International Journal of Mathematical Archive-7(5), 2016, 199-206

MHD BLOOD FLOW THROUGH STENOSED INCLINED TUBE OF EXPONENTIAL DIVERGING VESSEL WITH PERIODIC BODY ACCELERATION

V. P. RATHOD¹, RAVI.M*²

¹Department of Mathematics, Gulbarga University, Kalaburagi, India.

²Department of Mathematics, Govt. First Grade College, Raichur, Karnataka State, India.

(Received On: 09-05-16; Revised & Accepted On: 31-05-16)

ABSTRACT

The mathematical model for MHD blood flow through stenosed inclined tube of exponential diverging vessel with periodic body acceleration is studied. Using appropriate boundary conditions, analytical expressions for the velocity, volumetric flow rate, fluid acceleration and the wall shear stress have been derived. These expressions are computed numerically and the computational results are presented graphically.

Key Words: Blood flow, Stenosis, Exponential diverging vessels, Periodic body acceleration, magnetic field, inclined tubes

INTRODUCTION

Arterial MHD pulsatile flow of blood under the periodic body acceleration is studied by Das and Saha [1]. El-Shahawey *et al.* [2] studied the pulsatile flow of blood through a porous medium under periodic body acceleration. Effect of magnetic field on blood flow under periodic Body acceleration in porous medium is studied by Varun Mohan *et al.* [3]. Bhuvan and Hazarika [4] studied the effect of magnetic field on pulsatile flow of blood in a porous channel. Mishra *et al.*, [5] studied the study of oscillatory blood flow through porous medium in a stenosed artery. Blood flow through an artery having radially non-symmetric mild stenosis is studied by Singh *et al.* [6]. Flow of Casson fluid through an inclined tube of non-uniform cross-section with multiple stenoses is studied by Sreenadh *et al.* [7]. Rathod *et al.*, [8] studied the pulsatile flow of blood through a stenosed tube under periodic body acceleration with magnetic field. Sharma *et al.*, [10] studied the pulsatile unsteady flow of blood through porous medium in a stenosic artery under the influence of transverse magnetic field. Rathod and Ravi [11] studied the blood flow through stenosed inclined tubes with periodic body acceleration in the presence of magnetic field and it's applications to cardiovascular diseases. Singh and Rathee [12] studied the analysis of non-Newtonian blood flow through stenosed vessel in porous medium under the effect of magnetic field.

Using finite Hankel and Laplace transforms, analytical expressions for velocity profile, volumetric flow rate and wall shear stress have been obtained for an inclined stenosed tube and their natures are portrayed graphically for different parameters such as Hartmann number, phase angle, time etc.

MATHEMATICAL FORMULATION

Let us consider the axially symmetric and fully developed pulsatile flow of blood through a stenosed porous circular artery with body acceleration under the influence of uniform transverse magnetic field. Blood is assumed to be Newtonian and incompressible fluid. Also for mathematical model, we take the artery to be a long cylindrical tube along z-axis. The pressure gradient and body acceleration are respectively given by

$$-\frac{\partial P}{\partial z} = A_0 + A_1 \cos(\omega_P t) \tag{1}$$
$$G = a_0 \cos(\omega_P t + \phi) \tag{2}$$

Corresponding Author: Ravi.M*2

¹Department of Mathematics, Gulbarga University, Kalaburagi, India. ²Department of Mathematics, Govt. First Grade College, Raichur, Karnataka State, India.

V. P. Rathod¹, Ravi.M^{*2} / MHD Blood Flow Through Stenosed Inclined Tube of Exponential Diverging... / IJMA- 7(5), May-2016.

where A_0 and A_1 are pressure gradient of steady flow and amplitude of oscillatory part respectively, a_0 is the amplitude of body acceleration, $\omega_p = 2\pi f_p$, $\omega_b = 2\pi f_b$ with f_p is the pulse frequency and f_b is body acceleration frequency, ϕ is the phase angle of body acceleration G with respect to pressure gradient and time, t.

The governing equation of motion for flow in cylindrical polar coordinates is given by

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial z} + \rho G + \mu \nabla^2 u - \frac{\mu}{k} u - \sigma B_0^2 u + \rho g \sin \theta$$
(3)

where u is the axial velocity of blood; $\frac{\partial p}{\partial z}$ is pressure gradient; ρ is density of blood; μ is the viscosity of blood; k is the permeability of the isotropic porous medium; B_0 is the external magnetic field along the radial direction and σ is the conductivity of blood.

