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# Between $\alpha$-closed sets and $\tilde{g}_\alpha$ -closed sets

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***ABSTRACT***

*In this paper we introduce and study a new class of generalized closed sets called $\psi^*\alpha$ -closed sets in topological spaces. We analyze the relations between $\psi^*\alpha$ -closed sets with already existing closed sets. We discuss some basic properties of $\psi^*\alpha$ -closed sets The class of $\psi^*\alpha$ -closed sets is properly placed between the class of $\alpha$-closed sets and the class of $\tilde{g}_\alpha$ (resp. $\psi$)-closed sets. We prove that the class of $\psi^*\alpha$- closed sets form a topology.*

***Keywords:*** *$\alpha$ -closed sets, $\psi$-closed sets, $\psi$g-closed sets and $\psi^*\alpha$ -closed sets*

## 1. INTRODUCTION

Njastad [18] introduced the concept of an $\alpha$-open sets. Levine [13] introduced the notion of g-closed sets in topological spaces and studied their basic properties. Veerakumar [22] introduced and studied $\psi$-closed sets in topological spaces. Ramya and Parvathi [20] introduced a new concept of generalized closed sets called $\psi\hat{g}$ -closed sets and $\psi$g-closed sets in topological spaces. Jafari *et.al*[10] introduced the class of $\tilde{g}_\alpha$-closed sets. In this paper we introduce a new class of generalized closed sets called $\psi^*\alpha$ -closed sets in topological spaces. This class is obtained by generalizing $\alpha$ -closed sets via $\psi$g-open sets.

## 2. PRELIMINARIES

Throughout this paper (X, $\tau$) represents non-empty topological space on which no separation axioms are defined, unless otherwise mentioned. The interior, closure and complement of a subset A of a space (X, $\tau$) are denoted by int(A), cl(A) and $A^c$ respectively.

**Definition 2.1:** A subset A of a topological space (X, $\tau$) is called
(i) Semi-open set [12] if A $\subseteq$ cl(int(A))
(ii) $\alpha$ -open set [18] if A $\subseteq$ int(cl(int(A)))
(iii) Pre-open set [17] if A $\subseteq$ int(cl(A))
(iv) semi pre-open set [3] if A $\subseteq$ cl(int(cl(A)))

The complements of the above mentioned sets are called semi-closed, $\alpha$-closed, pre-closed and semi pre-closed sets respectively

The intersection of all semi-closed (resp.$\alpha$-closed, pre-closed and semi pre-closed) subsets of (X,$\tau$) containing A is called the semi-closure (resp. $\alpha$-closure, pre-closure and semi pre-closure) of A and is denoted by scl(A) (resp. $\alpha$cl(A), pcl(A) and spcl(A)). A subset A of (X, $\tau$) is called nowhere dense if int(cl(A))=$\phi$. A subset A of a topological space (X, $\tau$) is called semi-closed (resp.$\alpha$-closed) if and only if scl(A)=A(resp. $\alpha$cl(A)=A)

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**Definition 2.2:** A subset A of a topological space (X, τ) is called
(a)generalized closed set ( briefly g-closed) [13] if cl(A) ⊆ U whenever A⊆U and U is open in (X, τ).
(b) generalized semi-closed set (briefly gs-closed) [4] if scl(A) ⊆ U whenever A⊆U and U is open in (X, τ).
(c) semi-generalized closed set (briefly sg-closed) [5] if scl(A) ⊆ U whenever A⊆U and U is semi- open in (X, τ).
(d) generalized $\alpha$-closed set(briefly g$\alpha$-closed) [14] if αcl(A) ⊆ U whenever A⊆U and U is $\alpha$-open in (X, τ).
(e) $\alpha$-generalized closed set (briefly $\alpha$g-closed) [15] if αcl(A) ⊆ U whenever A⊆U and U is open in (X, τ).
(f) generalized semi-pre -closed set(briefly gsp-closed) [7] if spcl(A) ⊆ U whenever A⊆U and U is open in (X, τ).
(g) $\hat{g}$ - closed set [24] if cl(A) ⊆ U whenever A⊆U and U is semi-open in (X, τ).
(h) $g^*$-closed set[23] if cl(A) ⊆ U whenever A⊆U and U is g-open in (X, τ).
(i)$^*$g- closed set [30] if cl(A) ⊆ U whenever A⊆U and U is $\hat{g}$ -open in (X, τ).
(j) gp-closed set [16] if pcl(A) ⊆ U whenever A⊆U and U is open in (X, τ).
(k) $g^*$p -closed set [25] if pcl(A) ⊆ U whenever A⊆U and U is g-open in (X, τ).
(l) α$\hat{g}$ - closed set [1] if αcl(A) ⊆ U whenever A⊆U and U is $\hat{g}$ -open in (X, τ).
(m) αgs -closed set[19] if αcl(A) ⊆ U whenever A⊆U and U is semi-open in (X, τ).
(n) $g^\#$s-closed set [26] if scl(A) ⊆ U whenever A⊆U and U is αg-open in (X, τ).
(o) $^\#$gs-closed set [29] if scl(A) ⊆ U whenever A⊆U and U is $^*$g -open in (X, τ).
(p) $\tilde{g}$-closed set [9] if cl(A) ⊆ U whenever A⊆U and U is $^\#$ gs-open in (X, τ).
(q) $\tilde{g}_\alpha$-closed set [10] if αcl(A) ⊆ U whenever A⊆U and U is $^\#$ gs-open in (X, τ).
(r) $\tilde{g}$-semi-closed set[21] if scl(A) ⊆ U whenever A⊆U and U is $^\#$ gs-open in (X, τ).
(s) $\tilde{g}$-pre closed set[8] if pcl(A) ⊆ U whenever A⊆U and U is $^\#$ gs-open in (X, τ).
(t) $g^\#$- closed set[27] if cl(A) ⊆ U whenever A⊆U and U is $\alpha$g - open in (X, τ).
(u) $g^\#p^\#$ -closed set[2] if cl(A) ⊆ U whenever A⊆U and U is $g^\#$ - open in (X, τ).
(v) ψ-closed set [22] if scl(A) ⊆ U whenever A⊆U and U is sg-open in (X, τ).
(w) ψg-closed set [20] if ψcl(A) ⊆ U whenever A⊆U and U is open in (X, τ).
(x) $g^*\psi$ -closed set[28] if ψcl(A) ⊆ U whenever A⊆U and U is g-open in (X, τ).
(y) ψ$\hat{g}$ - closed set[20] if ψcl(A) ⊆ U whenever A⊆U and U is $\hat{g}$ -open in (X, τ).
(z) αψ- closed set [6] if ψcl(A) ⊆ U whenever A⊆U and U is $\alpha$-open in (X, τ).
The complements of the above mentioned sets are called their respective open-sets.

