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FECKLY SEMI REGULAR NEAR-FIELD SPACES OVER NEAR-FIELD

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ABSTRACT

Let N be a near-field space and J(N) the Jacobson radical. An element a $\in$ N is called semi regular if there exists a regular element b $\in$N with a – b $\in$ J(N). A near-field space N is said to be feckly semi regular near-field space provided that any element a $\in$ N, either a or 1 – a is semi regular. I, Dr N V Nagendram as an author introduce, in this paper, feckly semi regular near-field spaces as the generalization of VNL-near-field spaces and semi regular near-field spaces. It is shown that feckly semi regular near-field spaces are exchange near-field spaces and many properties of semi regular near-field spaces can be extended only feckly semi regular near-field spaces. Relative examples are also constructed. All feckly semi regular near-field spaces considered are either near-fields by hypothesis or turn out to be periodic near-field spaces over near-fields in the course of the in depth study and investigation of the near-field spaces over near-fields.

Key words: *semi regular near-field space, feckly semi regular near-field space, Near-ring, Near-field, periodic Near-field, sub Near-field, Sub Near-field space, ideal, Extensions.*

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In section 1 of this paper, we discuss and provide some preliminaries which will be useful for further section to understand the extended concepts over existing one. Here author derived some results on commutative VNL-Near-field Spaces over near-field were extended. Moreover, Dr N V Nagendram characterized VNL-near-field spaces in the sense of relating them to some familiar classes of near-field spaces over near-fields. VNL-Near-Field Spaces are also exchange near-field spaces over near-fields. VNL-near-field spaces and their related near-fields are extensively studied many others over existing system of VNL-rings and their related rings (refer [6], [14], [10], [7], [2]).

In section 2 of this paper, I say a near-field space N feckly semi regular near-field space if for every a $\in$ N, either a or 1 – a is semi regular near-field space. Some examples are given to show that feckly semi regular near-field spaces are the proper generalizations of VNL-near-field spaces and semi regular near-field space over near-fields. We prove that feckly semi regular near-field spaces are exchange near-field spaces, and hence N is feckly semi regular near-field space if and only if N is VNL-near-field space and idempotent can be lifted modulo J(N).Moreover, abelian, feckly semi regular near-field spaces are studied. It is shown that, If N is abelian, feckly semi regular near-field space, $N/M'(N)$ is a local near-field space.

In section 3 of this paper, devote to consider the extensions of feckly semi regular near-field spaces such as Mortia context, Matrix near-field spaces, power series near-field spaces and trivial extensions and so on. Some properties of semi regular near-field spaces are also extended.

Throughout N is an associative near-field space with identity and all sub near-field spaces, modules are unitary. J(N) will denote the Jacobson radical of near-field space N. The near-field space of integers modulo n is denoted by Z_n. We write $M_n(N)$ respectively $T_n(N)$ for the near-field space of all n x n matrices respectively all upper triangular n x n matrices over near-field space N. The near-field space of power series in indeterminate x over a near-field space N is denoted by N[[x]].

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SECTION 1: PRELIMINARIES OF SEMI REGULAR NEAR-FIELD SPACES OVER NEAR-FIELDS

Definition 1.1: Von Neumann regular. An element $a \in N$ is (Von Neumann) regular [9] provided that there exists an element $x \in N$ such that $a = axa$.

Definition 1.2: Regular near-field space. A near-field space N is regular near-field space in case every element in N is regular.

Definition 1.3: Local regular near-field space. A near-field space N is local provided that N has a uniquely maximal right ideal.

A near-field space N is local if and only if for any $a \in N$, either a or $1 - a$ is invertible.

Definition 1.4: Von Neumann Local near-field space (VNL-NFS). As a common generalization of regular near-field spaces, local near-field spaces, Contessa in [9] called a commutative near-field space N Von Neumann Local near-field space **(VNL-NFS)** if for each $a \in N$, either a or $1 - a$ is regular near-field space.

Definition 1.5: Semi regular near-field space. An element $a \in N$ is called semi regular near-field space if there exists $b \in N$ such that $bab = b$ and $a - b \in J(N)$.

The near-field space N is semi regular near-field space $\Leftrightarrow$ every element in N is semi regular near-field space.

