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η – DUALS OF SOME DIFFERENCE SEQUENCE SPACES DEFINED BY ORLICZ FUNCTION

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ABSTRACT

In this paper we introduce some difference sequence spaces defined by Orlicz function. Also we obtain η – duals and null duals of some difference sequence spaces, defined by means of a fixed sequence of multiplier and by an Orlicz function.

Keywords: *Orlicz function, η – dual, null dual, difference sequence spaces.*

INTRODUCTION

The notion for duals for sequence spaces introduced by Köthe and Toeplitz [8]. Later on it was studied by Maddox [13], Lascarides [11], Okutoyi [16], Chandra and Tripathy [2] and many others. It is also found in the monographs of Köthe [9], Maddox [14], Cook [3] and Kamthan and Gupta [6].

The notion of α – duals is generalized by Chandra and Tripathy [2] on introducing the notion of η – duals for sequence spaces.

The study of Orlicz sequence spaces was initiated with a certain specific purpose in Banach space theory. Indeed, Lindberg got interested in Orlicz spaces in connection with finding Banach spaces with symmetric Schauder bases having complementary subspaces isomorphic to c_0 or $\ell_p(1 \le p < \infty)$. Subsequently J. Lindenstrauss and L. Tzafriri studied these Orlicz sequence spaces in more detail, and solved many important and interesting structural problems in Banach spaces. Later on, different classes of sequence spaces defined by Orlicz function were studied by N. P. Nung, P. Y. Lee, J. Y. T. Woo, E. Savas, A. Esi, M. Et, B. Choudhary, V. K. Bhardwaj and many others.

It is a fundamental principle of functional analysis that investigations of space are often combined with those of dual spaces. From the point of view of the duality theory, the study of sequence spaces is much more profitable when we consider them equipped with linear topologies. However, in such cases it is rather cumbersome to obtain their topological duals. Even if we are successful in finding these topological duals, we would like to deal with only those duals whose members are representable as sequences; indeed, such situations present not much difficulty in the analysis. Köthe and Toeplitz were the first to recognize the problem, and to resolve it they introduced the notion of α – dual which turns out of be the same as the topological dual in quite many familiar and useful examples of sequence spaces endowed with their natural linear matrices. In the same paper they also introduced the notion of β −dual. Later on some other notions of dual were introduced as well as generalized some earlier notions of dual by D. J. H. Garling, W. H. Rukle, H. Kizmaz, D. Rath, B. C. Tripathy, C. A. Bektas, M. Et, R. Colak, M. Mursaleen, S. D. Parasar and many others.

DEFINITION AND PRELIMINARIES

A BK - space (introduced by Zeller [19]) (X, $\|.\|$) is a Banach space of complex sequences $x = (x_k)$ in which the co-ordinate maps are continuous, that is, $|x_k^n - x_k| \to 0$, whenever $\|x^n - x\| \to 0$ as $n \to \infty$, where $x^n = (x_k^n)$, for all $n \in$ N and $x = (x_k)$.

The difference sequence space $X(\Delta)$ was introduced by Kizmaz [7] as follows

$$X(\Delta) = \{x = (x_k) : (\Delta x_k) \in X\} \quad \text{for } X = \ell_\infty, c, c_0, \text{where } \Delta x = (\Delta x_k) = (x_k - x_{k+1}).$$

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After Et. and Colak [15] generalized the difference sequence spaces to the sequence spaces

$X(\Delta^m) = \{x = (x_k) : (\Delta^m x_k) \in X\}$ for $X = \ell_\infty, c, c_0$ where $m \in$ N, $\Delta^0 x = (x_k)$, $\Delta x = (x_k - x_{k+1})$,
$\Delta^m x = (\Delta^m x_k) = (\Delta^{m-1}x_k - \Delta^{m-1}x_{k+1})$, and so that $\Delta^m x_k = \sum_{\nu=0}^{m}(-1)^\nu \binom{m}{\nu} x_v$.

