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ON NANO GENERALIZED PRE-CONTINUOUS FUNCTIONS IN NANO TOPOLOGICAL SPACES

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ABSTRACT

The purpose of this paper is to define and study some properties of Nano generalized pre-continuous functions in Nano topological spaces.

Keywords: *Nano Topology, Ngp-closed sets, Ngp-closure, Ngp-interior, Ngp- continuous Function.*

1. INTRODUCTION

Continuous function is one of the main concepts of Topology. Balachandran *et al*. [2] and Mashour *et al*. [9] have introduced g-continuous and pre-continous function in topological spaces respectively. Arokiarani [1] introduced generalized pre-continuous functions and generalized pre-irresolute functions and compared with various stronger forms of the same functions. The notion of Nano topology was introduced by Lellis Thivagar[6] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and he also defined Nano closed sets, Nano-interior, Nano-closure, Nano continuous functions, Nano open mapping, Nano closed mapping and Nano Homeomorphism. Bhuvaneswari *et al.* [4] introduced and studied some properties of Nano generalized pre-closed sets in Nano topological spaces. In this paper, a new class of continuous functions called Nano generalized pre-continuous function is introduced and some of its properties in terms of Ng –closed sets, Ng-closure and Ng- interior are discussed.

2. PRELIMINARIES

Definition: 2.1 [8] A subset *A* of a topological space (X,τ) is said to be a generalized pre closed (briefly *gp* -closed), if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and *U* is open in (X,τ).

Definition: 2.2 [1] Let $f:(X,\tau)\to(Y,\sigma)$ be function and *f* is said to be *gp*-continuous if $f^{-1}(V)$ is *gp* -closed in *X* for every closed set *V* of *Y* .

Definition: 2.3 [10] Let *U* be a non-empty finite set of objects called the universe and *R* be an equivalence relation on *U* named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$.

(i) The lower approximation of *X* with respect to *R* is the set of all objects, which can be for certain classified as *X* with respect to *R* and its is denoted by $L_R(X)$.
That is, $L_R(X)=\bigcup_{x\in u}\{R(x):R(x)\subseteq X\}$ where $R(x)$ denotes equivalence class determined by *x*.

(ii) The upper approximation of *X* with respect to *R* is the set of all objects, which can be possibly classified as *X* with respect to *R* and it is denoted by $U_R(X)$ .That is, $U_R(X)=\bigcup_{x\in u}\{R(x):R(x)\cap X\neq\phi\}$

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(i) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$

Property: 2.4 [10] If (U, R) is an approximation space and X, $Y \subseteq U$, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$
(ii) $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$
(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
(viii) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$
(ix) $U_R U_R(X) = L_R U_R(X) = U_R(X)$
(x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$

Definition: 2.5 [6] Let U be the Universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 1.1.5, $\tau_R(X)$ satisfies the following axioms:

(i) U and $\phi \in \tau_R(X)$.
(ii) The union of the elements of any sub – collection of $\tau_R(X)$ is in $\tau_R(X)$
(iii) The intersection of the elements of any finite sub – collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X.

We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano-open sets. The elements of the complement of $\tau_R(X)$ are called as Nano-closed sets.

Definition: 2.6 [3] Let $(U, \tau_R(X))$ be a Nano topological space. A subset A of $(U, \tau_R(X))$ is called Nano generalized closed set (briefly Ng- closed) if $Ncl(A) \subseteq G$ where $A \subseteq G$ and G is Nano open in $(U, \tau_R(X))$.

Definition: 2.7 [4] Let $(U, \tau_R(X))$ be a Nano topological space. A subset A of $(U, \tau_R(X))$ is called Nano generalized pre-closed set (briefly Ngp-closed) if $Npcl(A) \subseteq G$ where $A \subseteq G$ and G is Nano open in $(U, \tau_R(X))$.

Definition: 2.7 [7] Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is Nano continuous on U if the inverse image of every Nano closed set in $(V, \tau_{R'}(Y))$ is Nano closed in $(U, \tau_R(X))$

Definition: 2.8 [5] Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is Nano generalized continuous (shortly Ng-continuous) function on U if the inverse image of every Nano closed set in $(V, \tau_{R'}(Y))$ is Ng- closed in $(U, \tau_R(X))$.

3. PROPERTIES OF NANO GENERALIZED PRE CONTINUOUS FUNCTION IN NANO TOPOLOGICAL SPACES

Definition: 3.1 Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is Nano generalized pre-continuous (shortly Ngp-continuous) function on U if the inverse image of every Nano closed set in $(V, \tau_{R'}(Y))$ is Ngp- closed in $(U, \tau_R(X))$.