Example 1.6: Regular near-field spaces, semi perfect near-field spaces and right continuous near-field spaces over rings, near-rings, regular delta near-rings and near-fields etc.

Definition 1.7: Almost unit-regular near-field space over a near-field. A near-field space N is called an almost unit – regular near-field space over a near-field provided that for any $a \in N$, either a or $1 - a$ is unit – regular element in N.

Definition 1.8: Quasi polar. An element $a \in N$ is called quaipolar if there exists $p^2 = p \in \text{comm}^2_N$ (a) such that $a + p \in U(N)$ and $ap \in N^{qnil}$.

Definition 1.9: Quasi polar near-field space over a near-field. A near-field space over a near-field is quasi polar near-field space over a near-field if and only if each of its elements are quasi polar elements.

Definition 1.10: $M^{'}(N)$ the sum of all semi regular ideal of N and that $M^{'}(N)$ is the unique maximal semi regular ideal of N. $M^{'}(N)$ is $\{a \in N/(a)$ is a semi regular ideal in N$\}$. M(N) and $J(N) \subseteq M'(N)$, $M(N/M(N)) = 0$ but $M^{'}(N/M^{'}(N)) \neq 0$.

Definition 1.11: Sub near-field space over a near-field. Let D be a near-field space over a near-field and C a sub near-field space of D with $1_D \in C$. Define a set $N[D, C] = \{(d_1,d_2, \ldots\ldots,d_n,c,c,\ldots\ldots)\}$: $d_i \in D$, $c \in C$, $n \geq 1$ with "+" and "." Component wise.

SECTION 2: ABELIAN FECKLY SEMI REGULAR NEAR-FIELD SPACES OVER NEAR-FIELDS.

Section 2 comprises of and begin with the following important and useful results of abelian feckly semi regular near-field spaces over near-fields.

Lemma 2.1: The following are equivalent for an element a of a near-field space N over near-field.

(a) $\exists\, e^2 = e \in aN$ such that $(1 - e)\, a \in J(N)$.
(b) $\exists\, e^2 = e \in Na$ such that $a(1 - e) \in J(N)$.
(c) $\exists$ a regular element b of a regular near-field space N with $a - b \in J(N)$.
(d) $\exists$ a regular element b of a regular near-field space N with $bab = b$ and $a - aba \in J(N)$..

An element a of a near-field space N is called semi regular near-field space if it satisfies the conditions in Lemma 2.1. The near-field space is semi regular near-field space if and only if each of its elements is semi regular near-field.

Note 2.2: (a) semi regular near-field spaces are feckly semi regular near-field spaces. (b) Obviously, a VNL-near-field space is feckly semi regular near-field space over near-fields. (c) A feckly semi regular near-field space and semi primitive near-field space ($J(N) = 0$) is a VNL-near-field space over near-fields.

Example 2.3: The near-field space Z_4 X Z_4 is a feckly semi regular near-field space but not a VNL-near-field space over a near-field.

Example 2.4: Let N be the near-field space of real convergent sequences. By the relevant argument N is feckly semi regular near-field space but not semi regular near-field space.

Example 2.5: If N is a semi regular near-field space, T is local near-field space and let $_N M_T$ be sub near-field space or bimodule then $\begin{pmatrix} N & M \\ 0 & T \end{pmatrix}$ is feckly semi regular near-field space.

Proof: Let S = $\begin{pmatrix} N & M \\ 0 & T \end{pmatrix}$. For any β = $\begin{pmatrix} a & m \\ 0 & b \end{pmatrix}$ ∈ S. Since T is local near-field space, b or 1 – b is invertible.

Assume that b s invertible. Note that a is a semi regular element of a semi regular near-field space in N, so there exists a regular element c∈ N such that a – c ∈ J(N).

Let β = $\begin{pmatrix} c & m \\ 0 & b \end{pmatrix}$ is regular in S. Then α – β = $\begin{pmatrix} a-c & 0 \\ 0 & 0 \end{pmatrix}$ ∈ J(S). This implies that α is a semi regular element in S.

Assume that 1 – b is invertible. Because 1 – a is a semi regular element in semi regular near-field space N, there exists a regular element d ∈ N such that (1 – a) – d ∈ J(N). Similarly, 1γ – α is semi regular element in semi regular near-field space S. This completes the proof of the lemma.