## 3. $\psi^*\alpha$ -CLOSED SETS

**Definition 3.1:** A subset A of a topological space (X, τ) is said to be $\psi^*\alpha$ -closed set if αcl(A) ⊆ U whenever A⊆U and U is ψg -open in (X, τ).

The class of all $\psi^*\alpha$ -closed sets of (X,τ) is denoted by $\psi^*\alpha$ C(X, τ).

**Proposition 3.2:** Every closed set in (X, τ) is $\psi^*\alpha$ -closed but not conversely.

**Proof:** Let A be a closed set and U be any ψg -open set containing A in X. Since every closed set is $\alpha$-closed, αcl(A) ⊆ cl(A) =A⊆ U. Therefore A is $\psi^*\alpha$ - closed.

**Example 3.3:** Let X={a, b, c}, τ ={ϕ,{a},{a, b},X}. Then the subset {b} is $\psi^*\alpha$ -closed but not closed in (X, τ).

**Proposition 3.4:** Every $\alpha$ -closed set in (X, τ) is $\psi^*\alpha$ -closed but not conversely.

**Proof:** Let A be an $\alpha$-closed set and U be any ψg -open set containing A in X. Since A is $\alpha$-closed, αcl(A) =A, αcl(A) =A⊆ U. Therefore A is $\psi^*\alpha$ -closed.

**Example 3.5:** Let X={a, b, c}, τ ={ϕ,{a, b},X}. Then the subset {a, c} is $\psi^*\alpha$ -closed but not $\alpha$ -closed in (X, τ).

**Lemma 3.6:** Every $^\#$gs -closed set in (X, τ) is ψg - closed but not conversely.

**Proof:** Let A be a $^\#$gs -closed set and U be any open set containing A in X. Since every open set is $^*$g-open and A is $^\#$gs -closed, scl(A)⊆U. For every subset A of X, ψcl(A)⊆scl(A) and so ψcl(A)⊆U. Hence A is ψg -closed.

**Example 3.7:** Let X={a, b, c}, τ ={ϕ,{a}, X}.Then the subset {a, b} is ψg -closed but not $^\#$gs -closed in (X, τ).

**Proposition 3.8:** Every $\psi^*\alpha$ -closed set in (X, τ) is $\tilde{g}_\alpha$- closed but not conversely.

**Proof:** Let A be a $\psi^*\alpha$-closed set and U be any $^\#$gs - open set containing A in X. Since every $^\#$gs -open set is ψg -open and A is $\psi^*\alpha$ -closed, αcl(A) ⊆ U. Hence A is $\tilde{g}_\alpha$- closed.

**Example 3.9:** Let X={a, b, c, d}, τ ={ϕ,{d},{a, b},{a, b, d},X}. Then the subset {b, c, d} is $\tilde{g}_\alpha$- closed but not $\psi^*\alpha$ -closed in (X, τ).

**Proposition 3.10:** Every $\psi^*\alpha$ -closed set in (X, τ) is gα(resp.αg, sg, gs, $\tilde{g}_s$)-closed but not conversely.

**Proof:** By [10], every $\tilde{g}_\alpha$- closed set is gα ( resp. αg, sg, gs, $\tilde{g}_s$)- closed set. Hence it holds.

**Example 3.11:** Let X={a, b, c, d}, τ ={ϕ,{d}, {a, b}, {a, b, d}, X}.Then the subset {a, c, d} is gα-closed αg-closed, sg-closed, gs-closed and $\tilde{g}_s$- closed but not $\psi^*\alpha$ -closed in (X, τ).

**Proposition 3.12:** Every $\psi^*\alpha$ -closed set in (X, τ) is $\tilde{g}$- pre closed but not conversely.

**Proof:** Follows from the fact that every $\tilde{g}_\alpha$- closed is $\tilde{g}$- pre closed.

**Example 3.13:** Let X={a, b, c}, τ ={ϕ,{a, b},X}. Then the subset {a} is $\tilde{g}$- preclosed but not $\psi^*\alpha$ -closed in (X, τ).

**Lemma 3.14:** Every semi -closed set in (X, τ) is ψg -closed but not conversely.

**Proof:** Let A be a semi- closed set and U be any open set containing A in X. Since A is semi- closed, scl(A)=A. For every subset A of X, ψcl(A)⊆scl(A) and so we have ψcl(A)⊆U. Hence A is ψg -closed.

**Example 3.15:** Let X={a, b, c}, τ ={ϕ,{a},X}. Then the subset {a, b} is ψg -closed but not semi -closed in (X, τ).

**Proposition 3.16:** Every $\psi^*\alpha$ -closed set in (X, τ) is αgs-closed but not conversely.

**Proof:** Let A be a $\psi^*\alpha$ -closed set and U be any semi-open set containing A in X. Since every semi-open set is ψg-open and A is $\psi^*\alpha$ -closed, αcl(A) ⊆ U. Hence A is αgs -closed.

**Example 3.17:** Let X={a, b, c}, τ ={ϕ,{a},{b, c},X}.Then the subset {a, c} is αgs-closed but not $\psi^*\alpha$ -closed in (X, τ).

**Lemma 3.18:** Every g-closed set in (X, τ) is ψg-closed but not conversely.

**Proof:** Let A be a g-closed set and U be any open set containing A in X. Since A is g-closed, cl(A)⊆U. For every subset A of X, ψcl(A)⊆cl(A) and so we have ψcl(A)⊆U. Hence A is ψg-closed.