An Orlicz Function is a function $M : [0,\infty) \to [0,\infty)$ which is continuous, non decreasing and convex with M (0) = 0, $M(x) > 0$ for $x > 0$ and $M(x) \to \infty$, as $x \to \infty$. If convexity of M is replaced by $M(x + y) \le M(x) + M(y)$ then it is called Modulus function which is defined and characterized by Ruckle [18].

An Orlicz function M satisfies the Δ_2 - *condition* ($M \in \Delta_2$ for short) if there exist constant $k \ge 2$ and $u_0 > 0$ such that

$$M(2u) \le KM(u)$$

whenever $|u| \le u_0$

An Orlicz function M can always be represented in the integral form $M(x) = \int_0^x q(t)dt$, where q known as the kernel of M, is right differentiable for t $\ge 0, q(t) > 0, q$ is non – decreasing and $q(t) \to \infty$ as $t \to \infty$.

Note that an Orlicz function satisfies the inequality

$$M(\lambda x) \le \lambda M(x) \text{ for all } \lambda \text{ with } 0 < \lambda < 1,$$

Since M is convex and $M(0) = 0$.

W. Orlicz used the idea of Orlicz function to construct the space (L^M). Lindenstrauss and Tzafriri [12] used the idea of Orlicz sequence space

$$\ell_M = \{x = (x_k) : \sum_{k=1}^{\infty} M(\tfrac{|x_k|}{\rho}) < \infty, \text{ for some } \rho > 0\}$$

which is Banach space with the norm

$$\|x\|_M = \inf\{\rho > 0 : \sum_{k=1}^{\infty} M(\tfrac{|x_k|}{\rho}) \le 1\}.$$

The space ℓ_M is closely related to the space ℓ_p, which is an Orlicz sequence space with

$$M(x) = x^p \text{ for } 1 \le p < \infty.$$

Let M be an Orlicz function. Then Qamaruddin, Khan and Mursaleen [17] defined the following sequence spaces

$$c_0(\Delta, M) = \{x = (x_k) : \lim_k M(\tfrac{|\Delta x_k|}{\rho}) = 0, \text{ for some } \rho > 0\},$$
$$c(\Delta, M) = \{x = (x_k) : \lim_k M(\tfrac{|\Delta x_k - l|}{\rho}) = 0, \text{ for some } \rho > 0, l \in C\},$$
$$\ell_\infty(\Delta, M) = \{x = (x_k) : sup_k\, M(\tfrac{|\Delta x_k|}{\rho}) < \infty, \text{ for some } \rho > 0\}.$$

In this paper our aim is to investigate some important structures of some spaces which are defined using an Orlicz function and a multiplier sequence. These spaces generalize the spaces $X(\Delta)$, for $X = \ell_\infty, c, c_0$ introduced and studied by Kizmaz [7].

Let $\Lambda = (\lambda_k)$ be a non – zero sequence of scalars. Then Hemen Dutta [4] defined the following sequence space for an Orlicz function M

$$c_0(M, \Lambda, \Delta) = \{x = (x_k) : \lim_k M(\tfrac{|\Delta\lambda_k x_k|}{\rho}) = 0, \text{ for some } \rho > 0\},$$
$$c(M, \Lambda, \Delta) = \{x = (x_k) : \lim_k M(\tfrac{|\Delta\lambda_k x_k - L|}{\rho}) = 0, \text{ for some } L \text{ and } \rho > 0\},$$
$$\ell_\infty(M, \Lambda, \Delta) = \{x = (x_k) : sup_k\, M(\tfrac{|\Delta\lambda_k x_k|}{\rho}) < \infty, \text{ for some } \rho > 0\},$$

where $\Delta\lambda_k x_k = \lambda_k x_k - \lambda_{k+1}x_{k+1}$, for all $k \in N$.