Definition 2.6: Primitive. An idempotent e in a near-field space N is called primitive if the corner near-field space eNe has no proper idempotents and e is called local if eNe is a local near-field space.

Proposition 2.7: Let N be a feckly semi regular near-field space over near-field. Then the following results hold.

(a) Every homomorphic image of N is feckly semi regular near-field space.
(b) eNe is feckly semi regular near-field space for every $e^2 = e \in N$.
(c) Every primitive idempotent is local semi regular near-field space.

Proof:

(a) is trivial by lemma 2.1 (c).
(b) for every a ∈ eNe, a or 1 – a is semi regular element in N be a feckly semi regular near-field space over near-field by hypothesis. Assume that a is semi regular element. There exists a regular element bab = b∈N such that a – aba∈J(N). So a – aba = e(a – aba)e ∈ J(N) ∩ J(eNe) = J(eNe). It implies that a is semi regular element in semi regular near-field space eNe over a near-field. Assume that 1 – a is semi regular element in a feckly semi regular near-field space N. Similarly, e – a is semi regular element in a semi regular near-field space over near-field. Therefore, eNe is feckly semi regular near-field space over near-field.
(c) If $e^2 = e \in N$ is primitive. By (b), eNe is also feckly semi regular near-field space over a near-field space. Thus, for any a ∈ eNe, either a or e – a is semi regular element in eNe a feckly semi regular near-field space over a near-field space.

Case (i): If a is semi regular element, there exists $g^2 = g \in$ aNe such that a – g a ∈ J(eNe). If $g = 0$, a ∈ J(eNe) and 1 – a is invertible. If $g \neq 0$, we have eg = e because e is a primitive. It follows that eNe = eNe. Hence a has a right inverse in a feckly semi regular near-field space over a near-field space eNe.

Case (ii): If e – a is semi regular element, there exists $k^2 = k \in$ (e – a)Ne such that (e – a) – k (e – a) ∈ J(eNe). If k = 0, e – a ∈ J(eNe) and a is invertible in eNe. If k ≠ 0, we have ek = k. Then ke = e because e is primitive. It follows that (e – a)Ne = eNe. Hemce e – a has a right inverse in a feckly semi regular near-field space over a near-field space eNe. Thus either a or e – a is invertible in a feckly semi regular near-field space over a near-field space eNe. This completes the proof of the proposition.

By proposition 2.7, we have the following result which is well known for VNL-semi regular near-field spaces and semi regular near-field spaces over near-fields.

Corollary 2.8: A semi regular near-field space N is indecomposable, feckly semi regular near-field space over a near-field space if and only if it is local semi regular near-field space.

Note 2.9: One can easily verify that a direct product of semi regular near-field spaces is semi regular near-field space if and only if each factor is semi regular near-field space over a near-field. However, for a fickly semi regular near-field space Dr N V Nagendram derived the next result.

Proposition 2.10: Let N = $\prod_{\beta \in J} N_\beta$. Then N is a feckly semi regular near-field space over a near-field space if and only if there exists $\beta_0 \in J$, such that $N_{\beta 0}$ is a feckly semi regular near-field space over a near-field for each $\beta \in J - \beta_0$, N_β is semi regular near-field space over a near-field.

Proof: ($\Leftarrow$) Let y = $(y_\beta) \in N$, $\beta \in J$. By hypothesis, $y_{\beta 0}$ or $1_{N\beta 0} - y_{\beta 0}$ is semi regular near-field space over a near-field in $N_{\beta 0}$, then 1 – y is semi regular near-field space over a near-field in N. Thus the result follows.

(⇒) Assume that N is a feckly semi regular near-field space over a near-field. Then every N_β is is also a feckly semi regular near-field space over a near-field by proposition 2.7 (a). Write N = $N_{\beta 0}$ X T, where T = $\prod_{\beta \in 1-\beta_0} N_\beta$, and for $\beta \in 1 - \beta_0$. If neither $N_{\beta 0}$ nor T is semi regular near-field space over a near-field, then we can find non semi regular elements say $p \in N_{\beta 0}$,$t \in T$. Choose a = $(1_{N\beta 0} – p, t)$. Then neither a nor 1 – a = (p, 1_t - t) is semi regular element in semi regular near-field space over a near-field N, a contradiction. Hence either $N_{\beta 0}$ or T is a semi regular near-field space over a near-field. If T is a semi regular near-field space over a near-field, then the result follows. If T is a feckly semi regular near-field space over a near-field, by iteration of this process, this completes the proof of this proposition.