**Example 3.19:** Let X={a, b, c}, τ ={ϕ,{a},{b},{a, b},X}. Then the subset {a} is ψg-closed but not g-closed in (X, τ).

**Proposition 3.20:** Every $\psi^*\alpha$ -closed set in (X, τ) is gp - closed ($g^*$p-closed) but not conversely.

**Proof:** Let A be a $\psi^*\alpha$ -closed set and U be any open (g-open) set containing A in X. Since every open (g-open) set is ψg-open and A is $\psi^*\alpha$ -closed, αcl(A) ⊆ U. For every subset A of X, pcl(A)⊆αcl(A)and so we have pcl(A)⊆U. Hence A is gp -closed ($g^*$p-closed).

**Example 3.21:** Let X={a, b, c}, τ ={ϕ,{a, b}, X}. Then the subset {a} is gp -closed ($g^*$p-closed) but not $\psi^*\alpha$ -closed in (X, τ).

**Lemma 3.22:** Every sg -closed set in (X, τ) is ψg - closed but not conversely.

**Proof:** Let A be a sg -closed set and U be any open set containing A in X. Since every open set is semi-open and A is sg-closed, scl(A)⊆U. For every subset A of X, ψcl(A)⊆scl(A) and so we have ψcl(A)⊆U. Hence A is ψg -closed.

**Example 3.23:** Let X={a, b, c}, τ ={ϕ,{a}, {a, b},X}. Then the subset {a, c} is ψg -closed but not sg -closed in (X, τ).

**Proposition 3.24:** Every $\psi^*\alpha$ -closed set in (X, τ) is ψ-closed but not conversely.

**Proof:** Let A be a $\psi^*\alpha$ - closed set and U be any sg -open set containing A in X. Since every sg-open set is ψg-open and A is $\psi^*\alpha$ -closed set, αcl(A) ⊆ U. For every subset A of X, scl(A)⊆αcl(A) and so we have scl(A)⊆U. Hence A is ψ -closed.

**Example 3.25:** Let X={a, b, c}, τ ={ϕ,{a},{b},{a, b},X}.Then the subset {a} is ψ-closed but not $\psi^*\alpha$ -closed in (X, τ).

**Proposition 3.26:** Every $\psi^*\alpha$ -closed set in (X, τ) is $\psi\hat{g}$ (resp. ψg, gsp)-closed but not conversely.

**Proof:** By [20], every ψ-closed set is $\psi\hat{g}$ (resp. ψg, gsp)-closed. Therefore it holds

**Example 3.27:** Let X={a, b, c, d}, τ ={ϕ,{a},X}.Then the subset {a, b} is $\psi\hat{g}$ -closed, ψg-closed, and gsp-closed but not $\psi^*\alpha$ -closed in (X, τ).

**Lemma 3.28:** Every αg -closed set in (X, τ) is ψg - closed but not conversely.

**Proof:** Let A be an αg -closed set and U be any open set containing A in X. Since A is αg -closed, αcl(A) ⊆ U. For every subset A of X, ψcl(A)⊆αcl(A) and so ψcl(A)⊆U. Hence A is ψg -closed.

**Example 3.29:** Let X={a, b, c}, τ ={ϕ,{a},{b},{a, b},X}. Then the subset {a} is ψg -closed but not αg -closed in (X, τ).

**Lemma 3.30:** Every gα -closed set in (X, τ) is ψg - closed but not conversely.

**Proof:** Let A be a gα -closed set and U be any open set containing A in X. Since every open set is α-open and A is gα -closed, αcl(A) ⊆ U. For every subset A of X, ψcl(A)⊆αcl(A) and so ψcl(A)⊆U. Hence A is ψg -closed.

**Example 3.31:** Let X={a, b, c}, τ ={ϕ,{a},{a, b},X}.Then the subset {a, c} is ψg -closed but not gα -closed in (X, τ).

**Proposition 3.32:** Every $\psi^*\alpha$ -closed set in (X, τ) is $g^\#s$ -closed but not conversely.

**Proof:** Let A be a $\psi^*\alpha$ -closed set and U be any αg -open set containing A in X. Since every αg open set is ψg -open and A is $\psi^*\alpha$-closed, αcl(A) ⊆ U. For every subset A of X, scl(A)⊆αcl(A) and so scl(A)⊆U. Hence A is $g^\#s$ -closed.

**Example 3.33:** Let X={a, b, c}, τ ={ϕ,{a},{b},{a, b},X}.Then the subset {a} is $g^\#s$ -closed but not $\psi^*\alpha$ -closed in (X, τ).

**Lemma 3.34:** Every $\hat{g}$ -closed set in (X, τ) is ψg -closed but not conversely.

**Proof:** Let A be a $\hat{g}$ -closed set and U be any open set containing A in X. Since every open set is semi open and A is $\hat{g}$ -closed, cl(A)⊆U. For every subset A of X, ψcl(A)⊆cl(A) and so we have ψcl(A)⊆U. Hence A is ψg -closed.

**Example 3.35:** Let X={a, b, c}, τ ={ϕ,{a},X}. Then the subset {a, b} is ψg -closed but not $\hat{g}$ - closed in (X, τ).

**Proposition 3.36:** Every $\psi^*\alpha$ -closed set in (X, τ) is $\alpha\hat{g}$ - closed but not conversely.

**Proof:** Let A be a $\psi^*\alpha$ -closed set and U be any $\hat{g}$ -open set containing A in X. Since every $\hat{g}$ - open set is ψg-open and A is $\psi^*\alpha$ -closed, αcl(A) ⊆ U. Hence A is $\alpha\hat{g}$-closed.

**Example 3.37:** Let X={a, b, c}, τ ={ϕ,{a},{a, b}, X}. Then the subset {a, c} is $\alpha\hat{g}$-closed but not $\psi^*\alpha$ -closed in (X, τ).

