ON NANO GENERALIZED PRE-CONTINUOUS FUNCTIONS IN NANO TOPOLOGICAL SPACES

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ABSTRACT

The purpose of this paper is to define and study some properties of Nano generalized pre-continuous functions in Nano topological spaces.

Keywords: Nano Topology, Ngp-closed sets, Ngp-closure, Ngp-interior, Ngp- continuous Function.

1. INTRODUCTION

Continuous function is one of the main concepts of Topology. Balachandran et al. [2] and Mashour et al. [9] have introduced g-continuous and pre-continuous function in topological spaces respectively. Arokiarani [1] introduced generalized pre-continuous functions and generalized pre-irresolute functions and compared with various stronger forms of the same functions. The notion of Nano topology was introduced by Lellis Thivagar[6] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and he also defined Nano closed sets, Nano-interior, Nano-closure, Nano continuous functions, Nano open mapping, Nano closed mapping and Nano Homeomorphism. Bhuvaneswari et al. [4] introduced and studied some properties of Nano generalized pre-closed sets in Nano topological spaces. In this paper, a new class of continuous functions called Nano generalized pre-continuous function is introduced and some of its properties in terms of Ng – closed sets, Ng-closure and Ng- interior are discussed.

2. PRELIMINARIES

Definition: 2.1 [8] A subset $A$ of a topological space $(X, \tau)$ is said to be a generalized pre closed (briefly gp -closed), if $pcl(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is open in $(X, \tau)$.

Definition: 2.2 [1] Let $f : (X, \tau) \to (Y, \sigma)$ be function and $f$ is said to be gp-continuous if $f^{-1}(V)$ is gp -closed in $X$ for every closed set $V$ of $Y$.

Definition: 2.3 [10] Let $U$ be a non-empty finite set of objects called the universe and $R$ be an equivalence relation on $U$ named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(U,R)$ is said to be the approximation space. Let $X \subseteq U$.

(i) The lower approximation of $X$ with respect to $R$ is the set of all objects, which can be for certain classified as $X$ with respect to $R$ and its is denoted by $L_R(X)$.

That is, $L_R(X) = \bigcup_{x \in X} \{R(x) : R(x) \subseteq X\}$ where $R(x)$ denotes equivalence class determined by $x$.

(ii) The upper approximation of $X$ with respect to $R$ is the set of all objects, which can be possibly classified as $X$ with respect to $R$ and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in X} \{R(x) : R(x) \cap X \neq \phi\}$

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(i) The boundary region of $X$ with respect to $R$ is the set of all objects, which can be classified neither as $X$ nor as not-$X$ with respect to $R$ and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$

**Property: 2.4** [10] If $(U, R)$ is an approximation space and $X, Y \subseteq U$, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$

(ii) $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$

(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$

(iv) $L_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$

(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$

(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$

(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$

(viii) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$

(ix) $U_RU_R(X) = L_RU_R(X) = U_R(X)$

(x) $L_RL_R(X) = U_RL_R(X) = L_R(X)$

**Definition: 2.5** [6] Let $U$ be the Universe, $R$ be an equivalence relation on $U$ and $\mathcal{T}_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 1.1.5, $\mathcal{T}_R(X)$ satisfies the following axioms:

(i) $U$ and $\phi \in \mathcal{T}_R(X)$.

(ii) The union of the elements of any sub – collection of $\mathcal{T}_R(X)$ is in $\mathcal{T}_R(X)$

(iii) The intersection of the elements of any finite sub – collection of $\mathcal{T}_R(X)$ is in $\mathcal{T}_R(X)$.

That is, $\mathcal{T}_R(X)$ is a topology on $U$ called the Nano topology on $U$ with respect to $X$.

We call $(U, \mathcal{T}_R(X))$ as the Nano topological space. The elements of $\mathcal{T}_R(X)$ are called as Nano-open sets. The elements of the complement of $\mathcal{T}_R(X)$ are called as Nano-closed sets.

**Definition: 2.6** [3] Let $(U, \mathcal{T}_R(X))$ be a Nano topological space. A subset $A$ of $(U, \mathcal{T}_R(X))$ is called Nano generalized closed set (briefly Ng- closed) if $Ncl(A) \subseteq G$ where $A \subseteq G$ and $G$ is Nano open in $(U, \mathcal{T}_R(X))$.

**Definition: 2.7** [4] Let $(U, \mathcal{T}_R(X))$ be a Nano topological space. A subset $A$ of $(U, \mathcal{T}_R(X))$ is called Nano generalized pre-closed set (briefly Ngp-closed) if $Npcl(A) \subseteq G$ where $A \subseteq G$ and $G$ is Nano open in $(U, \mathcal{T}_R(X))$.

