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MAGNETO-HYDRODYNAMIC COUPLESTRESS COSINE FORMCONVEX CURVED PLATES

SYED ARISHIYA NASEEM FATIMA[1], TRIMBAK BIRADAR[2],
S. T. FATHIMA*[3] AND HANUMAGOWDA B.N[4]

[1]Department of Mathematics,
K. C. T College of Engineering, Gulbarga-585104, India.

[2]Department of Mathematics,
Sharnbasveshwar College of science, Gulbarga-585103, India.

[3,4]Department of Mathematics,
Reva University, Bangalore-560064, India.

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ABSTRACT

On the basis of Stokes couplestress fluidmodel together with the hydromagnetic flow the study of MHD-couple stress cosine curved plates is studied and Modified Reynolds equation has been derived which is applied to the study of squeeze film characteristic including the non-Newtonian hydromagnetic effects. According to the results the combined effect of couplestresses and external magnetic fields provide an increase in the load capacity and the response time as compared to the classical Newtonian hydrodynamic convex curved plates.

Keywords: *convex curved plates, cosine function,couple-stress fluid, Magnetic field.*

1. INTRODUCTION

In recent years, the characteristic of Magneto-hydrodynamics (MHD) in flow analysis are important for many engineering and industrial applications. The MHD bearings with conducting fluids possess the high thermal-conductivity and high electrical-conductivity features over the conventional bearings. The squeeze film lubrication for different configuration of bearings under the action of transverse magnetic field has been discussed by several authors [1-6].

It is known that, the theory of couplestress fluid by Stokes [7-12] is a generalization of viscous fluid theory with couple stresses and body couples. Couplestress fluids are consequence of the assumption that, the interaction of one part of the body on another across a surface is equivalent to a force and momentum distribution. It consists of rigid randomly oriented particles suspended in a viscous medium such as electro-rheological fluids and synthetic fluids. The MHD bearings with conducting fluids possess numerous advantages over the conventional bearings.

Recently, Lin *et al.* [13] studied hydromagnetic non-Newtonian cylindrical squeeze film and its application to circular plates by derivation of modified lubrication equation. They found that the improved characteristics are further emphasized for circular plates operating with a larger magnetic field parameter and non-Newtonian parameter.

The MHD conducting couplestress squeeze-film characteristics of cosine form convex curved plates has not been studied so far. Hence, in this paper an attempt has been made to study the conducting couple stresses on the MHD squeeze-film characteristics of cosine form convex curved plates. Expressions for the MHD squeeze-film pressure, load carrying capacity and the time height relation are obtained.

Corresponding Author: S. T. Fathima[3]*
[3,4]Department of Mathematics, Reva University, Bangalore-560064, India.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

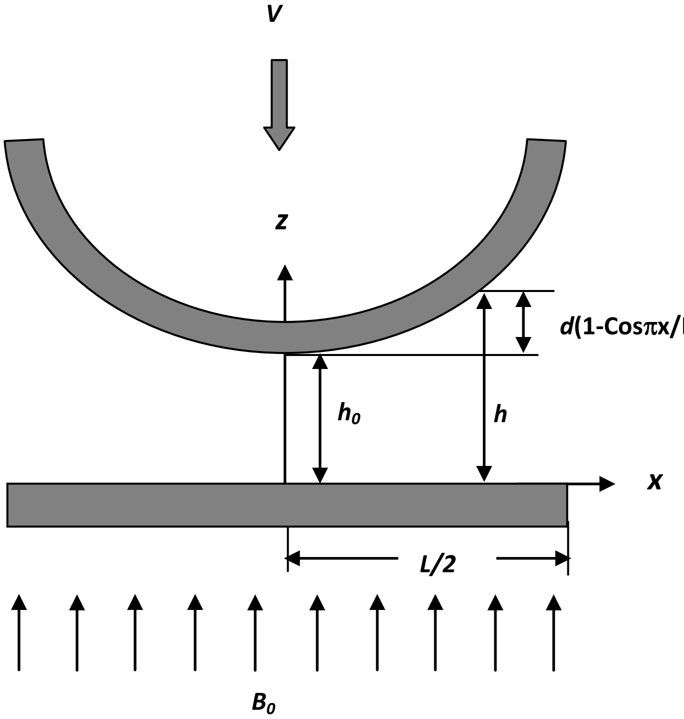

Figure-1: The physical configuration of convex curved plates in the presence of transverse magnetic field.