**Lemma 3.38:** Every $^*g$-closed set in (X, τ) is ψg-closed but not conversely.

**Proof:** Let A be a $^*g$ -closed set and U be any open set containing A in X. Since every open set is $\hat{g}$ -open and A is $^*g$ -closed, cl(A)⊆U. For every subset A of X, ψcl(A)⊆cl(A) and so we have ψcl(A)⊆U. Hence A is ψg-closed.

**Example 3.39:** Let X={a, b, c}, τ ={ϕ,{a},{b},{a, b}, X}.Then the subset {b} is ψg -closed but not $^*g$ -closed in (X, τ).

**Proposition 3.40:** Every $\psi^*\alpha$ -closed set in (X, τ) is $^\#gs$- closed but not conversely.

**Proof:** Let A be a $\psi^*\alpha$ -closed set and U be any $^*g$ -open set containing A in X. Since every $^*g$ -open set is ψg-open and A is $\psi^*\alpha$ -closed, αcl(A) ⊆ U. For every subset A of X, scl(A)⊆αcl(A) and so we have scl(A)⊆U. Hence A is $^\#gs$ -closed.

**Example 3.41:** Let X={a, b, c}, τ ={ϕ,{a},{a, b},X}.Then the subset {a, c} is $^\#gs$ -closed but not $\psi^*\alpha$ -closed in (X, τ).

**Proposition 3.42:** Every $\psi^*\alpha$ -closed set in (X, τ) is $g^*\psi$ -closed but not conversely.

**Proof:** Follows from the fact that every ψ-closed is $g^*\psi$ -closed

**Example 3.43:** Let X={a, b, c, d}, τ ={ϕ,{a}, {a, b},X}. Then the subset{a, c, d} is $g^*\psi$ -closed but not $\psi^*\alpha$ -closed in (X, τ).

**Proposition 3.44:** Every $\psi^*\alpha$ -closed set in (X, τ) is $\alpha\psi$ -closed but not conversely.

**Proof:** Let A be a $\psi^*\alpha$ -closed set and U be any $\alpha$ -open set containing A in X. Since every $\alpha$ -open set is ψg -open and A is $\psi^*\alpha$ -closed, $\alpha cl(A) \subseteq U$. For every subset A of X, $\psi cl(A) \subseteq \alpha cl(A)$. and so we have $\psi cl(A) \subseteq U$. Hence A is αψ-closed.

**Example 3.45:** Let X={a, b, c}, τ ={ϕ,{a},{b},{a, b},X}.Then the subset {a} is $\alpha\psi$ -closed but not $\psi^*\alpha$ -closed in (X, τ).

**Remark 3.46:** The following example shows that $\psi^*\alpha$ -closedness is independent from g-closedness, $g^*$-closedness, $^*g$-closedness and $g^\#p^\#$- closedness.

**Example 3.47:** Let X={a, b, c}, τ ={ϕ,{a},{a, b},X}.In this topology the set {a, c} is g-closed, $g^*$-closed, $^*g$ - closed and $g^\#p^\#$- closed but not $\psi^*\alpha$ -closed. The set {b} is $\psi^*\alpha$ -closed but not g-closed, $g^*$-closed, $^*g$ - closed and $g^\#p^\#$- closed.

**Remark 3.48:** The following examples show that $\psi^*\alpha$ -closedness is independent from semi-closedness.

**Example 3.49:** Let X={a, b, c}, τ ={ϕ,{a, b},X}.In this topology the set {b, c} is $\psi^*\alpha$ -closed but not semi-closed.

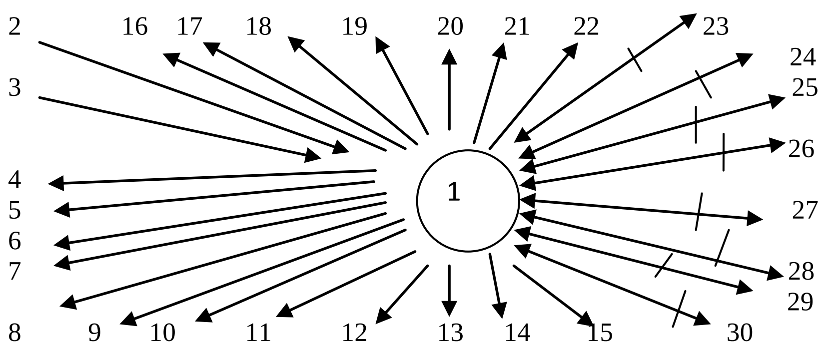
**Example 3.50:** Let X={a, b, c}, τ ={ϕ.{a},{b},{a, b},X}. In this topology the set{b} is semi-closed but not $\psi^*\alpha$ -closed.

**Remark 3.51:** The following examples show that $\psi^*\alpha$ -closedness is independent from ĝ-closedness, $g^\#$-closedness and g̃ -closedness.

**Example 3.52:** Let X={a, b, c} with τ ={ϕ,{a},{a, b},X}. In this topology the set{b} is $\psi^*\alpha$ -closed but not ĝ - closed, $g^\#$-closed and g̃-closed.

**Example 3.53:** Let X={a, b, c, d} with τ ={ϕ.{d},{a, b},{a, b, d}, X}. In this topology the set {a, c, d} is ĝ - closed, $g^\#$-closed and g̃-closed but not $\psi^*\alpha$ -closed.

**Remark 3.54:** The following diagram has shown the relationship of $\psi^*\alpha$ -closed sets with already existing various closed sets. where $A \rightarrow B$ represents A implies B but not conversely. where A ⟷ B represents A and B are independent of each other.




1. $\psi^*\alpha$ -closed 2. closed 3. α-closed 4. $\tilde{g}_\alpha$ -closed 5. gα-closed 6. αg -closed 7.sg-closed 8. gs-closed 9. g̃-semi- closed 10. g̃-pre-closed 11.αgs-closed 12.gp-closed 13. $g^*$p-closed 14. ψ-closed 15. ψĝ-closed 16.ψg- closed 17. gsp-closed 18. $g^\#$s-closed 19. αĝ -closed 20. $^\#$gs -closed 21. $g^*\psi$-closed 22.αψ-closed 23.g- closed 24. $g^*$-closed 25.$^*$g-closed 26 $g^\#p^\#$-closed 27. semi-closed 28. ĝ -closed 29. $g^\#$-closed 30. g̃ -closed

**Definition 3.55:** A subset A of a topological space (X, τ) is said to be $\psi^*\alpha$ -open if its complement $A^c$ is $\psi^*\alpha$ -closed.