The geometry of stenos is is shown in figure-1.

$$R(z) = \begin{cases} a - \delta \left(1 + \cos \frac{\pi z}{2z_0} \right), & -2z_0 \le z \le 2z_0 \\ a & otherwise \end{cases}$$

where R(z) is the radius of the stenosed artery and depends on δ , *a* is the radius of artery, $4z_0$ is the length of stenosis and 2δ is the maximum protuberance of the stenotic form of the artery wall.



Fig.1.Geometry of artery with stenosis

$$\xi = \frac{r}{R(z)}$$

The equation (3) becomes

$$\rho \frac{\partial u}{\partial t} = A_0 + A_1 \cos(\omega_p t) + \rho a_0 \cos(\omega_b t + \phi) + \frac{\mu}{R^2} \left[\frac{\partial^2 u}{\partial \xi^2} + \frac{1}{\xi} \frac{\delta u}{\delta \xi} \right] - \mu C^2 u + \rho g \sin \theta$$
(4)

where

$$C = \sqrt{\frac{1}{k} + \frac{M^2}{R^2}}, \ M = \sqrt{\frac{\sigma}{\mu}} RB_0(Hartmann number)$$



Fig.2.Flow geometry of an arterial stenosis

V. P. Rathod¹, Ravi.M^{*2} / MHD Blood Flow Through Stenosed Inclined Tube of Exponential Diverging... / IJMA- 7(5), May-2016.

It is considered that the blood is undergoing periodic pulsatile motion through a rigid circular channel with impermeable walls which diverge exponentially as, $R_1(z) = \frac{a}{2} \left(1 + e^{\frac{\varepsilon z}{z_0}} \right)$ where 'a' is the tube radius, ε the

divergence parameter over a characteristic axial distance Z_0 and is considered to be very less than one.

It is assumed that t < 0 only the pumping action of the heart is present and at t = 0, the flow in the artery corresponds to the instantaneous pressure gradient i.e.,

$$-\frac{\partial P}{\partial z} = A_0 + A_1$$

As a result, the flow velocity at t=0 is given by $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

$$u(\xi,0) = \frac{A_0 + A_1}{\mu C^2} \left[1 - \frac{I_0(CR\xi)}{I_0(CR\xi_1)} \right]$$
(5)

where I_0 is modified Bessel function of first kind of order zero and $\xi_1 = \frac{R_1(z)}{R_0}$

The initial and boundary conditions to the problem are

$$u(\xi, 0) = \frac{A_0 + A_1}{\mu C^2} \left[1 - \frac{I_0(CR\xi)}{I_0(CR\xi_1)} \right]$$

 $u = 0 \quad at \quad \xi = \xi_1$
 $u \quad is \quad finite \quad at \quad \xi = 0$
(6)

Solutions: Applying Laplace transform to equation (4) and first boundary condition of (6), we get

$$\rho s \overline{u} - \frac{\rho(A_0 + A_1)}{\mu C^2} \left[1 - \frac{I_0(CR\xi)}{I_0(CR\xi_1)} \right]$$
$$= \frac{A_0}{s} + \frac{A_1 s}{(s^2 + \omega_p^2)} + \frac{\rho a_0(s \cos \phi - \omega_b \sin \phi)}{(s^2 + \omega_b^2)} + \frac{\mu}{R^2} \left[\frac{\partial^2 \overline{u}}{\partial \xi^2} + \frac{1}{\xi} \frac{\delta \overline{u}}{\delta \xi} \right] - \mu C^2 \overline{u} + \frac{\rho g \sin \theta}{s}$$
(7)
where $\overline{u}(\xi, s) = \int_0^\infty e^{-st} u(\xi, t) dt(s \succ 0)$

Then applying the finite Hankel transform to equation (7), we obtain

$$\overline{u}^{*}(\lambda_{n},s) = \frac{J_{1}(\xi_{1}\lambda_{n})R^{2}}{\lambda_{n}[\rho s R^{2} + \mu(C^{2}R^{2} + \lambda_{n}^{2})]} \begin{bmatrix} \frac{A_{0}}{s} + \frac{A_{1}s}{(s^{2} + \omega_{p}^{2})} + \frac{\rho a_{0}(s\cos\phi - \omega_{b}\sin\phi)}{(s^{2} + \omega_{b}^{2})} + \frac{\rho(A_{0} + A_{1})R^{2}}{\mu(C^{2}R^{2} + \lambda_{n}^{2})} \\ + \frac{\rho g\sin\theta}{s} \end{bmatrix}$$
(8)
where $\overline{u}^{*}(\lambda_{n},s) = \int_{0}^{1} ru(r,s)J_{0}(r\lambda_{n})dr$ and λ_{n} are zeros of J_{0} , Bessel function of first kind and $v = \frac{\mu}{\rho}$

ρ

V. P. Rathod¹, Ravi.M^{*2} / MHD Blood Flow Through Stenosed Inclined Tube of Exponential Diverging... / IJMA- 7(5), May-2016.