It is obvious that $c_0(M, \Lambda, \Delta) \subset c(M, \Lambda, \Delta) \subset \ell_\infty(M, \Lambda, \Delta)$. Throughout the paper X will denote one of the sequence spaces c_0, c and ℓ_∞. The sequence spaces $X(M, \Lambda, \Delta)$ are Banach spaces normed by

$$\|x\|_\Delta = |\lambda_1 x_1| + \inf\{\rho > 0 : sup_k M(\tfrac{|\Delta\lambda_k x_k|}{\rho}) \le 1\}.$$

Now we shall write $\Delta^{-1}x_k = x_k - x_{k-1}$, for all $k \in N$. It is trivial that $\Delta\lambda_k x_k \in X(M)$ if and only if $(\Delta^{-1}\lambda_k x_k) \in X(M)$. Now for $x \in X(M, \Lambda, \Delta^{-1})$, we define

$$\|x\|_{\Delta^{-1}} = \inf\{\rho > 0 : sup_k(\tfrac{|\Delta^{-1}\lambda_k x_k|}{\rho}) \le 1\}.$$

It can be shown that $X(M,\Lambda,\Delta)$ is a $BK-$space under the norms $\|.\|_{\Delta}$ and $\|.\|_{\Delta^{-1}}$ respectively and it is obvious that the norms $\|.\|_{\Delta}$ and $\|.\|_{\Delta^{-1}}$ are equivalent.

Obviously $\Delta^{-1}\colon X(M,\Lambda,\Delta^{-1}) \to X(M)$, defined by $\Delta^{-1}x = y = (\Delta^{-1}\lambda_k x_k)$ is isometric isomorphism.

Hence $c_0(M,\Lambda,\Delta^{-1}), c(M,\Lambda,\Delta^{-1})$ and $\ell_\infty(M,\Lambda,\Delta^{-1})$ are isometrically isomorphic to $c_0(M), c(M)$ and $\ell_\infty(M)$ respectively. From abstract point of view $X(M,\Lambda,\Delta^{-1})$ is identical with $X(M)$, for $X = c_0, c$ and ℓ_∞.

The results obtained in the next section also hold for the spaces $c_0(M,\Lambda,\Delta^{-1})$, $c(M,\Lambda,\Delta^{-1})$ and $\ell_\infty(M,\Lambda,\Delta^{-1})$ as well as for the spaces associated with these three spaces.

Now define the spaces $\tilde{c}_0(M,\Lambda,\Delta)$, $\tilde{c}(M,\Lambda,\Delta)$ and $\tilde{\ell}_\infty(M,\Lambda,\Delta)$ as follows $\tilde{c}_0(M,\Lambda,\Delta)$ is a subspace of $c_0(M,\Lambda,\Delta)$ consisting of those $x \in c_0(M,\Lambda,\Delta)$ such that

$$\lim_k M\left(\frac{|\Delta\lambda_k x_k|}{d}\right) = 0 \; for\ each\ d > 0.$$

Similarly we can define $\tilde{c}(M,\Lambda,\Delta)$ and $\tilde{\ell}_\infty(M,\Lambda,\Delta)$ as subspace of $c(M,\Lambda,\Delta)$ and $\ell_\infty(M,\Lambda,\Delta)$ respectively. It is obvious that $\tilde{c}_0(M,\Lambda,\Delta) \subset \tilde{c}(M,\Lambda,\Delta) \subset \tilde{\ell}_\infty(M,\Lambda,\Delta)$. Also as above we can show that $\tilde{c}_0(M,\Lambda,\Delta)$, $\tilde{c}(M,\Lambda,\Delta)$ and $\tilde{\ell}_\infty(M,\Lambda,\Delta)$ are isometrically isomorphic to $\tilde{c}_0(M), \tilde{c}(M)$ and $\tilde{\ell}_\infty(M)$ respectively.