The class of all $\psi^*\alpha$ -open sets in (X, τ) is denoted by $\psi^*\alpha O(X, \tau)$.

**Proposition 3.56:** Every open (respectively α-open) set is $\psi^*\alpha$ -open.

**Proposition 3.57:** Every $\psi^*\alpha$ -open set is $\tilde{g}_\alpha$ - open (respectively gα-open, αg-open, sg-open, gs-open, $\tilde{g}$-semi- open, $\tilde{g}$-pre- open, αgs-open, gp- open, $g^*$p-open, ψ-open, $\psi\hat{g}$- open, ψg-open, gsp- open, $g^\#$s-open, $\alpha\hat{g}$ -open, $^\#$gs- open, $g^*\psi$-open and αψ-open)

## 4. PROPERTIES OF $\psi^*\alpha$ -CLOSED SETS AND $\psi^*\alpha$ -OPEN SETS

**Theorem 4.1:** If A and B are $\psi^*\alpha$ -closed sets in a topological space (X, τ), then $A\cup B$ is $\psi^*\alpha$ -closed set in (X, τ).

**Proof:** Let A and B be any two $\psi^*\alpha$ -closed sets in (X, τ) and U be any ψg -open set containing A and B. We have $\alpha cl(A) \subseteq U$ and $\alpha cl(B) \subseteq U$. Always $\alpha cl(A\cup B) = \alpha cl(A) \cup \alpha cl(B) \subseteq U$. Hence $A \cup B$ is $\psi^*\alpha$ -closed in (X, τ).

**Theorem 4.2:** Let A be a $\psi^*\alpha$ -closed set in (X, τ). Then αcl(A)-A contains no non-empty closed set in (X, τ).

**Proof:** Suppose that A is $\psi^*\alpha$-closed. Let F be a closed subset of αcl(A)-A. Then $F^c$ is open and hence ψg -open such that $A\subseteq F^c$. Since A is a $\psi^*\alpha$ -closed set, $\alpha cl(A) \subseteq F^c$. Thus $F\subseteq(\alpha cl(A))^c$. Since every closed set is $\alpha$ -closed, F is $\alpha$-closed. Hence $F\subseteq\alpha cl(A)$. Therefore $F\subseteq \alpha cl(A) \cap (\alpha cl(A))^c = \phi$. Hence $F=\phi$.

**Remark 4.3:** The converse of the above theorem is not true as seen from the following example.

**Example 4.4:** Let X={a, b, c}, τ={ϕ, {a}, {b, c}, X}. If A={b} then αcl(A)-A ={b, c}-{b}={c}does not contain non-empty closed set. However A is not a $\psi^*\alpha$-closed subset of (X, τ).

**Theorem 4.5:** A set A is $\psi^*\alpha$ –closed in (X, τ) if and only if αcl(A)-A contains no non-empty ψg -closed set in (X, τ).

**Proof: (Necessity):** Suppose that A is $\psi^*\alpha$ -closed. Let F be a ψg -closed set contained in αcl(A)-A. Now $F^c$ is a ψg -open set in X such that $A\subseteq F^c$. Since A is a $\psi^*\alpha$ -closed set in X, $\alpha cl(A) \subseteq F^c$. Thus $F\subseteq(\alpha cl(A))^c$. Also $F\subseteq\alpha cl(A)$-A. Therefore $F\subseteq \alpha cl(A) \cap (\alpha cl(A))^c = \phi$. Hence $F= \phi$.

**Sufficiency:** Suppose that αcl(A)-A contains no non empty ψg -closed set. Let $A\subseteq G$ and G be ψg-open. If αcl(A) is not a subset of G then $\alpha cl(A) \cap G^c$ is a non-empty ψg -closed subset of αcl(A)-A, which is a contradiction. Therefore $\alpha cl(A)\subseteq G$ and hence A is $\psi^*\alpha$ -closed.

**Proposition 4.6:** If A is ψg -open and $\psi^*\alpha$ -closed subset of (X, τ).Then A is an α-closed set of (X, τ).

**Proof:** Since A is ψg -open and $\psi^*\alpha$ -closed, $\alpha cl(A) \subseteq A$. Hence A is α-closed.

**Theorem 4.7:** If a set A is $\psi^*\alpha$ -closed and ψg -open and F is α-closed in (X,τ), then $A \cap F$ is α-closed.

**Proof:** Since A is $\psi^*\alpha$ -closed and ψg -open, A is α-closed by **Proposition 4.6** Since F is $\alpha$ -closed in X, $A \cap F$ is $\alpha$ -closed in X.

**Theorem 4.8:** If A is a $\psi^*\alpha$ -closed set in (X,τ) and $A\subseteq B\subseteq\alpha cl(A)$ . Then B is also a $\psi^*\alpha$ -closed set in (X, τ).

**Proof:** Let U be a ψg -open set of (X, τ) such that $B\subseteq U$. Then $A\subseteq U$. Since A is a $\psi^*\alpha$ -closed set, $\alpha cl(A) \subseteq U$. Also since $B\subseteq\alpha cl(A)$, $\alpha cl(B)\subseteq \alpha cl(\alpha cl(A))= \alpha cl(A)$. Hence $\alpha cl(B)\subseteq U$. Therefore B is also a $\psi^*\alpha$ -closed set in (X, τ).

**Theorem 4.9:** Let A be a $\psi^*\alpha$ -closed set of (X,τ). Then A is α-closed if and only if αcl(A)-A is ψg -closed.

**Proof: (Necessity):** Let A be an α-closed subset of (X,τ). Then αcl(A)=A and therefore αcl(A)-A=ϕ which is ψg-closed in (X, τ).