The Laplace and Hankel inversions of equation (8) give the final solution for blood velocity as

Г

$$u(\xi,t) = 2\sum_{n=1}^{\infty} \frac{1}{\xi_{1}\lambda_{n}} \frac{J_{0}(\lambda_{n}\xi)}{J_{1}(\xi_{1}\lambda_{n})} \left\{ -e^{-\left(\frac{v}{R^{2}}\right)(\lambda_{n}^{2}+C^{2}R^{2})t} \left\{ \frac{\frac{A_{1}R^{2}[v(\lambda_{n}^{2}+C^{2}R^{2})\cos\omega_{p}t + \omega_{p}R^{2}\sin\omega_{p}t]}{\rho[R^{4}\omega_{p}^{2} + v^{2}(\lambda_{n}^{2}+C^{2}R^{2})^{2}]} \right\} - e^{-\left(\frac{v}{R^{2}}\right)(\lambda_{n}^{2}+C^{2}R^{2})t} \left\{ \frac{\frac{-A_{1}\omega_{p}^{2}R^{6}}{\mu(\lambda_{n}^{2}+C^{2}R^{2})[R^{4}\omega_{p}^{2} + v^{2}(\lambda_{n}^{2}+C^{2}R^{2})^{2}]}{R^{4}\omega_{b}^{2} + v^{2}(\lambda_{n}^{2}+C^{2}R^{2})\cos\phi + \omega_{b}R^{2}\sin\phi]} \right\}$$
(9)

which can be written in the form

$$u(\xi,t) = \frac{2A_{0}R^{2}}{\mu} \sum_{n=1}^{\infty} \frac{1}{\xi_{1}\lambda_{n}} \frac{J_{0}(\lambda_{n}\xi)}{J_{1}(\xi_{1}\lambda_{n})} \left\{ \frac{A_{0} + \rho g \sin \theta}{A_{0} (\lambda_{n}^{2} + C^{2}R^{2})} + \frac{\varepsilon[(\lambda_{n}^{2} + C^{2}R^{2})\cos \omega_{p}t + \alpha^{2}\sin \omega_{p}t]}{(\lambda_{n}^{2} + C^{2}R^{2})^{2} + \alpha^{4}} \right\} \\ + \frac{\rho a_{0}}{A_{0}} \left\{ \frac{(\lambda_{n}^{2} + C^{2}R^{2})\cos(\omega_{p}t + \phi) + \beta^{2}\sin(\omega_{p}t + \phi)}{(\lambda_{n}^{2} + C^{2}R^{2})^{2} + \beta^{4}} \right\} \\ - e^{-\left(\frac{\nu}{R^{2}}\right)(\lambda_{n}^{2} + C^{2}R^{2})t} \left\{ \frac{\frac{-\varepsilon\alpha^{4}}{(\lambda_{n}^{2} + C^{2}R^{2})(\alpha^{4} + (\lambda_{n}^{2} + C^{2}R^{2})^{2})}{(\lambda_{n}^{2} + C^{2}R^{2})(\alpha^{4} + (\lambda_{n}^{2} + C^{2}R^{2})^{2})} \right\} \\ + \frac{\frac{\rho a_{0}}{A_{0}} \{(\lambda_{n}^{2} + C^{2}R^{2})^{2} + \beta^{4}}{(\lambda_{n}^{2} + C^{2}R^{2})^{2} + \beta^{4}} \\ + \frac{\rho^{2}g \sin \theta}{A_{0}} \frac{1}{(\lambda_{n}^{2} + C^{2}R^{2})} \right\}$$
(10)
where $\alpha^{2} = \frac{\omega_{p}R^{2}}{\nu} = \operatorname{Re}_{p} \int_{0}^{\beta^{2}} = \frac{\omega_{b}R^{2}}{\nu} = \operatorname{Re}_{b} \int_{0}^{\infty} \frac{-\frac{A_{1}}{A_{0}}}{\beta_{0}} \int_{0}^{\infty} \frac{-\frac{A_{1}}{A_{0}}} \int_{0}^{\infty} \frac{-\frac{A_{1}}{A_{0}}}{\beta_{0}} \int_{0}^{\infty} \frac{A_{1}}{A_{0}}} \int_{0}^{\infty} \frac{-\frac{A_{1}}{A_{0}}}{\beta_{0}} \int_{0}^{\infty} \frac{-\frac{A_{1}}{A_{0}}}{\beta_{0}} \int_{0}^{\infty} \frac{-\frac{A_{1}}{A_{0}}}{\beta_{0}} \int_{0}^{\infty} \frac{-\frac{A_{1}}{A_{0}}}{\beta_{0}} \int_{0}^{\infty} \frac{-\frac{A$