Figure.1 shows the squeeze film geometry between the cosine form convex curved plates. The film thickness *h* for the squeeze film can be generated by the form of a cosine function $h = h_0 + d\left\{1-\cos\left(\frac{\pi}{L}x\right)\right\}$.In the equation, *L* is the length of the plates, *d* is the amplitude of the cosine function, and h_0 is the minimum film thickness. At the central position: *x* = 0, the film thickness is equal to the minimum film thickness h_0. At the edge position: $x = \frac{L}{2}$, the film thickness is equal to the sum of the minimum film thickness and the amplitude $h_0 + d$. A uniform transverse magnetic field B_0 is applied to the bearing in the Z -direction as shown in the above Figure1.It is assumed that the fluid film is thin, the body forces and the body couples are negligible.

Under the usual assumption of hydromagnetic lubrication theory applicable to thin films and Stokes theory for couplestresses, the continuity equation and the Magneto-hydrodynamic (MHD) momentum equation in Cartesian form becomes

$$\mu\frac{\partial^2 u}{\partial z^2} - \eta\frac{\partial^4 u}{\partial z^4} - \sigma B_0^2 u = \frac{\partial p}{\partial x} \tag{1}$$

$$\frac{\partial p}{\partial z} = 0 \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{3}$$

Where $M = \frac{M_0}{h_0}$ and $M_0 = B_0 h_0\left(\frac{\sigma}{\mu}\right)^{1/2}$ is the Hartmann number.

The third term in Eq.[1] is a Lorentz body force coming from $J_y B_0$ under the assumption that the induced magnetic field is much less than the applied magnetic field as described by Kuzma [14]. Then, from Ohm's law the axial current density becomes $J_y = \sigma B_0 u$. Therefore the term $\sigma B_0{}^2 u$ appears in the Eq. [1].

The relevant boundary conditions for the velocity components are

i) At the upper surface $z=h$

$$u = 0(no\ Slip)\ \frac{\partial^2 u}{\partial y^2} = 0\ \ (Vanishing\ of\ \ couplestresses) \tag{4a}$$

$$w = V = dh/dt\,(SqueezingVelocity) \tag{4b}$$

ii) At the lower surface $z=0$

$$u = 0(no\ Slip)\ \frac{\partial^2 u}{\partial y^2} = 0(Vanishing\ of\ \ couplestresses) \tag{5a}$$

$$w = 0 \tag{5b}$$

Where $M_0 = B_0 h_0 (\sigma/\mu)^{1/2}$ is the Hartmann number.

The solution of equation (1) subject to the boundary conditions eqns. (4) and (5) is given by

$$u = -\frac{h_0^2}{\mu M_0^2}\frac{\partial p}{\partial x}\left\{\frac{1}{(A^2-B^2)}\left(\frac{B^2 Cosh\frac{A(2z-h)}{2l}}{Cosh\frac{Ah}{2l}} - \frac{A^2 Cosh\frac{B(2z-h)}{2l}}{Cosh\frac{Bh}{2l}}\right)+1\right\} \tag{6}$$

Substituting (6) in (3) and integrating across the film thickness h gives the modified form of Reynolds equation

$$\frac{\partial}{\partial x}\left[\frac{h_0^2}{\mu}\frac{\partial p}{\partial x}f(h,l,M_0)\right] = -\frac{dh}{dt} \tag{7}$$