**Definition: 2.7** [7] Let $(U, \mathcal{T}_R(X))$ and $(V, \mathcal{T}_R(Y))$ be two Nano topological spaces. Then a mapping $f : (U, \mathcal{T}_R(X)) \rightarrow (V, \mathcal{T}_R(Y))$ is Nano continuous on $U$ if the inverse image of every Nano closed set in $(V, \mathcal{T}_R(Y))$ is Nano closed in $(U, \mathcal{T}_R(X))$.

**Definition: 2.8** [5] Let $(U, \mathcal{T}_R(X))$ and $(V, \mathcal{T}_R(Y))$ be two Nano topological spaces. Then a mapping $f : (U, \mathcal{T}_R(X)) \rightarrow (V, \mathcal{T}_R(Y))$ is Nano generalized continuous (shortly Ng-conti-nuous) function on $U$ if the inverse image of every Nano closed set in $(V, \mathcal{T}_R(Y))$ is Ng- closed in $(U, \mathcal{T}_R(X))$.

3. PROPERTIES OF NANO GENERALIZED PRE CONTINUOUS FUNCTION IN NANO TOPOLOGICAL SPACES

**Definition: 3.1** Let $(U, \mathcal{T}_R(X))$ and $(V, \mathcal{T}_R(Y))$ be two Nano topological spaces. Then a mapping $f : (U, \mathcal{T}_R(X)) \rightarrow (V, \mathcal{T}_R(Y))$ is Nano generalized pre-continuous (shortly Ngp-continuous) function on $U$ if the inverse image of every Nano closed set in $(V, \mathcal{T}_R(Y))$ is Ngp- closed in $(U, \mathcal{T}_R(X))$. 
Example: 3.2 Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then $\tau_R(X) = \{\Phi, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, \{a, c\}, \{a, d\}, \{b\}, \{b, c\}, \{b, d\}, \{c\}, \{c, d\}, \{d\}, \{a, c, d\}, \{a, d, b\}, \{b, c, d\}, \{a, b, c, d\}, \emptyset\}$. Then Ngp-closed sets are $\Phi$, $\{b\}$, $\{c\}$, $\{d\}$, $\{a, c\}$, $\{b, d\}$, $\{a, b\}$, $\{a, d\}$, $\{b, c\}$, $\{a, c, d\}$, $\{a, d, b\}$, $\{b, c, d\}$, $\{a, b, c, d\}$, $\emptyset$. Since $\tau_R(X)$ is Ngp-continuous.

Hence $f^{-1}(b) = \{a, c\}$, $f^{-1}(b) = \{a\}$ and $f^{-1}(U) = U$. That is the inverse image of every Nano closed set in $V$ is Ngp-closed in $U$. Therefore $f$ is Ngp-continuous.

Theorem: 3.3 A function $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is Ngp-continuous if and only if the inverse image of every Nano open set in $(V, \tau_R(Y))$ is Ngp-open in $(U, \tau_R(X))$.

Proof: Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ be Ngp-continuous and $F$ be Nano open in $(V, \tau_R(Y))$. Then $F^C$ is Nano closed in $(V, \tau_R(Y))$. Since $f$ is Ngp-continuous, $f^{-1}(F^C)$ is Ngp-closed in $(U, \tau_R(X))$. But $f^{-1}(F^C) = (f^{-1}(F))^C$. Therefore $f^{-1}(F)$ is Ngp-open in $(U, \tau_R(X))$. Thus the inverse image of every Nano open set in $(V, \tau_R(Y))$ is Ngp-open in $(U, \tau_R(X))$. If $f$ is Ngp-continuous on $(U, \tau_R(X))$. Conversely, assume that $f^{-1}(F)$ is Ngp-open in $(U, \tau_R(X))$ for each Nano open set $F$ in $(V, \tau_R(Y))$. Let $G$ be a Nano closed set in $(V, \tau_R(Y))$. Then $G^C$ is Nano open in $(V, \tau_R(Y))$ and by assumption, $f^{-1}(G^C)$ is Ngp-open in $(U, \tau_R(X))$. Since $f^{-1}(G^C) = (f^{-1}(G))^C$, we have $f^{-1}(G)$ is Ngp-closed in $(U, \tau_R(X))$. Therefore $f$ is Ngp-continuous.

Theorem: 3.4 Every Nano continuous function is Ngp-continuous but not conversely.