The analytical expression of u consists of four parts. The first and second parts correspond to steady and oscillatory parts of pressure gradient, the third term indicates body acceleration and the last term is the transient term .As $t \rightarrow \infty$, the transient term approaches to zero. Then from equation (10), we get _

,

$$u(\xi,t) = \frac{2A_0R^2}{\mu} \sum_{n=1}^{\infty} \frac{1}{\xi_1 \lambda_n} \frac{J_0(\lambda_n \xi)}{J_1(\xi_1 \lambda_n)} \left\{ \frac{\left\{ \frac{A_0 + \rho g \sin \theta}{A_0 (\lambda_n^2 + C^2 R^2)} + \frac{\varepsilon[(\lambda_n^2 + C^2 R^2) \cos \omega_p t + \alpha^2 \sin \omega_p t]}{(\lambda_n^2 + C^2 R^2)^2 + \alpha^4} \right\} + \frac{\rho a_0}{A_0} \left\{ \frac{(\lambda_n^2 + C^2 R^2) \cos(\omega_b t + \phi) + \beta^2 \sin(\omega_b t + \phi)}{(\lambda_n^2 + C^2 R^2)^2 + \beta^4} \right\}$$
(11)

The volumetric flow rate Q is given by $Q(\xi, t) = 2\pi \int_{0}^{K} ru \, dr$

$$Q(\xi,t) = \frac{4\pi A_0 R^2}{\mu} \sum_{n=1}^{\infty} \frac{1}{\lambda_n^2} \left[\begin{cases} \frac{A_0 + \rho g \sin \theta}{A_0 (\lambda_n^2 + C^2 R^2)} + \frac{\mathcal{E}[(\lambda_n^2 + C^2 R^2) \cos \omega_p t + \alpha^2 \sin \omega_p t]}{(\lambda_n^2 + C^2 R^2)^2 + \alpha^4} \\ + \frac{\rho a_0}{A_0} \left\{ \frac{(\lambda_n^2 + C^2 R^2) \cos(\omega_b t + \phi) + \beta^2 \sin(\omega_b t + \phi)}{(\lambda_n^2 + C^2 R^2)^2 + \beta^4} \right\} \right]$$
(12)

The fluid acceleration F is given by $F(\xi, t) = \frac{\partial u}{\partial t}$

$$F(\xi,t) = \frac{2a_0}{\rho} \sum_{n=1}^{\infty} \frac{1}{\xi_1 \lambda_n} \frac{J_0(\lambda_n \xi)}{J_1(\xi_1 \lambda_n)} \left[\begin{cases} \frac{\alpha^2 \varepsilon \{-(\lambda_n^2 + C^2 R^2) \sin \omega_p t + \alpha^2 \cos \omega_p t\} \}}{(\lambda_n^2 + C^2 R^2)^2 + \alpha^4} \\ + \frac{\rho a_0 \beta^2}{A_0} \{ \frac{-(\lambda_n^2 + C^2 R^2) \sin (\omega_b t + \phi) + \beta^2 \cos (\omega_b t + \phi)}{(\lambda_n^2 + C^2 R^2)^2 + \beta^4} \} \end{cases} \right]$$
(13)

The expression for the wall shear stress τ_w can be obtained from $\tau_w = \mu \left(\frac{\partial u}{\partial r}\right)_{r=R}$

$$\tau_{w}(\xi,t) = -\frac{2A_{0}R^{2}}{\xi_{1}}\sum_{n=1}^{\infty} \left[\begin{cases} \frac{A_{0} + \rho g \sin \theta}{A_{0} (\lambda_{n}^{2} + C^{2}R^{2})} + \frac{\mathcal{E}[(\lambda_{n}^{2} + C^{2}R^{2})\cos \omega_{p}t + \alpha^{2}\sin \omega_{p}t]}{(\lambda_{n}^{2} + C^{2}R^{2})^{2} + \alpha^{4}} \\ + \frac{\rho a_{0}}{A_{0}} \left\{ \frac{(\lambda_{n}^{2} + C^{2}R^{2})\cos(\omega_{b}t + \phi) + \beta^{2}\sin(\omega_{b}t + \phi)}{(\lambda_{n}^{2} + C^{2}R^{2})^{2} + \beta^{4}} \right\}$$
(14)







Fig.-3: Velocity decreases with increase in ξ_1 for increase in magnetic field for particular value of $\phi = 90^\circ, \theta = 30^\circ$ and t = 2

Fig.-4: Velocity increases with increase in t for increase in magnetic field for particular value of $\phi = 90^{\circ}$, $\theta = 30^{\circ}$ and $\xi_1 = 0.2$

Fig.-5: If we plot F vs ξ_1 for increasing t for particular value of $\phi = 90^\circ, \theta = 30^\circ and M = 2$ then the nature of the curve is as shown in the figure.