Where $f(h,l,M_0) = \frac{1}{M_0^2}\left\{\frac{2l}{A^2-B^2}\left(\frac{B^2}{A}\tanh\frac{Ah}{2l} - \frac{A^2}{B}\tanh\frac{Bh}{2l}\right)+h\right\}$

Introducing non-dimensional quantities, the expressions for the pressure distribution and the load-supporting capacity in dimensionless form are

$$x^* = \frac{x}{L}, h^* = \frac{h}{h_0}, l^* = \frac{2l}{h_0}, p^* = \frac{p h_0^3}{\mu L^2(-dh/dt)}, \delta = \frac{d}{h_0}e^{i\theta}$$

$$\frac{\partial}{\partial x^*}\left\{\frac{\partial p^*}{\partial x^*}F(h^*,l^*,M_0)\right\} = -1 \tag{8}$$

$$F(h^*,l^*,M_0) = \frac{1}{M_0^2}\left\{\frac{l^*}{A^{*2}-B^{*2}}\left(\frac{B^{*2}}{A^*}\tanh\frac{A^*h^*}{l^*} - \frac{A^{*2}}{B^*}\tanh\frac{B^*h^*}{l^*}\right)+h^*\right\}$$

$$h^* = 1+\delta\left\{1-\cos(\pi x^*)\right\}$$

The pressure conditions are: $p^* = 0$ at $x^* = \pm\frac{1}{2}$ and $\frac{dp^*}{dx^*} = 0$ at $x^* = 0$.

Integrating (8) and using the pressure conditions, one can obtain

$$p^* = -\int_{1/2}^{x^*}\frac{x^*}{F(h^*,l^*,M_0)}dx^*$$

$$p^* = \int_{x^*}^{1/2}\frac{x^*}{F(h^*,l^*,M_0)}dx^* \tag{9}$$

Integrating the film pressure, one can obtain the load-carrying capacity

$$W = b\int_{-L/2}^{L/2} p\,dx \tag{10}$$

where b denotes the width of the curved plates. The non-dimensional form of load supporting capacity is given by

$$W^* = \frac{h_0^3 W}{\mu L^3 b(-dh/dt)} = \int_{-1/2}^{1/2} p^* dx^* \tag{11}$$

After performing the integration, one can obtain the non-dimensional load capacity.

$$W^* = \int_{-1/2}^{1/2}\left\{\int_{x^*}^{1/2} \frac{x^*}{F(h^*, l^*, M_0)}dx^*\right\}dx^* \tag{12}$$

The non-dimensional form of squeeze film time is given by

$$T^* = \frac{Wh_0^2 t}{\mu L^3 b} \tag{13}$$

The elapsed time required for the upper curved plate to approach the lower plate is given by

$$T^* = \int_{h_0^*}^{1}\left[\int_{-1/2}^{1/2}\left\{\int_{x^*}^{1/2} \frac{x^*}{F(h^*, l^*, M_0)}dx^*\right\}dx^*\right]dh_0^* \tag{14}$$

For $\quad T^*, h^* = h_0^* + \delta\{1 - \cos(\pi x^*)\}$.

Using the numerical method of integration, the film pressure (9), the load capacity (12) and the elapsed time (14) can be calculated.

3. RESULTS AND DISCUSSIONS

The effect of MHD on the conducting couplestress fluid lubrication between convex curved plates is observed and the squeeze film characteristics are analysed with respect to the non-dimensional parameters namely the Hartmann number M_0^* and conducting couplestress parameter l^*.

3.1 Squeeze film pressure:

Figure 2, 3 represents the variation of non-dimensional p^* with horizontal coordinate x^* for different values of l^* and M_0. It is observed that effect of couplestress is to enhance the pressure at the centre in the presence of magnetic field as compared to Newtonian case. Increase in p is more pronounced for larger values of l^*. The dotted lines represents the non magnetic case, and solid lines represents the magnetic case. It is pragmatic that effect of magnetic field is to enhance the pressure as compared to non-magnetic case.

