paper. Only similar activity of activity no.-2 has been taken into consideration in the text books of both W.B.B.S.E. & N.C.E.R.T. but the presentation of it is not systematic. Therefore, it is a gap in the syllabus of mathematics.
2. This study will help the teachers and learners to find the area of a circle through activity based learning.
3. Collection of multiple numbers of activities will help the teachers to choose the appropriate activity for the learners considering the learners’ ability levels, time limits, availability of working materials and class room ambience etc.
4. This study will also help to prepare a proper syllabus, to develop a good text book and to improve the quality of teaching-learning process of ‘area of circle’ of mathematics.
5. These type of activities will help the children to enjoy learning mathematics so that the phobia in mathematics will be reduced and stop the drop out of learners.
6. Special interest towards mathematics can be enhanced which will be helpful for entire science education.
7. As these activities are presented step by step i.e. in an iterative sequential form. This can be appropriate in preparing text material through computer based learning.
7. FURTHER STUDY

Some more activities may be developed and the accuracy level may be increased by taking large number of division.

REFERENCE


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