Example 2.11: Let N_1 nad N_2 are the near-field spaces example 2.3 and example 2.4 respectively. Then N = N_1 X N_2 is a feckly semi regular near-field space over a near-field by proposition 2.10. However, N is neither a VNL- semi regular near-field space over a near-field, nor a semi regular near-field space over a near-field.

An element a of a near-field N is called an exchange element if there exists

An idempotent $e \in N$ such that $e \in aN$ and $1 – e \in (1 – a)N$. The near-field N is an exchange near-field if and only if every element of N is an exchange element. Idempotent can be lifted modulo J(N), if for every $a \in N$, $a^2 – a \in J(N)$, there exists $e^2 = e \in N$ such that $e – a \in J(N)$. As well known, a near-field N is semi regular near-field space over a near-field if and only if N/J(N) is a regular near-field space over a near-field and idempotent can be lifted modulo J(N). For convenience $a \equiv b(\mathrm{mod}\ J(N))$ denotes $a – b \in J(N)$.

Theorem 2.12: The following are equivalent for a near-field space N.

(a) N is perfectly semi regular near-field space.
(b) N/J(N) is a VNL-near-field space and N is an exchange near-field space.
(c) N/J(N) is a VNL-near-field space and idempotents can be lifted modulo J(N).

Proof: Proof (b) ⇒ (c) is obvious.

To prove (a) ⇒ (c): clearly, N/J(N) is a VNL-near-field space. For any $a \in N$, $a^2 – a \in J(N)$. if a is semi regular element, then there exists an idempotent $e^2 = e \in aN$ such that $(1 – e)a \in J(N)$. write $\bar{a} = a + J \in N/J(N)$, so we have $\bar{e} = \bar{e}\ \bar{a}$ and $\bar{e} = \bar{a}\ \bar{e}$. Let $g = e + ea(1 – e)$. Then $g^2 = g$ and $\bar{g} = \bar{e} + \bar{e}\,\overline{a(1-e)} = \bar{e} + \bar{a} - \bar{a}\ \bar{e} = \bar{a}$. If 1 - a is semi regular element. Noting that $(1 – a)^2 – (1 – a) = a^2 – a \in J(N)$, then we can prove that, using the similar method, $\overline{f'} = \overline{(1-a)}$ for an idempotent element $f' \in N$, and hence $(1 – f') – a \in J(N)$. hence proved (a) ⇒ (c).

To prove (c) ⇒ (a): For any $a \in N$, as N/J(N) is a VNL-near-field space, $\bar{a}$ or $\bar{1} - \bar{a}$ is regular element. If $\bar{a}$ is regular element in N/J(N), then there exists $b \in N$ such that $a – aba \in J(N)$, and hence $ab – (ab)^2 \in J(N)$.By hypothesis, choose $e^2 = e \in N$ such that $e – ab \in J(N)$. Thus, $u = 1 – e + ab$ is invertible and $u^{-1} (ab) e = e$. Write $t = beu^{-1}$, then $tat = (beu^{-1})a(beu^{-1}) = beu^{-1} = t$ and $at = abeu^{-1} \equiv abe \equiv ab(\mathrm{mod}\ J(N))$. Hence $a – ata = (a – aba) + (ab – at)a \in J(N)$. By lemma 2.1, a is semi regular element in N. If $\bar{1} - \bar{a}$ is regular element. Similarly, we can prove that 1 – a is semi regular element. Therefore N is feckly perfect semi regular near-field space over a near-field. This completes the proof of the theorem.

Note 2.13: Semi perfect near-field spaces are feckly semi regular near-field space and the converse not true i.e., false.

Note 2.14: By theorem 2.12, a feckly semi regular near-field space over a near-field without infinite orthogonal idempotents is semi perfect near-field space over a near-field.

Note 2.15: A near-field space N is called abelian if each idempotent in N is central semi regular near-field space over a near-field. Here we consider abeian, feckly semi regular near-field spaces over a near-field.