3.2 Load carrying capacity:

Figure.4, and 5 shows the variation of non-dimensional W^* with amplitude ratio δ for different values of l^* and M_0. It is seen that effect of magnetic field is prominent for increasing values of magnetic parameter M_0 in presence of couplestress fluid. It is observed that with the increasing values of l^* and M_0 there is significant increase in the load carrying capacity W^*. The effects are more pronouncedin convex curved plates for CCSF (conducting couplestress fluids) in the presence of transverse magnetic field. The increase in W is more evident for larger values of l^*.

3.3 Non-dimensional Squeeze film time:

Figure 6, and 7 represents Variation on non-dimensional squeeze film time T^* with h_0^* for different values of l^* and M_0.It is observed that with the increasing values of l^* and M_0 there is significant increase in squeeze film time T^*. It is observed that the effect of magnetic parameter is to enhance the squeezing effect for convex curved plates in the presence of CCSF.

The effect of couplestress effect in the presence of transverse magnetic field on the squeeze film characteristics is evaluated by relative percentage difference. The increase in the non-dimensional load carrying capacity R_{W^*} and the non-dimensional squeeze film time R_{T^*} are defined by

$$R_{W^*} = \left\{ \left(W^*_{couplestress} - W^*_{Newtonian} \right) \Big/ W^*_{Newtonian} \right\} \times 100$$

And $$R_{T^*} = \left\{ \left(T^*_{couplestress} - T^*_{Newtonian} \right) \Big/ T^*_{Newtonian} \right\} \times 100$$

The values of R_{W^*} and R_{T^*} are listed in Table. 1 for various values of M_0, l^*. It is clear that an increase of nearly 4.6%, 24.6% in W^* and T^* is observed for $l^* = 0.2$ and $M_0 = 2$. It is seen that the relative difference in load R_{W^*} is quite significant for increasing values of l^*. Further the effect of l^* clearly indicates the increasing values of relative squeeze film time R_{T^*} indicating the enhanced performance characteristics.

4. CONCLUSION

The study of Magneto-Hydrodynamic Couplestress Cosine form Convex Curved Plates is studied on the basis of Stokes couplestress fluid model together with the hydromagnetic flow equations Modified Reynolds equation has been derived and the following conclusions are made.

1. The non-dimensional Pressure, load carrying capacity and squeeze film time increases for the couple stress lubricants as compared to the corresponding Newtonian case.
2. The increasing values of magnetic parameter M_0 is to increase the performance characteristics of conducting cosine form Convex Curved Plates.
3. The relative load carrying capacity and squeeze film time increases with increasing values of couplestress parameter as shown in Table.1
2. Furthur it is observed that the squeeze film time is significant for increasing values of couplestress parameter l^* and magnetic parameter M_0 .

NOMENCLATURE

B_0	Applied magnetic field in the $z-$direction
H	Film thickness
L	length of the plates,
d	amplitude of the cosine function
b	width of the curved plates
h_0	Minimum film thickness
h_0^*	Dimensionless film thickness after time Δt
h_s	Stochastic film thickness
M_0	Magnetic Parameter $\left(= B_0 h_0 (\frac{\sigma}{\mu})^{1/2} \right)$
l	Couplestress fluids $\left(\frac{\eta}{\mu} \right)^{1/2}$
l^*	Dimensionless parameter $\frac{2l}{h_0}$
u, w	Velocity components in film region
x, y, z	Local Cartesian co-ordinates
p^*	Dimensionless pressure $\left(= \frac{p h_0^3}{\mu L^2 (-dh/dt)} \right)$
t	Mean time of approach

T^* Dimensionless time of approach $\left(= \frac{h_0^2 W t}{\mu L^3 b} \right)$

V Squeezing Velocity $\left(-\frac{dh}{dt} \right)$

W^* Dimensionless load carrying capacity $\left(= \frac{h_0^3 W}{\mu L^3 b(-dh/dt)} \right)$