Moreover $X(M,\Lambda) \subset X(M,\Lambda,\Delta)$ and $\tilde{X}(M,\Lambda) \subset \tilde{X}(M,\Lambda,\Delta)$ which can be shown by using the following inequality

$$M\left(\frac{|\Delta\lambda_k x_k|}{2\rho}\right) \le \frac{1}{2} M\left(\frac{|\lambda_k x_k|}{\rho}\right) + \frac{1}{2} M\left(\frac{|\lambda_{k+1} x_{k+1}|}{\rho}\right).$$

MAIN RESULTS

In this section we compute η – dual and Null or $N-$dual of some difference sequence spaces as described in the preceding section.

Let E and F be two sequence spaces. Then the F dual of E is defined as

$$E^F = \{x = (x_k) : (x_k y_k) \in F \text{ for all } (y_k) \in E\}.$$

The notion of duals of sequence space was introduced by G. Köthe and O. Toeplitz [8].

Let X be a sequence space and define

$$X^\alpha = \{a = (a_k) : \textstyle\sum_{k=1}^{\infty} |a_k x_k| < \infty, \forall x \in X\},$$
$$X^\beta = \{a = (a_k) : \textstyle\sum_{k=1}^{\infty} a_k x_k \text{ is convergent}, \forall x \in X\},$$
$$X^\gamma = \{a = (a_k) : sup_n \,|\textstyle\sum_{k=1}^{n} a_k x_k| < \infty, \forall x \in X\},$$
$$X^N = \{a = (a_k) : \lim_k a_k x_k = 0, \forall x \in X\},$$
$$X^\eta = \{a = (a_k) : \textstyle\sum_{k=1}^{\infty} |a_k x_k|^r < \infty, \forall x \in X\}, \text{ where } r \ge 1.$$

Taking r =1 in above definition we get α – dual of X.

Then X^α, X^β, X^γ, X^N, and X^η are called the $\alpha-$, $\beta-$, $\gamma-$, $N-(or\ null)\ and\ \eta-duals$ of X, respectively.

Theorem 1: $x \in \ell_\infty(M,\Lambda,\Delta)$ $\; implies \;$ $sup_k M\left(\frac{|k^{-1}\lambda_k x_k|}{\rho}\right) < \infty$, for some $\rho > 0$.

Proof: Let $x \in \ell_\infty(M,\Lambda,\Delta)$, then

$$sup_k \mathrm{M}\left(\frac{|\lambda_k x_k - \lambda_{k+1}x_{k+1}|}{\rho}\right) < \infty, \text{ for some } \rho > 0$$

Then there exists a $U > 0$ such that

$$M\left(\frac{|\lambda_k x_k - \lambda_{k+1}x_{k+1}|}{\rho}\right) < U, \text{ for all } k \in N.$$

Taking $\mu = k\rho$, for an arbitrary fixed positive integer k, by the subadditivity of modulus, the monotonicity and convexity of M

$$M\left(\frac{|\lambda_1 x_1 - \lambda_{k+1}x_{k+1}|}{\mu}\right) < \frac{1}{k}\sum_{l=1}^{k} M\left(\frac{|\lambda_l x_l - \lambda_{l+1}x_{l+1}|}{\rho}\right) < U.$$

Then the above inequality, the inequality

$$\frac{|\lambda_{k+1}x_{k+1}|}{(k+1)\rho} \le \frac{1}{k+1}\left(\frac{|\lambda_1 x_1|}{\rho} + k\frac{|\lambda_1 x_1 - \lambda_{k+1}x_{k+1}|}{k\rho}\right)$$

and the convexity of M imply

$$M\left(\frac{|\lambda_{k+1}x_{k+1}|}{(k+1)\rho}\right) \le \frac{1}{k+1}\left(M\left(\frac{|\lambda_1 x_1|}{\rho}\right) + kM\left(\frac{|\lambda_1 x_1 - \lambda_{k+1}x_{k+1}|}{k\rho}\right)\right)$$
$$\le \max\{M\left(\frac{|\lambda_1 x_1|}{\rho}\right), U\} < \infty$$

Hence we have the desired result.