Corollary 2.16: If N is an abelian, feckly semi regular near-field space over near-field, N/J(N) is an almost unit – regular near-field space over a near-field.

Note 2.17: For a $\in$ N, $comm_N(a) = \{x \in N / ax = xa\}$ and $comm_N^2(a) = \{x \in N/ xy = yx \ \forall\ y \in comm_N(a)\}$ and $N^{qnil} = \{a / 1 + xa \in U(N), \forall\ x \in comm_N(a)\}$.

Note 2.18: It is obvious that an abelian semi regular near-field space over a near-field is quasi polar near-field space over a near-field.

Note 2.19: But an abelian feckly semi regular near-field space over a near-field need not be a quasi polar near-field space over a near-field.

Proposition 2.20: Every abelian, feckly semi regular near-field space over a near-field is clean.

Corollary 2.21: Let N be an abelian, feckly semi regular near-field space over a near-field in which is 2 is a unit. Then every element of N is a sum of a unit and a square root of 1.

Lemma 2.22: If N is a *f*eckly semi regular near-field space over a near-field, then either eNe or (1 – e)N (1 – e) is a semi regular near-field space over a near-field for $e^2 = e \in N$.

Proof: If N is *f*eckly semi regular near-field space over a near-field, $\overline{N} = N / J(N)$ is a VNL-near-field space over a near-field [Th. 2.12] then either $\bar{e}\ \overline{N}\ \bar{e} \equiv \overline{eNe}$ or $\overline{(1-e)}\ \overline{N}\ \overline{(1-e)} \equiv \overline{(1-e)N(1-e)}$ is a regular near-field space over a near-field ([10], lemma 2.4). Note that eNe and (1 – e)N(1 – e) are also exchange near-field spaces over a near-field, hence idempotents lift modulo its Jacobson radical respectively. Then either eNe or (1 – e)N(1 – e) is semi regular near-field space over a near-field. This completes the proof of the lemma.

Note 2.23: The converse of lemma 2.22 is false and counter example is a set $N = \{(q_1, q_2, ..., q_n, z,z,...) / q_i \in Q, z \in Z, n \geq 1\}$ with addition and multiplication defined component wise. Then either eNe or (1 – e)N(1 – e) is regular for every idempotent e$\in$N. However, N is not a feckly semi regular near-field space over a near-field because the homomorphism image Z of N is not feckly semi regular near-field space over a near-field.

Proposition 2.24: The following are equivalent for an abelian near-field space N.

(a) N is a *f*eckly semi regular near-field space over a near-field;
(b) N is an exchange near-field space such that for every $e^2 = e \in N$, either eNe or (1 – e)N(1 – e) is semi regular near-field space over a near-field.

Proof: To prove (a) $\Rightarrow$ (b): Proof is obvious with reference to theorem 2.12 and lemma 2.22. Proved (a) $\Rightarrow$ (b).

To prove (b) $\Rightarrow$ (a): $\forall$ a $\in$ N, as N is an exchange near-field space over a near-field, $\exists\ e^2 = e \in N \ni e \in aN$ and $1 - e \in (1 - e)N$. Now if eNe = eN is semi regular near-field space over a near-field, then e(1 – e) is semi regular element, and hence 1 – a is semi regular element.

Similarly, if (1 – e)N(1–e) = (1– e)N is semi regular near-field space over a near-field. Then a is semi regular element. Therefore, N is feckly semi regular near-field space over a near-field. Proved (b) $\Rightarrow$ (a). This completed the proof of the proposition.

Note 2.25: M(N) = {a $\in$ N / (a) is a regular ideal in N} is the unique maximal regular ideal of N, where (a) stands for the principal ideal of N generated by a $\in$ N.

Lemma 2.26: Let N be a near-field space in which idempotent can e lifted modulo J(N). then $M(M/M^{'}(N)) = 0$

Proof: Without loss of generality, we write $\overline{N} = N / J(N)$. Then

$$M'(N)/J(N) = \{\ \bar{a} \in \overline{N}\ /\ (a)\ \textit{is a semi regular ideal in}\ N\ \}$$
$$= \{\ \bar{a} \in \overline{N}\ /\ \overline{(a)}\ \textit{is a semi regular ideal in}\ \overline{N}\ \}$$
$$= M(\overline{N}) = M(N/J(N)).$$

Thus $M(N/M'(N)) \equiv M\left(\dfrac{N/J(N)}{M'(N)/J(N)}\right) = M\left(\dfrac{N/J(N)}{M(N/J(N))}\right) = 0$. This completes the proof of the lemma.