Example: 3.2 Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X = {a, b}. Then $\tau_R(X)$ = {Φ, {a}, {a, b, d}, {b, d}, U}. Then Ngp-closed sets are {Φ, {b}, {c}, {d}, {a, c}, {b, c}, {c, d}, {a, b, c} , {b, c, d} ,{a, c, d}, U}. Let V= {x, y, z, w} with V/R' = {{x},{y, z},{w}} and Y={x, z}.Then $[\tau_{R'}(Y)]^C$ ={Φ,{y, z, w}, {w},{x, w}, V}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ as $f(a)$ = x, $f(b)$ = y, $f(c)$ = w, $f(d)$ = z. Then $f^{-1}(y,z,w)$={b, c, d}, $f^{-1}(w)$ = {c}, $f^{-1}(x,w)$ = {a, c} and $f^{-1}(V)$ = U. That is the inverse image of every Nano closed set in V is Ngp-closed in U. Therefore f is Ngp- continuous.

Theorem: ***3.3*** A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Ngp-continuous if and only if the inverse image of every Nano open set in $(V,\tau_{R'}(Y))$ is Ngp-open in $(U,\tau_R(X))$.

Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be Ngp-continuous and F be Nano open in $(V,\tau_{R'}(Y))$. Then F^C is Nano closed in $(V,\tau_{R'}(Y))$. Since f is Ngp-continuous, $f^{-1}(F^C)$ is Ngp-closed in $(U,\tau_R(X))$. But $f^{-1}(F^C)$ = $(f^{-1}(F))^C$. Therefore $f^{-1}(F)$ is Ngp-open in $(U,\tau_R(X))$. Thus the inverse image of every Nano open set in $(V,\tau_{R'}(Y))$ is Ngp-open in $(U,\tau_R(X))$, if f is Ngp-continuous on $(U,\tau_R(X))$.
Conversely, assume that $f^{-1}(F)$ is Ngp-open in $(U,\tau_R(X))$ for each Nano open set F in $(V,\tau_{R'}(Y))$. Let G be a Nano closed set in $(V,\tau_{R'}(Y))$. Then G^C is Nano open in $(V,\tau_{R'}(Y))$ and by assumption, $f^{-1}(G^C)$ is Ngp-open in $(U,\tau_R(X))$. Since $f^{-1}(G^C)=(f^{-1}(G))^C$, we have $f^{-1}(G)$ is Ngp-closed in $(U,\tau_R(X))$. Therefore f is Ngp-continuous.

Theorem: 3.4 Every Nano continuous function is Ngp-continuous but not conversely.

Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be Nano continuous on $(U,\tau_R(X))$. Since f is Nano continuous on $(U,\tau_R(X))$, the inverse image of every Nano closed set in $(V,\tau_{R'}(Y))$ is Nano closed in $(U,\tau_R(X))$.But every Nano closed set is Nano generalized pre-closed set. Hence the inverse image of every Nano closed set in $(V,\tau_{R'}(Y))$ is Ngp-closed in $(U,\tau_R(X))$. Thus f is Ngp- continuous.

Remark: 3.5 The converse of the above theorem is not true as seen from the following example.

Example: 3.6 Let U = {a, b, c} = V. Then $\tau_R(X)$ = {Φ, {a, c}, U} with U/R = {{a, c}, {b}} and X = {a, c} and $\tau_{R'}(Y)$ = {Φ, {a, c},{b}, V} with V/R' = {{b},{a ,c}} and Y = {a, b}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ as $f(a)=c$, $f(b)=a$, $f(c)=b$ which is Ngp-continuous. But for the Nano closed set {a, c} in $(V,\tau_{R'}(Y))$, its inverse image $f^{-1}(a,c)=\{a,b\}$ is not Nano closed in $(U,\tau_R(X))$ .Hence f is not Nano continuous.

Theorem: 3.7 Every Nano pre-continuous function is Ngp-continuous but not conversely.

Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be Nano pre-continuous on $(U,\tau_R(X))$. Since f is Nano pre-continuous on $(U,\tau_R(X))$, the inverse image of every Nano open set in $(V,\tau_{R'}(Y))$ is Nano pre-open in $(U,\tau_R(X))$ .But every Nano pre-open set is Nano generalized pre-open set. Hence the inverse image of every Nano open set in $(V,\tau_{R'}(Y))$ is Ngp-open in $(U,\tau_R(X))$. Thus f is Ngp- continuous.

Remark: 3.8 The converse of the above theorem is not true as seen from the following example
.
Example: 3.9 Let U = {a, b, c} =V. Then $\tau_R(X)$ = {Φ, {a}, U} with U/R = {{a}, {b, c}} and X = {a} and $\tau_{R'}(Y)$ = {Φ, {a, c},{b}, V} with V/R' = {{b},{a ,c}} and Y = {a, b}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ as an identity mapping. Then f is Ngp-continuous. But for the Nano open set {b} in $(V,\tau_{R'}(Y))$, its inverse image $f^{-1}(b)=\{b\}$ is not Nano pre-open in $(U,\tau_R(X))$ .Hence f is not Nano pre-continuous.

Theorem: 3.10 If a function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Ng-continuous, then it is Ngp-continuous.

Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be Ng-continuous. Suppose F is any Nano closed set in $(V,\tau_{R'}(Y))$, then the inverse image $f^{-1}(F)$ is Ng-closed in $(U,\tau_R(X))$. Since every Ng-closed set is Ngp-closed, $f^{-1}(F)$ is Ngp-closed in $(U,\tau_R(X))$. Therefore f is Ngp-continuous.

Remark: 3.11 The converse of the above theorem is not true as seen from the following example.

Example: 3.12 Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ as in the example 7.2.10. Then for the Nano closed set {b} in $(V,\tau_{R'}(Y))$, its inverse image $f^{-1}(b)=\{c\}$ is not Ng-closed in $(U,\tau_R(X))$.Hence f is not Ng- continuous.

Theorem: 3.13 A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Ngp-continuous if and only if, $f(Ngp-cl(A))\subseteq Ncl(f(A))$ for every subset A of $(U,\tau_R(X))$.

Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be Ngp-continuous and A⊆U. Then $f(A)\subseteq V$. Hence $Ncl(f(A))$ is Nano closed in V. Since f is Ngp-continuous and $Ncl(f(A))$ is Nano closed in *V*, $f^{-1}(Ncl(f(A)))$ is Ngp-closed in *U*. Since $f(A)\subseteq Ncl(f(A))$, $f^{-1}(f(A))\subseteq f^{-1}(Ncl(f(A)))$, then $A\subseteq f^{-1}(Ncl(f(A)))$.Thus $f^{-1}(Ncl(f(A)))$ is Ngp-closed set containing A. But $Ngp-cl(A)$ is the smallest Ngp-closed set containing A. Hence we have $Ngp-cl(A)\subseteq f^{-1}(Ncl(f(A)))$ which implies $f(Ngp-cl(A))\subseteq Ncl(f(A))$.
Conversely, let $f(Ngp-cl(A))\subseteq Ncl(f(A))$ for every subset A of $(U,\tau_R(X))$. Let F be a Nano closed set in $(V,\tau_{R'}(Y))$.Now $f^{-1}(F)\subseteq U$. Hence, $f(Ngp-cl(f^{-1}(F)))\subseteq(Ncl(f(f^{-1}(F))))=Ncl(F)$.That is $(Ngp-cl(f^{-1}(F)))\subseteq f^{-1}(Ncl(F))=f^{-1}(F)$ as F is Nano closed. But $f^{-1}(F)\subseteq Ngp-cl(f^{-1}(F))$. Therefore $Ngp-cl(f^{-1}(F))=$ $f^{-1}(F)$ which implies that $f^{-1}(F)$ is Ngp-closed in $(U,\tau_R(X))$ for every Nano closed set F in $(V,\tau_{R'}(Y))$. That is f is Ngp-continuous.

Remark: 3.14 Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be Ngp-continuous. Then $f(Ngp-cl(A))$ is not necessarily equal to $Ncl(f(A))$ where $A\subseteq U$.

Example: 3.15 Let U = {a, b, c, d} with $\tau_R(X)$ = {Φ, {a, b, d}, {b, d}, {a}, *U*}.Let *V* = {x, y, z, w} with $\tau_{R'}(Y)$ = {Φ, {x, y, w}, {x, y}, {w}, *V*}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ as $f(a)=y$, $f(b)=x$, $f(c)=z$, $f(d)=w$. Then f is Ngp-continuous. Then $f(Ngp-cl(A))$ ={x, z, w}. But $Ncl(f(A))$= {x, y, z, w}. That is $f(Ngp-cl(A))\neq Ncl(f(A))$ even though f is Ngp-continuous.

Theorem: *3.16* Let $(U,\tau_R(X))$ and $(V,\tau_{R'}(Y))$ be two Nano topological spaces where $X\subseteq U$ and $Y\subseteq V$. Then $\tau_{R'}(Y)=\{V,\phi,L_{R'}(Y),U_{R'}(Y),B_{R'}(Y)\}$ and its basis is given by $B_{R'}=\{V,L_{R'}(Y),B_{R'}(Y)\}$. A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Ngp- continuous if and only if the inverse image of every member of $B_{R'}$ is Ngp-open in U.

Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be Ngp–continuous on $(U,\tau_R(X))$. Let $B\in B_{R'}$.Then B is Nano open in $(V,\tau_{R'}(Y))$. Since f is Ngp–continuous, $f^{-1}(B)$ is Ngp–open in U and $f^{-1}(B)\in\tau_R(X)$. Hence the inverse image of every member of $B_{R'}$ is Ngp–open in U.
Conversely, let the inverse image of every member of $B_{R'}$ be Ngp–open in U. Let G be Nano open in V. Now $G=\bigcup\{B:B\in B_1\}$ where $B_1\subset B_{R'}$. Then $f^{-1}(G)=f^{-1}[\bigcup\{B:B\in B_1\}]=\bigcup\{f^{-1}(B):B\in B_1\}$ where each $f^{-1}(B)$ is Ngp–open in U and their union which is $f^{-1}(G)$ is also Ngp–open in U. By definition of Ngp–continuous function, $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Ngp–continuous on $(U,\tau_R(X))$.

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