GREEK SYMBOLS

σ Conductivity of fluid

$\bar{\sigma}$ Standard deviation

η Material constant responsible for couplestress

μ Lubricant viscosity

ξ Random variable

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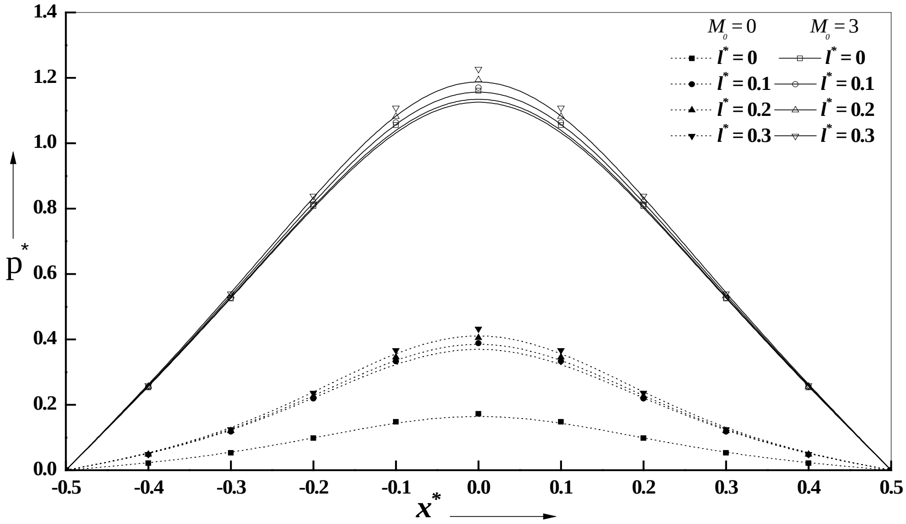

Fig. 2. Variation of non-dimensional P with x^* for different values of l^* and M_0 with δ=1.5

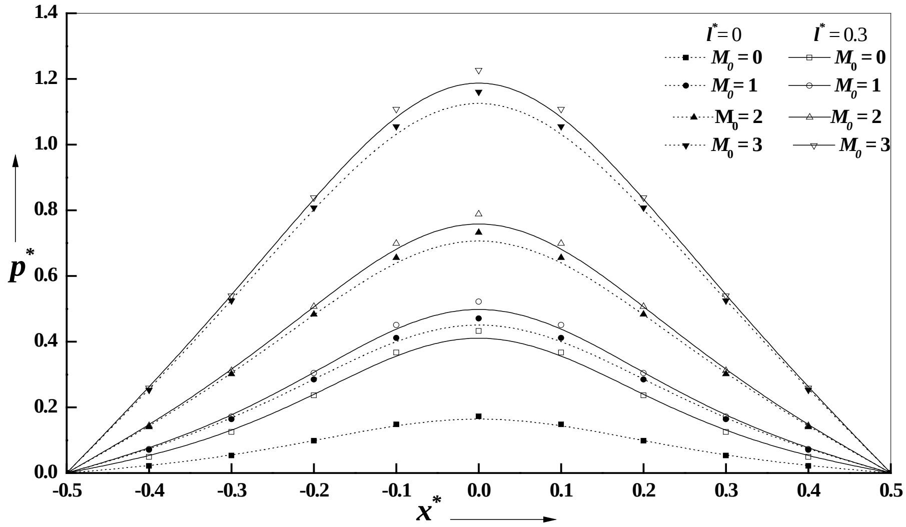

Fig. 3. Variation of non-dimensional p^* with x^* for different values of l^* and M_0 with δ=1.5