**Sufficiency:** Let αcl(A)-A be a ψg -closed set. Since A is $\psi^*\alpha$ -closed by **theorem 4.5,** αcl(A)-A contains no non-empty ψg-closed set which implies αcl(A)-A=ϕ. That is αcl(A)=A. Hence A is α-closed.

**Definition 4.10:** Let $(X,\tau)$ be a topological space and let $B\subseteq A\subseteq X$ Then B is $\psi^*\alpha$ -closed relative to A if $(\alpha cl)_A(B)\subseteq U$, whenever $B\subseteq U$, U is $\psi$g-open in A.

**Theorem 4.11:** Let $B\subseteq A\subseteq X$ and suppose that B is $\psi^*\alpha$ -closed in $(X, \tau)$, then B is $\psi^*\alpha$ -closed relative to A. The converse is true if A is $\alpha$ - open and $\psi^*\alpha$ -closed in $(X, \tau)$.

**Proof:** Suppose that B is a $\psi^*\alpha$ -closed in $(X, \tau)$. Let $B\subseteq U$, U is $\psi$g - open in A. Since U is $\psi$g - open set in A, $U=V \cap A$, where V is $\psi$g - open in X. Hence $B\subseteq U\subseteq V$. Since B is $\psi^*\alpha$ -closed in X. $\alpha cl(B)\subseteq V$. Hence $\alpha cl(B) \cap A \subseteq V \cap A$ which in turn implies that $(\alpha cl)_A(B)\subseteq V \cap A=U$. Therefore B is $\psi^*\alpha$ -closed relative to A.

Now, to prove the converse, assume that $B\subseteq A\subseteq X$ where A is $\alpha$- open and $\psi^*\alpha$ -closed in X and B is a $\psi^*\alpha$ -closed relative to A. Let $B\subseteq U$ and U be $\psi$g-open in X. Then $A \cap U$ is $\psi$g- open in A. Since $B\subseteq A$ and $B\subseteq U$, $B\subseteq A \cap U$ Since B is a $\psi^*\alpha$ -closed relative to A, $(\alpha cl)_A(B)\subseteq A \cap U$. Since A is $\alpha$-open, it is $\psi$g-open in X. Since $A\subseteq A$ and A is $\psi^*\alpha$ -closed in X, $\alpha cl(A)\subseteq A$. Since $B\subseteq A$, $\alpha cl(B)\subseteq \alpha cl(A)$.Hence $\alpha cl(B)\subseteq A$. Therefore $\alpha cl(B)\cap A\subseteq \alpha cl(B)$ implies $(\alpha cl)_A(B)=\alpha cl(B)$. Hence $\alpha cl(B) \subseteq A \cap U=U$. Thus B is $\psi^*\alpha$ -closed in X.

**Theorem 4.12:** In a topological space $(X, \tau)$, for each $x\in X$, either $\{x\}$ is $\psi$g-closed or $X-\{x\}$ is $\psi^*\alpha$ closed set in $(X,\tau)$.

**Proof:** suppose that $\{x\}$ is not $\psi$g-closed in X. Then $X-\{x\}$ is not $\psi$g-open in X. Hence X is the only $\psi$g-open set containing $X-\{x\}$. That is $(X-\{x\}) \subseteq X$ Therefore $\alpha cl(X-\{x\}) \subseteq X$ which implies that $X-\{x\}$ is $\psi^*\alpha$ -closed in $(X, \tau)$.

**Definition 4.13:** The intersection of all $\psi$g-open subsets of $(X, \tau)$ containing A is called $\psi$g- kernal of A and is denoted by $\psi$g-ker(A)
i.e $\psi g\text{-}ker(A)=\cap\{U / U$ is $\psi$g-open in $(X, \tau)$ and $A \subseteq U\}$

**Theorem 4.14:** A subset A of a topological space $(X, \tau)$ is $\psi^*\alpha$ -closed in $(X,\tau)$ if and only if $\alpha cl(A) \subseteq \psi g\text{-}ker(A)$.

**Proof: (Necessity):** Suppose that A is $\psi^*\alpha$ -closed set in $(X,\tau)$ and $x\in\alpha cl(A)$. If $x\notin\psi g\text{-}ker(A)$, then there exists a $\psi$g - open set U in $(X,\tau)$ such that $A\subseteq U$ and $x\notin U$. Since U is $\psi$g - open set containing A and A is $\psi^*\alpha$-closed, we have $\alpha cl(A) \subseteq U$, which is a contradiction to $x\in\alpha cl(A)$ and $x\notin U$.

**Sufficiency:** Suppose that $\alpha cl(A) \subseteq\psi g\text{-}ker(A)$. If U is any $\psi$g-open set containing A, then $\psi g\text{-}ker(A) \subseteq U$ so we have $\alpha cl(A) \subseteq U$. Hence A is $\psi^*\alpha$ -closed.

**Remark 4.15:** Jankovic and Reilly [11] stated that "If x is any point in a topological space $(X, \tau)$, then every singleton $\{x\}$ is either nowhere dense or preopen in $(X, \tau)$". Also this provides another decomposition namely $X=X_1\cup X_2$ where $X_1=\{x\in X:\{x\}$ is nowhere dense$\}$ and $X_2=\{x\in X:\{x\}$ is preopen$\}$.

**Proposition 4.16:** For any subset A of a topological space $(X,\tau)$, $X_2 \cap \alpha cl(A) \subseteq\psi g\text{-}ker(A)$.

**Proof:** Let $x\in X_2\cap \alpha cl(A)$ and if $x\notin\psi g\text{-}ker(A)$.Then there is a $\psi$g-open set U containing A such that $x\notin U$. Then $U^c$ is $\psi$g-closed set containing x. Since $x\in\alpha cl(A)$, $\alpha cl(\{x\})\subseteq\alpha cl(A)$. Since $x\in X_2$, $\{x\}\subseteq int(cl(\{x\})$, hence $int(cl(\{x\}) \neq\phi$. Also $x\in\alpha cl(A)$, so $A \cap int(cl(\{x\}))\neq\phi$. Hence there is some point $y\in A \cap int(cl(\{x\}))$ and therefore $y\in A \cap U^c$ , which is a contradiction.