Theorem 2: $x \in \ell_\infty(M, \Lambda, \Delta)$ *implies* $sup_k k^{-1} | \lambda_k x_k | < \infty$.

Proof: Proof is obvious by using theorem 1.

Remark: Similar results as in theorem 1 and theorem 2 hold for $\tilde{\ell}_\infty(\mathrm{M}, \Lambda, \Delta)$ also, where the statement for some $\rho > 0$ should be replaced by for every $\rho > 0$.

Theorem 3: Let M be an Orlicz function and let

$$D_1 = \{ a = (a_k) : \sum_{k=1}^{\infty} k^r | \lambda_k^{-1} a_k |^r < \infty\},$$
$$D_2 = \{ b = (b_k) : sup_k k^{-r} | \lambda_k b_k |^r < \infty\}.$$

Then

(i) $[c(M, \Lambda, \Delta)]^\eta = [\ell_\infty(M, \Lambda, \Delta)]^\eta = D_1$,
(ii) $[\tilde{c}(M, \Lambda, \Delta)]^\eta = [\tilde{\ell}_\infty(M, \Lambda, \Delta)]^\eta = D_1$,
$(iii) D_1^\eta = D_2$.

Proof: (i) *Let* $a \in D_1$, then $\sum_{k=1}^{\infty} | k\lambda_k^{-1} a_k |^r < \infty$. Now for any $x \in \ell_\infty(M, \Lambda, \Delta)$ we have $sup_k | k^{-1}\lambda_k x_k |^r < \infty$.

Then we have

$$\sum_{k=1}^{\infty} | a_k x_k |^r \le sup_k | k^{-1}\lambda_k x_k |^r \sum_{k=1}^{\infty} | k\lambda_k^{-1} a_k |^r < \infty.$$

Hence $a \in [\ell_\infty(M, \Lambda, \Delta)]^\eta$.

Thus

$$D_1 \subseteq [\ell_\infty(M, \Lambda, \Delta)]^\eta \tag{3.1}$$

Again we know

$$[\ell_\infty(M, \Lambda, \Delta)]^\eta \subseteq [c(M, \Lambda, \Delta)]^\eta \subseteq [c_0(M, \Lambda, \Delta)]^\eta \tag{3.2}$$

Conversely suppose that $a \in [c(M, \Lambda, \Delta)]^\eta$. Then $\sum_{k=1}^{\infty} | a_k x_k |^r < \infty$, for each $x \in [c(M, \Lambda, \Delta)]^\eta$. So we take

$$x_k = \lambda_k^{-1} k, \qquad k \ge 1$$

then

$$\sum_{k=1}^{\infty} | k\lambda_k^{-1} a_k |^r = \sum_{k=1}^{\infty} | a_k x_k |^r < \infty.$$

This implies that $a \in D_1$. Thus

$$[c(M, \Lambda, \Delta)]^\eta \subseteq D_1 \tag{3.3}$$

Combining (3.3) with (3.1), (3.2) it follows

$$[c(M, \Lambda, \Delta)]^\eta = [\ell_\infty(M, \Lambda, \Delta)]^\eta = D_1$$

This completes the proof of part (i).

(ii) Proof is similar to that of part (i).

(iii) The proof of the inclusion $D_1^\eta \supseteq D_2$ is similar to that of $D_1 \subseteq [\ell_\infty(M, \Lambda, \Delta)]^\eta$.

For the converse part suppose $a \in D_1^\eta$ and $a \notin D_2$. Then we have

$$sup_k | k^{-1}\lambda_k x_k |^r = \infty$$

Hence we can find a strictly increasing sequence (k_j) of positive integers k_j such that

$$|k_j^{-1}\lambda_{k_j}a_{k_j}| > j^2 \text{ for all } j \geq 1.$$

We define the sequence x by

$$x_k = \begin{cases} |a_{k_j}^{-1}|, if\ k = k_j \\ 0, \quad otherwise \end{cases}$$

Then $x \in D_1$, because

$$\sum_{k=1}^{\infty} |k\lambda_k^{-1}x_k|^r = \sum_{j=1}^{\infty} |k_j\lambda_{k_j}^{-1}a_{k_j}^{-1}|^r \leq \sum_{j=1}^{\infty} j^{-2r} < \infty$$

Thus $x \in D_1$ but $\sum_{k=1}^{\infty} |a_k x_k|^r = \sum_{j=1}^{\infty} |a_{k_j}x_{k_j}|^r = \infty$. This is a contradiction to $a \in D_1^{\eta}$

Hence $a \in D_2$. This completes the proof.

