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(i, j)-I_{rwg} – CLOSED SETS IN IDEAL BITOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to introduce the concept of (i, j)-regular weakly generalized closed sets, (i, j)-regular weakly generalized open sets and study their basic properties in ideal bitopological spaces.

Key words: *(i, j)-I_{rwg}-closed sets, (i, j)-I_{rwg}-open sets, τ_i-regular open sets and τ_i -regular closed sets.*

1. INTRODUCTION

The concept of bitopological space was introduced by J.C.Kelly [8]. Generalised closed sets with respect to an ideal in bitopological spaces was introduced by T.Noiri, N.Rajesh [9].

In this paper, regular weakly generalized closed and open sets with respect to ideal in bitopological spaces are introduced.

A non-empty collection I of subsets on a topological space (X, τ) is called a topological ideal if it satisfies the following two conditions:

(i) If A$\in$ I and B$\subset$ A implies B$\in$I (heredity)

(ii) If A$\in$ I and B $\in$I, then A$\cup$B $\in$I (finite additivity)

Let (X, τ_1, τ_2, I) or simply X denote an ideal bitopological space. For any subset A$\subseteq$ X, τ_i - int(A) and τ_i-cl(A) denote the interior and closure of a set A with respect to the topology τ_i respectively. The closure and interior of B relative to A with respect to the topology τ_I are written as τ_i – cl_A(B) and τ_i – int_A(B) respectively.

2. PRELIMINARIES

Definition 2.1: ([2], [3], [5], [7], [11]). A set A of a bitopological space (X, τ_1, τ_2) is called

(a) $\tau_i\tau_j$ - semi open if there exists a τ_i -open set U such that U$\subseteq$A$\subseteq$ τ_j -cl(U), i, j = 1,2 and i $\neq$ j.

(b) $\tau_i\tau_j$ - semi closed if X-A is $\tau_i\tau_j$ -semi open. Equivalently, a set A of a bitopological space (X, τ_1, τ_2) is called $\tau_i\tau_j$ - semi closed if there exists a τ_i - closed set F such that τ_j - int (F) $\subseteq$A $\subseteq$ F.

(c) $\tau_i\tau_j$ - regular closed if τ_i - cl[τ_j - int(A)] = A.

(d) $\tau_i\tau_j$ - regular open if τ_i - int[τ_j - cl(A)] = A.

(e) $\tau_i\tau_j$ - regular generalised closed ($\tau_i\tau_j$ -rg closed) in X if τ_j - cl(A)$\subseteq$U whenever A$\subseteq$U and U is $\tau_i\tau_j$ -regular open in X.

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(f) $\tau_i\tau_j$ - regular generalized open ($\tau_i\tau_j$ -rg open) in X if F$\subseteq \tau_j$ - int(A) whenever F$\subseteq$A and F is $\tau_i\tau_j$ -regular closed in X.

(g) $\tau_i\tau_j$- regular generalized star closed ($\tau_1\tau_2$- rg^* closed) in X if and only if τ_2 -rcl(A)$\subseteq$U whenever A$\subseteq$U and U is $\tau_1\tau_2$ -regular open in X.

(h) $\tau_1\tau_2$- regular generalized star open ($\tau_1\tau_2$- rg^* open) in X if and only if its complement is $\tau_1\tau_2$ - regular generalized star closed ($\tau_1\tau_2$ - rg^* closed) in X.

(i) $\tau_1\tau_2$ - generalized star regular closed ($\tau_1\tau_2$ - g^*r closed) in X if and only if τ_2 -rcl(A)$\subseteq$U whenever A$\subseteq$U and U is τ_1 - open in X.

(j) $\tau_1\tau_2$ - generalized star open ($\tau_1\tau_2$- g^*r open) in X if and only if its complement is $\tau_1\tau_2$- generalized star closed ($\tau_1\tau_2$- g^*r closed) in X.

Lemma 2.1: [2] Let a be an τ_i – open set in (X, τ_1 , τ_2) and let U be τ_i τ_j – regular open in A. Then U = A ∩ W for some τ_i τ_j – regular open set W in X, i,j = 1,2 and i ≠ j.