**Theorem 4.17:** A subset A of a topological space $(X,\tau)$ is $\psi^*\alpha$ -closed in $(X,\tau)$ if and only if $X_1 \cap \alpha cl(A) \subseteq A$

**Proof: (Necessity):** Suppose that A is $\psi^*\alpha$ -closed in $(X,\tau)$ and $x\in X_1\cap \alpha cl(A)$ but $x\notin A$. Since $x\in X_1$, $int(cl(\{x\})) =\phi$ so we have $int(cl(\{x\})) =\phi \subseteq\{x\}$. Therefore $\{x\}$ is semi-closed. Since every semi-closed set is $\psi$g -closed, $\{x\}$ is $\psi$g -closed and hence $U=X-\{x\}$ is $\psi$g-open set containing A and so $\alpha cl(A) \subseteq U$. Since $x\in\alpha cl(A)$ so we have $x\in U$, which is a contradiction.

**Sufficiency:** Suppose that $X_1 \cap \alpha cl(A) \subseteq A$. Since $A \subseteq\psi g\text{-}ker(A)$, $X_1 \cap \alpha cl(A) \subseteq\psi g\text{-}ker(A)$. Therefore $\alpha cl(A)= X\cap \alpha cl(A)=( X_1\cup X_2) \cap \alpha cl(A)=( X_1\cap \alpha cl(A)) \cup (X_2\cap \alpha cl(A))$.By hypothesis $X_1 \cap \alpha cl(A) \subseteq\psi g\text{-}ker(A)$ and by

**Proposition 4.16:** $X_2\cap \alpha cl(A) \subseteq\psi g\text{-}ker(A)$.Hence $\alpha cl(A) \subseteq\psi g\text{-}ker(A)$. Therefore by **Theorem 4.14** A is $\psi^*\alpha$ -closed.

**Theorem 4.18:** Arbitrary intersection of $\psi^*\alpha$ -closed sets in a topological space (X, τ) is $\psi^*\alpha$- closed in (X, τ).

**Proof:** Let F={$A_i$ : i∈ Λ }be a family of $\psi^*\alpha$ -closed sets and $A=\cap_{i\in\Lambda} A_i$. Since A ⊆$A_i$ for each i∈ Λ , $X_1 \cap \alpha cl(A) \subseteq X_1 \cap \alpha cl(A_i)$ for each i∈ Λ , using **theorem 4.17** for each $\psi^*\alpha$ -closed set $A_i$, we have $X_1 \cap \alpha cl(A) \subseteq X_1 \cap \alpha cl(A_i) \subseteq A_i$, for each i∈ Λ. Thus $X_1 \cap \alpha cl(A) \subseteq \cap_{i\in\Lambda} A_i$. = A That is $X_1 \cap \alpha cl(A) \subseteq A$ and so by **theorem 4.17** A is $\psi^*\alpha$-closed in (X,τ)

**Remark 4.19:** Thus from **theorem 4.1**and **theorem 4.18** leads us into another class of closed sets namely $\psi^*\alpha$ -closed sets which are closed under finite union and arbitrary intersection. Hence the class of $\psi^*\alpha$ -closed sets form a topology.

**Lemma 4.20:** For a subset A of (X, τ), αcl(X-A) =X-αint(A)

**Theorem 4.21:** A subset A of a topological space (X, τ) is $\psi^*\alpha$ -open if and only if U⊆αint(A) whenever U⊆A and U is ψg -closed.

**Proof: (Necessity)** Assume that A is $\psi^*\alpha$ -open. Then $A^c$ is $\psi^*\alpha$ -closed. Let U be a ψg -closed set in (X, τ) contained in A. Then $U^c$ is a ψg -open set in (X, τ) containing $A^c$. Since $A^c$ is $\psi^*\alpha$ -closed, $\alpha cl(A^c) \subseteq U^c$ equivalently U⊆αint(A).

**Sufficiency:** Assume that U is contained in αint(A) whenever U is contained in A and U is ψg -closed in (X, τ). Let $A^c$ be contained in U, where U is ψg -open. Then $U^c$ is contained in A. By criteria, $U^c \subseteq \alpha int(A)$. This implies $(\alpha int(A))^c \subseteq U$ that is $\alpha cl(A^c) \subseteq U$. Therefore $A^c$ is $\psi^*\alpha$ -closed. Hence A is $\psi^*\alpha$ -open in (X, τ).

**Proposition 4.22:** If αint(A) ⊆B⊆A and A is $\psi^*\alpha$ –open, then B is $\psi^*\alpha$ –open.

**Proof:** Follows from lemma **4.20 and Theorem 4.8**

**Theorem 4.23:** If A and B are $\psi^*\alpha$ -open sets in (X, τ), then A ∩ B is $\psi^*\alpha$ -open in (X, τ).

**Proof:** Let A and B be $\psi^*\alpha$-open sets in (X, τ). Then X-A and X-B are $\psi^*\alpha$-closed sets and (X-A)∪(X-B)= X-(A ∩B) is $\psi^*\alpha$-closed in (X, τ). Hence A ∩ B is $\psi^*\alpha$-open.

**Theorem 4.24:** If a set A is $\psi^*\alpha$-open in (X, τ) if and only if G=X whenever G is ψg-open and $\alpha int(A) \cup A^c \subseteq G$.

**Proof: (Necessity):** Let A be $\psi^*\alpha$-open and G is ψg-open and $\alpha int(A) \cup A^c \subseteq G$. This gives $G^c \subseteq (\alpha int(A) \cup A^c)^c = (\alpha int(A))^c \cap A = (\alpha int(A))^c - A^c = \alpha cl(A^c) - A^c$. Since $A^c$ is $\psi^*\alpha$- closed and $G^c$ is ψg -closed by **theorem 4.5**, it follows that $G^c$=ϕ. Therefore G=X.