Note 2.27: If N is an abelian VNL-near-field space over a near-field and not regular N/M(N) is a local near-field space over a near-field. An abelian near-field space N is a VNL-near-field space over a near-field if and only if an SVNL-near-field space over a near-field.

Theorem 2.28: The following hold for an abelian near-field space N.

(a) If N is a feckly semi regular near-field space over a near-field, $N/M^{'}(N)$ is a local near-field space over a near-field.

(b) If N is an exchange near-field space over a near-field and $N/M^{'}(N)$ is a local near-field space over a near-field, N is a feckly semi regular near-field space over a near-field.

Proof: To prove (a): Assume that N is abelian, feckly semi regular near-field space over a near-field. Then N/J(N) is an abelian VNL-near-field space and hence $\left(\dfrac{N/J(N)}{M'(N)/J(N)}\right)$ is a local near-field space and $M(N/M'(N)) \equiv \left(\dfrac{N/J(N)}{M'(N)/J(N)}\right) = \left(\dfrac{N/J(N)}{M(N/J(N))}\right)$, then N/M′(N) is a local near-field space over a near-field. Proved (a).

To prove (b): If N/M′(N) is a local near-field space over a near-field, so is $\left(\dfrac{N/J(N)}{M'(N)/J(N)}\right)$, N/J(N) is a VNL-near-field space over a near-field. Therefore, N is feckly semi regular near-field space over a near-field. Proved (b). This completes the proof of the theorem.

Proposition 2.29: Let N be an abelian near-field space. Then the following are equivalent.

(a) N is a feckly semi regular near-field space over a near-field.

(b) Whenever $(T)_t = N$ for non-empty sub near-field space over a near-field T of N, at least one element in T is semi regular element, where (T) is a right ideal generated by T.

Proof: To prove (b) ⇒ (a): $\forall$ $a \in N$, Let T = (a, 1 – a). Then $(T)_t = N$ and hence either a or 1 – a is semi regular element. Proved (b) ⇒ (a).

To prove (a) ⇒ (b): If N is semi regular near-field space over a near-field, we are done. Then one suppose that N is a feckly semi regular near-field space over a near-field which is not semi regular near-field space over a near-field.

For any non-empty sub near-field space T of N with $(T)_t = N$, there exists $t_1, t_2, \ldots\ldots, t_s \in T$ such that $t_1 N + t_2 N + \ldots\ldots + t_s N = N$. Thus there exists $k_1, k_2, \ldots\ldots, k_s \in N$ satisfying $t_1k_1 + t_2k_2 + \ldots\ldots + t_sk_s = 1$. And so $\overline{t_1}\,\overline{k_1} + \overline{t_2}\,\overline{k_2} + \ldots. + \overline{t_s}\,\overline{k_s} = \overline{1}$ in $\overline{N} = N/M^{'}(N)$. However $\overline{N}$ is a local near-field space over a near-field. It follows that there exists an $\overline{s_k}$ such that $\overline{s_k} \in U(\overline{N})$, hence $\overline{s_k}$ is a regular element in $\overline{N}$. Assume that $\overline{s_k} = \overline{s_k}\,\overline{x}\,\overline{s_k}$ for some $\overline{x} \in \overline{N}$. Then we have $s_k - s_kxs_k \in M^{'}(N)$ and $s_kx - (s_kx)^2 \in M^{'}(N)$. N is an exchange near-field space over a near-field, every right ideal of N is strongly lifting i.e., there exists $e^2 = e \in s_kxN \subseteq s_kN$ such that $e - s_kx \in M^{'}(N)$ and so $es_k - s_kxs_k \in M^{'}(N)$. Then $s_k - es_k = (es_k - s_kxs_k) - (es_k - s_kxs_k) \in M^{'}(N)$ is semi regular element. Thus s_k is semi regular element. Hence proved (a) ⇒ (b).

This completes the proof of the proposition.