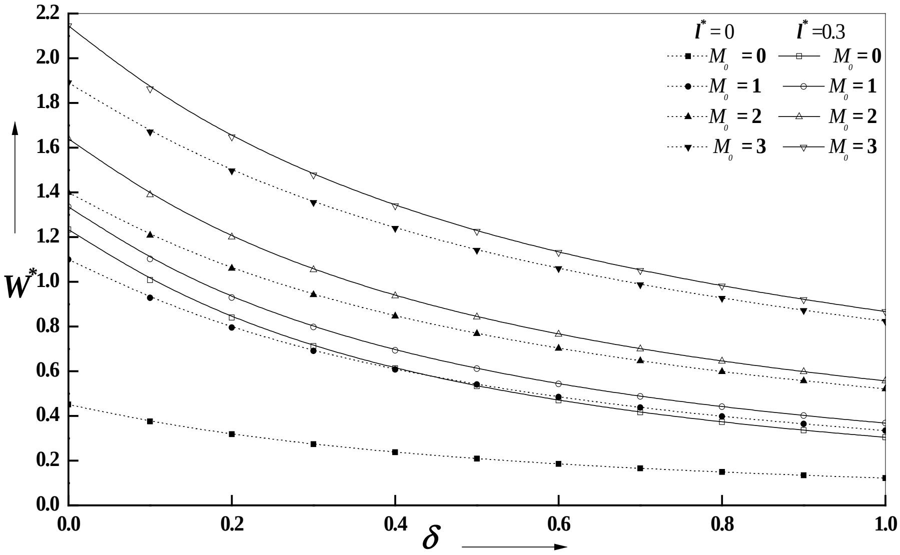

Fig. 4. Variation of non-dimensional W^* with amplitude ratio δ for different values of l^* and M_0.

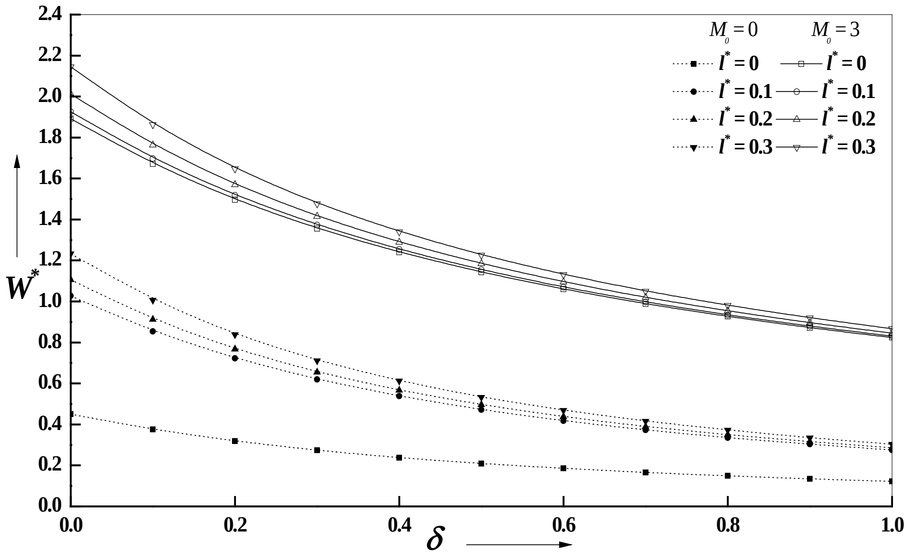

Fig. 5. Variation of non-dimensional W^* with amplitude ratio δ for different values of l^* and M_0.