Corollary 1: $For\ X = c\ and\ \ell_\infty$,
(i) $[X(M,\Delta)]^{\eta} = [\tilde{X}(M,\Delta)]^{\eta} = H_1$,
(ii) $H_1^{\eta} = H_2$,
$where$

$$H_1 = \{a = (a_k) : \sum_{k=1}^{\infty} |ka_k|^r < \infty\}$$

and

$$H_2 = \{b = (b_k) : sup_k |k^{-1}b_k|^r < \infty\}.$$

Theorem 4: $Let\ M\ be\ an\ Orlicz\ function\ and$ let $G_1 = \{a = (a_k) : \lim_k k\lambda_k^{-1}a_k = 0\}$. Then
(i) $[c(M,\Lambda,\Delta)]^N = [\ell_\infty(M,\Lambda,\Delta)]^N = G_1$
(ii) $[\tilde{c}(M,\Lambda,\Delta)]^N = \left[\tilde{\ell}_\infty(M,\Lambda,\Delta)\right]^N = G_1$

Proof:
(i) Proof is immediate using theorem 2.
(ii) Proof is similar to that of part (i).

Corollary 2: $For X = c\ and\ \ell_\infty$,

$$[X(M,\Delta)]^N = [\tilde{X}(M,\Delta)]^N = L_1,$$

$where$ $L_1 = \{a = (a_k) : \lim_k ka_k = 0\}$.

Theorem 5: If M satisfies the $\Delta_2 - condition, then\ X(M,\Lambda,\Delta) = \tilde{X}(M,\Lambda,\Delta), for\ every\ X = c_0, c\ and\ \ell_\infty$.

Proof: We give the proof for $X = \ell_\infty$ and for other spaces it will follow on applying similar arguments.

To prove the theorem, it is enough to show that $\ell_\infty(M,\Lambda,\Delta)$ is a subspace of $\tilde{\ell}_\infty(M,\Lambda,\Delta)$.

Let $x \in \ell_\infty(M,\Lambda,\Delta)$ then for some $\rho > 0$,

$$sup_k M\left(\frac{|\Delta\lambda_k x_k|}{\rho}\right) < \infty$$

Therefore

$$M\left(\frac{|\Delta\lambda_k x_k|}{\rho}\right) < \infty, \text{ for every } k \in N.$$

Choose an arbitrary $\mu > 0$. If $\rho \leq \mu$ then $M\left(\frac{|\Delta\lambda_k x_k|}{\mu}\right) < \infty$ for every $k \in N$. Let now $\mu < \rho$ and put $l = \frac{\rho}{\mu} > 1$.

Since M satisfies the Δ_2 −condition, there exists a constant K such that

$$M\left(\frac{|\Delta\lambda_k x_k|}{\mu}\right) \leq K\left(\frac{\rho}{\mu}\right)^{log_2 K} M\left(\frac{|\Delta\lambda_k x_k|}{\rho}\right) < \infty \text{ for every } k \in N.$$

Now let us denote

$$S = sup_k M\left(\frac{|\Delta\lambda_k x_k|}{\rho}\right) < \infty, \text{ for the fixed } \rho > 0.$$

Then it follows that for every $\mu > 0$, we have

$$sup_k M\left(\frac{|\Delta\lambda_k x_k|}{\mu}\right) \leq K\left(\frac{\rho}{\mu}\right)^{log_2 K}.S < \infty.$$

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