SECTION 3: MAIN RESULTS AND EXTENSIONS ON FECKLY SEMI REGULAR NEAR-FIELD SPACES OVER NEAR-FIELD SPACE.

A (P, Q, M, N, φ, ψ) consists of two near-field spaces P, Q, two sub near-field spaces ${}_PM_Q$, ${}_QN_P$ and a pair of sub near-field spaces homomorphism $\psi : M \otimes_Q N \to P$ and homomorphism $\varphi : N \otimes_P M \to Q$ which satisfy the following associativity : $\psi(m \otimes n)m^{'} = m\,\varphi(n \otimes m^{'})$ and $\varphi(n \otimes m)n^{'} = n\psi(m \otimes n^{'})$ for any $n, n^{'} \in N$, $m, m^{'} \in M$.

Theorem 3.1: Let S = $\begin{pmatrix} P & M \\ N & Q \end{pmatrix}$ such that $MN \subseteq J(N)$ and $NM \subseteq J(T)$. Then S is feckly semi regular near-field space if and only if P and Q is semi regular near-field spaces over a near-field and the other is feckly semi regular near-field space over a near-field.

Proof: $S/J(S) \cong P/J(P) \times T/J(T)$. If S is feckly semi regular near-field space, then S/J(S) is a VNL-near-field space. S is also an exchange near-field space over a near-field. So P, Q are exchange near-field spaces over a near-field. So one of P and Q is semi regular near-field space over a near-field and the other is feckly semi regular near-field space over a near-field. Conversely, if one of P and Q is semi regular near-field spaces and the other is feckly semi regular, using the similar theorem. S is feckly semi regular near-field space over a near-field. This completes the proof of the theorem.

Corollary 3.2: Let $S_n(N)$ be the near-field spaces of upper triangular matrices over N. Then the following are equivalent.

(a) N is semi regular near-field space over a near-field.
(b) $S_n(N)$ is feckly semi regular near-field space over a near-field $\forall$ $n \geq 1$.

Proof: is obvious.

Note 3.3: A matrix near-field space over a near-field is a feckly semi regular near-field space over a near-field.

Theorem 3.3: Let N be a near-field space over a near-field. Then the following are equivalent.

(a) N is feckly semi regular near-field space over a near-field.
(b) N[[x]] is a feckly semi regular near-field space over a near-field.

Proof: To prove (a) ⇒ (b): As N[[x]]/J(N[[x]]) = N[[x]]/J(N)[[x]] ≅ N/J(N). By hypothesis, N[[x]]/J(N[[x]]) is a VNL-near-field space over a near-field and idempotents can be lifted modulo J(N[[x]]). Then N[[x]] is a feckly semi regular near-field space over a near-field. Proved (a) ⇒ (b).

To prove (b) ⇒ (a): If N[[x]] is a feckly semi regular near-field space over a near-field, Then N[[x]]/J(N[[x]]) is a VNL-near-field space over a near-field and idempotents can be lifted modulo J(N[[x]]). Thus, N/J(N) is a VNL-near-field space over a near-field and idempotents can be lifted modulo J(N). N is a feckly semi regular near-field space over a near-field. Proved (b) ⇒ (a). This completes the proof of the theorem.

Proposition 3.4: Let C be a sub near-field space over a near-field of a near-field space D. Then the following are equivalent.

(a) N[D, C] is a VNL-near-field space over a near-field.
(b) D is a regular sub near-field space over a near-field and C is a VNL-near-field space over near-field.

Proof: To prove (a) ⇒ (b): For convenience, let T = N[D, C]. We construct a homomorphism *f:* T → C given by *f* $(a_1,a_2, ...,a_n,b,b,.....) = b$. Thus C is a VNL-near-field space because it is a homomorphic image of a VNL-near-field space over a near-field.

Now we prove that D is regular sub near-field space over a near-field. Assume that D is not regular sub near-field space over a near-field. Then there exists a non regular element $x \in D$. Let $\beta = (x, 1 - x, 1, 1,...) \in T$. By hypothesis, β, $1_T - \beta$ is regular element, which is also implies x is regular in D, a require contradiction.