Proof: Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ be Nano continuous on $(U, \tau_R(X))$. Since $f$ is Nano continuous on $(U, \tau_R(X))$, the inverse image of every Nano closed set in $(V, \tau_R(Y))$ is Nano closed in $(U, \tau_R(X))$. But every Nano closed set is Nano generalized pre-closed set. Hence the inverse image of every Nano closed set in $(V, \tau_R(Y))$ is Ngp-closed in $(U, \tau_R(X))$. Thus $f$ is Ngp-continuous.

Remark: 3.5 The converse of the above theorem is not true as seen from the following example.

Example: 3.6 Let $U = \{a, b, c\} = V$. Then $\tau_R(X) = \{\Phi, \{a\}, \{b\}, \emptyset\}$ with $U/R = \{\{a\}, \{b\}\}$ and $X = \{a\}$ and $\tau_R(Y) = \{\Phi, \{a\}, \{b\}, \emptyset\}$ with $V/R' = \{\{b\}\}$ and $Y = \{a\}$. Define $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ as $f(a) = c$, $f(b) = a$, $f(c) = b$ which is Ngp-continuous. But for the Nano closed set $\{a\}$ in $(V, \tau_R(Y))$, its inverse image $f^{-1}(a, c) = \{a, b\}$ is not Nano closed in $(U, \tau_R(X))$. Hence $f$ is not Nano continuous.

Theorem: 3.7 Every Nano pre-continuous function is Ngp-continuous but not conversely.

Proof: Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ be Nano pre-continuous on $(U, \tau_R(X))$. Since $f$ is Nano pre-continuous on $(U, \tau_R(X))$, the inverse image of every Nano open set in $(V, \tau_R(Y))$ is Nano pre-open in $(U, \tau_R(X))$. But every Nano pre-open set is Nano generalized pre-open set. Hence the inverse image of every Nano open set in $(V, \tau_R(Y))$ is Ngp-open in $(U, \tau_R(X))$. Thus $f$ is Ngp-continuous.

Remark: 3.8 The converse of the above theorem is not true as seen from the following example.

Example: 3.9 Let $U = \{a, b, c\} = V$. Then $\tau_R(X) = \{\Phi, \{a\}, \emptyset\}$ with $U/R = \{\{a\}\}$ and $X = \{a\}$ and $\tau_R(Y) = \{\Phi, \{a\}, \{b\}, \emptyset\}$ with $V/R' = \{\{b\}\}$ and $Y = \{a\}$. Define $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ as an identity mapping. Then $f$ is Ngp-continuous. But for the Nano open set $\{b\}$ in $(V, \tau_R(Y))$, its inverse image $f^{-1}(b) = \{b\}$ is not Nano pre-open in $(U, \tau_R(X))$. Hence $f$ is not Nano pre-continuous.
Theorem: 3.10 If a function \( f : (U, \tau_R(X)) \to (V, \tau_R(Y)) \) is Ng-continuous, then it is Ngp-continuous.

Proof: Let \( f : (U, \tau_R(X)) \to (V, \tau_R(Y)) \) be Ng-continuous. Suppose \( F \) is any Nano closed set in \((V, \tau_R(Y))\), then the inverse image \( f^{-1}(F) \) is Ng-closed in \((U, \tau_R(X))\). Since every Ng-closed set is Ngp-closed, \( f^{-1}(F) \) is Ngp-closed in \((U, \tau_R(X))\). Therefore \( f \) is Ngp-continuous.

Remark: 3.11 The converse of the above theorem is not true as seen from the following example.

Example: 3.12 Define \( f : (U, \tau_R(X)) \to (V, \tau_R(Y)) \) as in the example 7.2.10. Then for the Nano closed set \{b\} in \((V, \tau_R(Y))\), its inverse image \( f^{-1}(b) \) is not Ng-closed in \((U, \tau_R(X))\). Hence \( f \) is not Ng-continuous.

Theorem: 3.13 A function \( f : (U, \tau_R(X)) \to (V, \tau_R(Y)) \) is Ngp-continuous if and only if,
\[
f(\text{Ngp-cl}(A)) \subseteq \text{Ncl}(f(A)) \quad \text{for every subset } A \text{ of } (U, \tau_R(X)).
\]