3. (i, j)-I_{rwg}-CLOSED SETS

Definition 3.1: Let (X, τ_1, τ_2, I) be a bitopological space and I be an ideal on X. A subset A of X is said to be (i, j)-regular weakly generalized closed set with respect to an ideal I(shortly (i, j)- I_{rwg}-closed set) if and only if τ_j-cl*(int(A)) ⊆ U whenever A ⊆ U and U is τ_i-regular open in X, i, j = 1, 2 and i ≠ j.

Example 3.2: Let X = {a, b, c}, τ_1 = {∅, X, {b}, {a, b}}, τ_2 = {∅, X, {a},{a,c}}, I = {∅, {b}}. Then, ∅, X, {a}, {a,c}, {b,c}, {a,b} are (i, j)- I_{rwg}-closed sets in (X, τ_1, τ_2, I).

Theorem 3.3: Let (X, τ_1, τ_2, I) be an ideal bitopological space. Then every (i, j)- rg closed set is (i, j)-I_{rwg} -closed in X, i, j = 1,2 and i ≠ j.

Proof: Let A be (i, j) -rg-closed subset of X. Let A⊆ U and U is τ_i -regular open in X, i, j = 1, 2 and i ≠ j. Then τ_j-cl(int(A)) ⊆ τ_j-cl(A) ⊆ U. Hence τ_j-cl(int(A)) – U= ∅ ∈ I. Therefore A is (i, j)-rg closed.

Remark 3.4: The converse of the above theorem is not be true from the following example.

Example 3.5: Let X = {a, b, c}, τ_1={∅, X, {a}, {a,b}}, τ_2 = {∅, X, {b}, {b, c}}, I = {∅, {b},{c},{b,c}}. Then {a} (i, j) -I_{rwg}-closed but not (i, j)- rg closed set in X.

Theorem 3.6: Let A be a subset of an ideal bitopological space (X, τ_1, τ_2, I). If A is (i, j)-I_{rwg}-closed then τ_j-cl*(int(A)) – A does not contain τ_i-regular closed sets such that F ∉ I, i, j = 1, 2 and i ≠ j.

Proof: Suppose that A is (i, j)-I_{rwg}-closed, i, j = 1, 2 and i ≠ j. Let F be an τ_i - regular closed set such that F ⊆ τ_j-cl*(int(A)) – A. Since F ⊆ τ_j-cl*(int(A)) – A, we have F ⊆ [τ_j-cl*(int(A)) – A] ∩ (X – A). Consequently F ⊆ X – A and F ⊆ τ_j -cl*(int(A)). Since F ⊆ X – A, we have A ⊆ X – F. Since F is τ_i - regular closed set, X – F is τ_i-regular open. Since A is (i, j)-I_{rwg}-closed, we have τ_j-cl*(int(A)) – (X – F) = τ_j-cl*(int(A)) ∩ F = F ∈ I. Thus τ_j-cl*(int(A)) – A does not contain τ_i-regular closed sets such that F ∉ I.

Theorem 3.7: The union of two (i, j)-I_{rwg}-closed sets in (X, τ_1, τ_2, I) is also an (i, j)-I_{rwg}-closed set.

Proof: Let A and B be (i, j)-I_{rwg}-closed sets in X, i, j = 1, 2 and i ≠ j. We have to prove that A∪ B is also (i, j)-I_{rwg}-closed. Let A∪ B ⊆ U and U is τ_i-regular open. Since A∪ B ⊆ U, we have A ⊆ U and B ⊆ U. Since A ⊆ U then U is τ_i-regular open, we have τ_j-cl*(int(A)) ⊆ U (since A is (i, j)-I_{rwg}-closed). Similarly B ⊆ U and U is τ_i-regular open, we have τ_j-cl*(int(B)) ⊆ U. Therefore τ_j-cl*(int(A∪ B)) = (τ_j- cl*(int(A)) ∪ (τ_j-cl*(int(B)) ⊆ U. Hence A ∪ B is (i, j)-I_{rwg}-closed set.