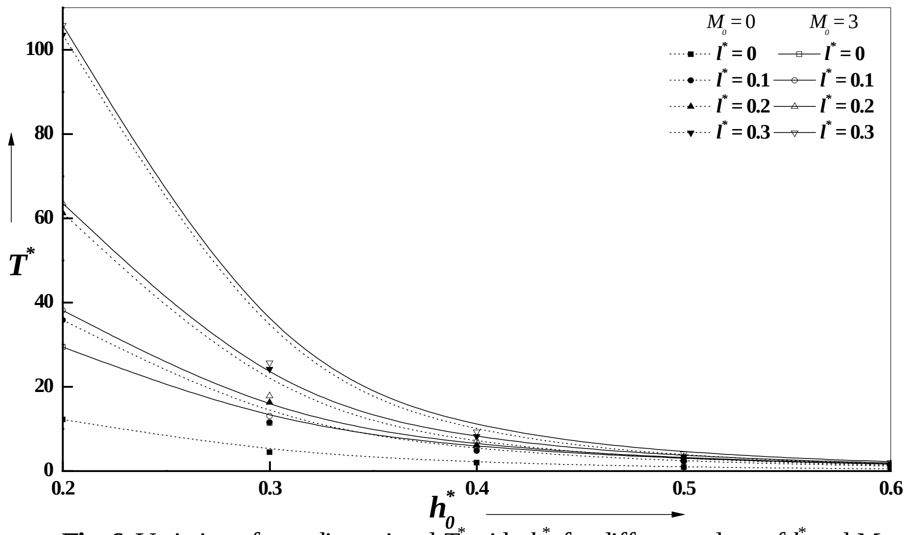

Fig. 6. Variation of non-dimensional T^* with h_0^* for different values of l^* and M_0 at δ=0.2.

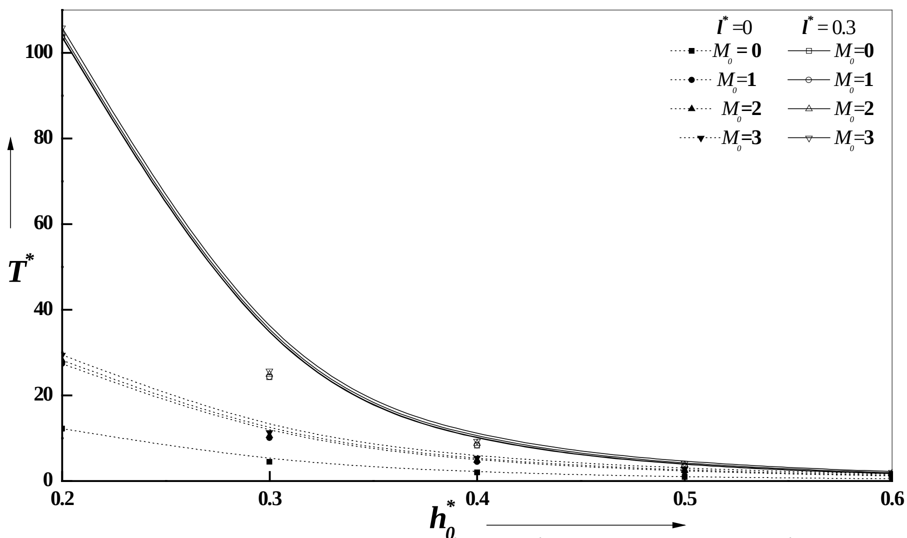

Fig. 7. Variation of non-dimensional T^* with h_0^* for different values of l^* and M_0 at δ=0.2.

Table1: Relative non-dimensional load carrying capacity $W^*$for different values of M_0 and l^* at $\delta = 0.5$.

		$R_W{}^*$	$R^*{}_T$
M_0 =0	l^* =0.1	1.6996	7.214
	l^* =0.2	6.7462	27.937
	l^* =0.3	14.8912	62.015
M_0 =1	l^* =0.1	1.5866	7.0707
	l^* =0.2	6.0125	27.082
	l^* =0.3	13.0822	59.952
M_0 =2	l^* =0.1	1.2651	6.4819
	l^* =0.2	4.6135	24.601
	l^* =0.3	9.7784	54.262
M_0 =3	l^* =0.1	1.0648	5.742
	l^* =0.2	3.6549	21.445
	l^* =0.3	7.4131	46.986

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