Proof: Let \( f : (U, \tau_R(X)) \to (V, \tau_R(Y)) \) be Ngp-continuous and \( A \subseteq U \). Then \( f(A) \subseteq V \). Hence \( \text{Ncl}(f(A)) \) is Nano closed in \( V \). Since \( f \) is Ngp-continuous, \( \text{Ncl}(f(A)) \) is nano closed in \( V \). Since \( f(A) \subseteq \text{Ncl}(f(A)), f^{-1}(\text{Ncl}(f(A))) \) is Ngp-closed in \( U \). Therefore \( f^{-1}(\text{Ncl}(f(A))) \) is Ngp-closed set containing \( A \). But \( \text{Ngp-cl}(A) \) is the smallest Ngp-closed set containing \( A \). Hence we have \( \text{Ngp-cl}(A) \subseteq f^{-1}(\text{Ncl}(f(A))) \) which implies \( f(\text{Ngp-cl}(A)) \subseteq \text{Ncl}(f(A)) \). Conversely, let \( f(\text{Ngp-cl}(A)) \subseteq \text{Ncl}(f(A)) \) for every subset \( A \) of \((U, \tau_R(X))\). Let \( F \) be a Nano closed set in \((V, \tau_R(Y))\). Now \( f^{-1}(F) \subseteq U \). Hence, \( f(\text{Ngp-cl}(f^{-1}(F))) \subseteq \text{Ncl}(f(f^{-1}(F))) = \text{Ncl}(F) \). That is \( \text{Ngp-cl}(F) \subseteq f^{-1}(\text{Ncl}(F)) = f^{-1}(F) \) as \( F \) is nano closed. But \( f^{-1}(F) \subseteq \text{Ngp-cl}(F) \). Therefore \( \text{Ngp-cl}(F) = f^{-1}(F) \) which implies that \( f^{-1}(F) \) is Ngp-closed in \((U, \tau_R(X))\) for every Nano closed set \( F \) in \((V, \tau_R(Y))\). That is \( f \) is Ngp-continuous.

Remark: 3.14 Let \( f : (U, \tau_R(X)) \to (V, \tau_R(Y)) \) be Ngp-continuous. Then \( f(\text{Ngp-cl}(A)) \) is not necessarily equal to \( \text{Ncl}(f(A)) \) where \( A \subseteq U \).

Example: 3.15 Let \( U = \{a, b, c, d\} \) with \( \tau_R(X) = \{\Phi, \{a, b, d\}, \{b, d\}, \{a\}, U\} \). Let \( V = \{x, y, z, w\} \) with \( \tau_R(Y) = \{\Phi, \{x, y, w\}, \{x, y\}, \{w\}, V\} \). Define \( f : (U, \tau_R(X)) \to (V, \tau_R(Y)) \) as \( f(a) = y, f(b) = x, f(c) = z, f(d) = w \). Then \( f \) is Ngp-continuous. Then \( f(\text{Ngp-cl}(A)) = \{x, z, w\} \). But \( \text{Ncl}(f(A)) = \{x, y, z, w\} \). That is \( f(\text{Ngp-cl}(A)) \neq \text{Ncl}(f(A)) \) even though \( f \) is Ngp-continuous.

Theorem: 3.16 Let \((U, \tau_R(X))\) and \((V, \tau_R(Y))\) be two Nano topological spaces where \( X \subseteq U \) and \( Y \subseteq V \). Then \( \tau_R(Y) = \{V, \phi, L_R(Y), U_R(Y), B_R(Y)\} \) and its basis is given by \( B_R = \{V, L_R(Y), B_R(Y)\} \). A function \( f : (U, \tau_R(X)) \to (V, \tau_R(Y)) \) is Ngp-continuous if and only if the inverse image of every member of \( B_R \) is Ngp-open in \( U \).

Proof: Let \( f : (U, \tau_R(X)) \to (V, \tau_R(Y)) \) be Ngp-continuous on \((U, \tau_R(X))\). Let \( B \in B_R \). Then \( B \) is Nano open in \((V, \tau_R(Y))\). Since \( f \) is Ngp-continuous, \( f^{-1}(B) \) is Ngp-open in \( U \) and \( f^{-1}(B) \in \tau_R(X) \). Hence the inverse image of every member of \( B_R \) is Ngp-open in \( U \).

Conversely, let the inverse image of every member of \( B_R \) be Ngp-open in \( U \). Let \( G \) be Nano open in \( V \). Now \( G = \bigcup \{B : B \in B_R\} \) where \( B_i \subseteq B_R \). Then \( f^{-1}(G) = f^{-1}\left[\bigcup \{B : B \in B_i\}\right] = \bigcup \{f^{-1}(B) : B \in B_i\} \) where each \( f^{-1}(B) \) is Ngp-open in \( U \) and their union which is \( f^{-1}(G) \) is also Ngp-open in \( U \). By definition of Ngp-continuous function, \( f : (U, \tau_R(X)) \to (V, \tau_R(Y)) \) is Ngp-continuous on \((U, \tau_R(X))\).
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