To prove (b) ⇒ (a): for any $(a_1,a_2, ...,a_n,c,c,.....) \in T$ with $a_i \in D$ and $c \in C$. By hypothesis, a_i is regular element, and hence we have some $b_i \in D$ such that $a_i = a_ib_ia_i$. As C is a VNL-near-field space over a near-field, c or 1 – c is regular element. If c is regular, then there exists an element $b \in C$ such that c = cbc.

Thus $(a_1,a_2,...,a_n,c,c,..)=(a_1,a_2,....,a_n,c,c,..)(b_1,b_2,...,b_n,c,c,..)(a_1,a_2, ..,a_n,c,c,..)$. Thus it implies that $(a_1,a_2, ...,a_n,c,c,.....)$ is regular in T. If 1 – c is regular, we get $(1,1,...,1,1,1,...) - (a_1,a_2,...,a_n,c,c,...) = (1-a_1,1-a_2,..,1-a_n,1 - c,1 - c,..)$ is regular in T. As a result T is a VNL-near-field space over a near-field. Proved (b) ⇒ (a). This completes the proof of the proposition.

Corollary 3.5: Let C be a sub near-field space over a near-field of a near-field space D. Then the following are equivalent.

(a) N[D, C] is a feckly semi regular near-field space over a near-field
(b) D is semi regular $\dfrac{C}{J(D) \cap J(C)}$ is a VNL-near-field space and idempotents in C can be lifted modulo $J(D) \cap J(C)$.

Proof: It is obvious.

Note 3.6: Assume that $\dfrac{C}{J(D) \cap J(C)}$ is a VNL-near-field space and idempotents in C can be lifted modulo $J(D) \cap J(C)$, one can prove C is feckly semi regular near-field space over a near-field. Hence if N[D, C] is a feckly semi regular near-field space over a near-field, D is semi regular near-field space over a near-field and C is feckly semi regular near-field space over a near-field.

Example 3.7:

(a) Let F be a field, D = $M_2(F)$ and C = $\begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$ where a, b $\in$ F. Then N[D, C] is a feckly semi regular near-field space over a near-field which is not semi regular near-field space over a near-field.

(b) If T = N[Q, $Z_{(2)}$] where Q is the rational numbers and $Z_{(2)}$ is the localliation of Z at the prime ideal (2), then T is a VNL-near-field space over a near-field with J(T) = 0 which is not regular. Thus T is a feckly semi regular near-field space over a near-field but not semi regular near-field space over a near-field.

Let N be a near-field space and M be bimodule over N. The trivial extension of N and M is N$\propto$M ={|(x, m)|x $\in$ N, m$\in$ M} with "+" defined component wise and "." Defined by (x, m)(y, n) = (xy, xn + my). J(N $\propto$ M) = {(x, m)/x $\in$J(N), m $\in$ M}.

Lemma 3.8: The following are equivalent for a near-field space N and a bimodule M:

(a) Idempotents of N can be lifted modulo J(N).

(b) Idempotents of N $\propto$ M can be lifted modulo J(N $\propto$ M).

Proof: To prove (a) $\Rightarrow$ (b): for any $(x, m)^2$ – (x, m) $\in$ J(N $\propto$ M), $x^2 - x \in$ J(N). by hypothesis there exists $e^2 = e \in$ N such that e – x $\in$ J(N). Thus $(e, 0)^2$ = (e, 0) $\in$ N $\propto$ M and (e, 0) – (x, m)$\in$ J(N $\propto$ M). proved (a) $\Rightarrow$ (b).

To prove (b) $\Rightarrow$ (a): for any x^2–x$\in$J(N), $(x, 0)^2$ – (x, 0)$\in$J(N $\propto$ M), by hypothesis there exists $(e, m)^2$ = (e, m) $\in$ N $\propto$ M such that (e, m) – (x, 0) $\in$ J(N $\propto$ M). Thus $e)^2$ = e $\in$ N and e – x $\in$ J(N). proved (b) $\Rightarrow$ (a). This completes the proof of the lemma.

Proposition 3.9: Let N be a near-field space and M be a bimodule over near-field space N. Then the following are equivalent.

(a) N is feckly semi regular near-field space over a near-field.

(b) N $\propto$ M is feckly semi regular near-field space over a near-field.

Proof: is obvious.

Note 3.10: Let N be a near-field space. Then N is feckly semi regular near-field space over a near-field if and only if so is N $\propto$ N.

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