Fig.-6: If we plot F vs t for increasing M for particular value of $\phi = 90^{\circ}$, $\theta = 30^{\circ}$ and $\xi_1 = 0.2$ then the nature of the curve is as shown in the figure.

Fig.-7: If we draw Q vs t for different M, as M increases Q decreases for increasing t for particular $\phi = 45^{\circ}$, $\theta = 30^{\circ}$

Fig.-8: τ increases with increase in M for different t and the nature of the curve is as shown in fig. for particular $\phi = 45^{\circ}, \theta = 30^{\circ} and \xi_1 = 0.4$

© 2016, IJMA. All Rights Reserved

REFERENCES

- 1. K.Das and G.C. Saha, "Arterial MHD Pulsatile flow of blood under the periodic body acceleration", Bull. Soc. Math . Banja Luka, Vol.16(2009), 21-42
- 2. El-Shahawey ,E.F. Elsayed, M.E.et al., "Pulsatile flow of blood through a porous medium under periodic body acceleration", Int. J .Theoretical Physics39(1),183-188(2000)
- Varun Mohan, Dr. Virendra Prasad, Dr.N.K.Varshney and Dr. Pankaj Kumar Gupta, "Effect of Magnetic Field on Blood Flow (Elastico- Viscous) Under Periodic Body Acceleration in Porous Medium", IOSR Journal of Mathematics (IOSR-JM) e-ISSN: 2278-5728,p-ISSN: 2319-765X, Volume 6, Issue 4 (May. - Jun. 2013), PP 43-48
- 4. Bhuvan.B.C.and Hazarika .G.C. ,"Effect of magnetic field on Pulsatile flow of blood in a porous channel", Bio-Science research Bulletin.17(2),105-111(2001)
- 5. Gaurav Mishra, Ravindra Kumar and K.K.Singh, "A study of Oscillatory blood flow through porous medium in a stenosed artery, Ultra Scientist.Vol.24(2)A,369-373(2012)
- 6. Bijendra Singh, Padma Joshi and B. K. Joshi, "Blood Flow through an Artery Having Radially Non-Symmetric Mild Stenosis", Applied Mathematical Sciences, Vol. 4, 2010, no. 22, 1065 – 1072
- S.Sreenadh, A.Raga Pallavi and Bh.Satyanarayana, "Flow of Casson fluid through an inclined tube of nonuniform cross section with multiple stenoses". Pelagiya Research Library ,Advances in Applied Science Research, 2011,2(5),340-349
- 8. Rathod.V.P, Shakara Tanveer ,Itagi Sheeba Rani, G.G. Rajput, "Pulsatile flow of blood under the periodic body acceleration with magnetic field", Ultra Scientist of Physical Sciences,17(1)M,7-16(2005)
- 9. Rathod .V.P. and Gopichand, "Pulsatile flow of blood through a stenosed tube under periodic body acceleration with magnetic field", Ultra Scientist of Physical Science, Vol(1)M,pp109-118(2005)
- 10. Mukesh Kumar Sharma , Kuldip Bansal and Seema Bansal, "Pulsatile unsteady flow of blood through porous medium in a stenotic artery under the influence of transverse magnetic field", Korea-Australia rheology journal,September 2012,Volume 24,Issue 3, 181-189
- 11. V.P.Rathod, Ravi.M, "Blood Flow Through Stenosed Inclined Tubes With Periodic Body Accelaration in The Presence Of Magnetic field and it's Applications to Cardiovascular Diseases", IJRET: International Journal of Research in Engineering and Technology eISSN: 2319-1163 | pISSN: 2321-7308, Volume: 03 Special Issue: 03 | May-2014 | NCRIET-2014
- 12. Jagdish Singh and Rajbala Rathee, "Analysis of non-Newtonian blood flow through stenosed vessel in porous medium under the effect of magnetic field", International Journal of the Physical Sciences, Vol.6(10), pp.2497-2506, 18May, 2011.

Source of support: Nil, Conflict of interest: None Declared

[Copy right © 2016. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]