(**Sufficiency):** Suppose that F is ψg -closed and F⊆A. Then $\alpha int(A) \cup A^c \subseteq \alpha int(A) \cup F^c$. As open implies α-open implies ψg-open, we get αint(A) is ψg-open and $F^c$ ψg-open. Henceαint(A) ∪ $F^c$ ψg-open. It follows by the hypothesis that $\alpha int(A) \cup F^c = X$ and hence F⊆αint(A). Therefore by **theorem 4.21,** A is $\psi^*\alpha$- open in (X, τ).

## 5. $\psi^*\alpha$ –CLOSURE

**Definition 5.1:** The $\psi^*\alpha$ -closure of A (briefly $\psi^*\alpha cl(A)$) of a topological space (X,τ) is defined as follows.
$\psi^*\alpha cl(A) = \cap$ {F⊆X: A ⊆F and F is $\psi^*\alpha$ -closed in (X,τ)}

**Proposition 5.2:** For a subset A of a topological space (X, τ), $A \subseteq \psi^*\alpha cl(A) \subseteq cl(A)$

**Proof:** Follows from proposition **3.2**

**Remark 5.3:** If A is $\psi^*\alpha$ -closed in (X,τ), then $\psi^*\alpha cl(A)$=A.

**Theorem 5.4:** Let A be a subset of X and x∈X, then $x \in \psi^*\alpha cl(A)$ if and only if for every $\psi^*\alpha$-open set U containing x, U ∩ A ≠ϕ.

**Proof: (Necessity):** Let $x \in \psi^*\alpha cl(A)$ and there exists a $\psi^*\alpha$-open set U containing x such that U ∩ A = φ. Since $A \subseteq U^c$, $\psi^*\alpha cl(A) \subseteq U^c$ and hence $x \notin \psi^*\alpha cl(A)$,which is a contradiction. Hence U ∩ A ≠ϕ.
(**Sufficiency):** Assume the given condition. Suppose that $x \notin \psi^*\alpha cl(A)$. Then there exists a $\psi^*\alpha$ - closed set F containing A such that x∉F. Then $x \in F^c$ and $F^c$ is $\psi^*\alpha$-open. By assumption, $F^c \cap A \neq \phi$. Since A ⊆ F, $F^c \cap A = \phi$, which is a contradiction. Therefore $x \in \psi^*\alpha cl(A)$.

**Proposition 5.5:** Let A and B be any two subsets of (X, τ). Then the following statements are true
(a) $\psi^*\alpha cl(\phi)=\phi$ and $\psi^*\alpha cl(X)=X$.
(b) If $A \subseteq B$, then $\psi^*\alpha cl(A) \subseteq \psi^*\alpha cl(B)$.
(c) $\psi^*\alpha cl(A) \cup \psi^*\alpha cl(B)= \psi^*\alpha cl(A \cup B)$
(d) $\psi^*\alpha cl(A\cap B) \subseteq \psi^*\alpha cl(A) \cap \psi^*\alpha cl(B)$
(e) $\psi^*\alpha cl(\psi^*\alpha cl(A)) =\psi^*\alpha cl(A)$.

**Proof: (a)** and **(b)** follow from the definition of $\psi^*\alpha$-closure.
**(c)** Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$, by **(b)** $\psi^*\alpha cl(A) \subseteq \psi^*\alpha cl(A \cup B)$ and $\psi^*\alpha cl(B) \subseteq \psi^*\alpha cl(A \cup B)$.Hence $\psi^*\alpha cl(A) \cup \psi^*\alpha cl(B) \subseteq \psi^*\alpha cl(A \cup B)$. To prove the reverse inclusion, let $x\in\psi^*\alpha cl(A\cup B)$ and suppose that $x\notin\psi^*\alpha cl(A) \cup \psi^*\alpha cl(B)$.Then $x\notin\psi^*\alpha cl(A)$ and $x\notin\psi^*\alpha cl(B)$. Therefore there exist a $\psi^*\alpha$ - closed sets U and V in X such that $A \subseteq U$, $B \subseteq V$, $x\notin U$ and $x\notin V$. Hence we have $A \cup B \subseteq U \cup V$ and $x\notin U \cup V$. By **theorem 4.1,** $U \cup V$ is a $\psi^*\alpha$-closed set and hence $x\notin\psi^*\alpha cl(A \cup B)$, which is a contradiction. Hence $\psi^*\alpha cl(A\cup B)\subseteq\psi^*\alpha cl(A)\cup \psi^*\alpha cl(B)$.Therefore $\psi^*\alpha cl(A) \cup \psi^*\alpha cl(B)= \psi^*\alpha cl(A \cup B)$.

**(d)** Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$, by **(b)** $\psi^*\alpha cl(A \cap B) \subseteq \psi^*\alpha cl(A)$ and $\psi^*\alpha cl(A \cap B) \subseteq \psi^*\alpha cl(B)$. Hence $\psi^*\alpha cl(A\cap B) \subseteq \psi^*\alpha cl(A) \cap \psi^*\alpha cl(B)$.

**(e)** Follows from the definition of $\psi^*\alpha$-closure.

**Remark 5.6:** The reverse inclusion of **(d)** is not true in general as seen from the following example.

**Example 5.7:** Let X={a, b, c, d}, τ={ϕ, {a}, {a, b, c}, X}.If A={a} and B={d}, then $\psi^*\alpha cl(A)=X$ and $\psi^*\alpha cl(B)=\{d\}$, $A \cap B=\phi$, $\psi^*\alpha cl(A \cap B)= \phi$. But $\psi^*\alpha cl(A) \cap \psi^*\alpha cl(B)=\{d\}$.

**Theorem 5.8:** The $\psi^*\alpha$ –closure is a Kuratowski closure operator on (X,τ).

**Proof:** From $\psi^*\alpha cl(\phi)=\phi$, $A \subseteq\psi^*\alpha cl(A)$, $\psi^*\alpha cl(A) \cup \psi^*\alpha cl(B)= \psi^*\alpha cl(A \cup B)$ and $\psi^*\alpha cl(\psi^*\alpha cl(A)) =\psi^*\alpha cl(A)$ we can say that $\psi^*\alpha$ –closure is a Kuratowski closure operator on (X,τ).

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