Remark 3.8: The intersection of two (i, j)-I_{rwg}-closed sets is not an (i, j)-I_{rwg}- closed set in general as seen from the following example.

Example 3.9: Let X = {a, b, c}, τ_1 = {∅, X, {a}, {a, b}}, τ_j = {∅, X, {b}, {b, c}}, I = {∅, {b},{c}, {b,c}}, A = {a,b}, B = {a, c} are (i, j)- I_{rwg}-closed sets, but A ⋂ B ={a} is not an (i, j)- I_{rwg}-closed set.

Lemma 3.10: Let A be an τ_i-open set in (X, τ_1, τ_2) and let U be τ_i-regular open in A. Then U = A ⋂ W for some τ_i-regular open set W in X, i, j= 1, 2 and i ≠ j.

Lemma 3.11: If A is $\tau_i\tau_j$-open and U is τ_i-regular open in X then U ⋂ A is τ_i- regular open in A, i, j= 1, 2 and i ≠ j.

Lemma 3.12: If A is $\tau_i\tau_j$-open in (X, τ_1, τ_2), then τ_j-cl_A(B) ⊆ A ⋂ τ_j-cl(B) for any subset B of A, i, j = 1, 2 and i ≠ j.

Theorem 3.13: Let I be an ideal in X. Let B ⊆ A where A is τ_i-regular open, τ_j-regular open and (i, j)-I_{rwg}-closed. Then B is (i, j)-I_{rwg}-closed relative to A with respect to an ideal I_A = {F ⊆ A \ F ∈ I} if B is (i, j)-I_{rwg}-closed in X, i, j = 1,2 and i ≠ j.

Proof: Suppose that B is (i, j)-I_{rwg}-closed in X, i, j = 1, 2 and i ≠ j. We have to prove that B is (i, j)-I_{rwg}-closed relative to A. Let B ⊆ U and U is τ_i-regular open in A. Since A is $\tau_i\tau_j$-open in X and U is τ_i-regular open in A, we have U = A ∩ W for some τ_i-regular open set W in X (by Lemma 3.10). Since A is $\tau_i\tau_j$-open in X and W is τ_i-regular open in X, we have U = A ∩ W is τ_i-regular open set in X (by Lemma 3.11). Hence B ⊆ U and U is τ_i-regular open in X. Since B is (i, j)-I_{rwg}- closed in X, τ_j-cl^*(int(B) ⊆ U. Therefore τ_j-cl^*(int(B)) ∩(X− U) ∈ I. Consequently, τ_j - cl^*(int(B) ∩ A ∩ (X − U) ∈ I_A. Since A is τ_i τ_j-open in X, we have τ_j- cl^*(int(B) ∩ A = τ_j-cl_A^* int(B). Hence τ_j-cl_A^* int(B) ⊆ U. Therefore B is (i, j)-I_{rwg}-closed relative to A.

Theorem 3.14: If A is (i, j)-I_{rwg}-closed, and A ⊆ B ⊆ τ_j-cl^*(int(A)) in (X, τ_1, τ_2, I) then B is (i, j)-I_{rwg}-closed, i, j = 1, 2 and i ≠ j.

Proof: Let A and B be subsets such that A ⊆ B ⊆ τ_j-cl^*(int(A)). Suppose that A is (i, j)-I_{rwg}-closed, i, j = 1, 2 and i ≠ j. Let B ⊆ U and U is τ_i-regular open in X. Since A ⊆ B and B ⊆ U, we have A ⊆ U. Hence A ⊆ U and U is τ_i-regular open in X. Since A is (i, j)-I_{rwg}-closed, we have τ_j -cl^*(int(A)) ⊆ U. Since B ⊆ τ_j-cl^*(int(A)), then τ_j-cl^*(int(B))⊆ τ_j-cl^*(int(A)). Henceτ_j-cl^*(int(B))⊆ τ_j-cl^*(int(A))⊆U. Therefore B is (i, j)-I_{rwg}-closed.

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