International Journal of Mathematical Archive-7(8), 2016, 37-44
Available online through www.ijma.info ISSN 2229 – 5046

A NEW NOTION OF GENERALIZED CLOSED SETS IN SOFT TOPOLOGICAL SPACES

S. PIOUS MISSIER*[1], S. JACKSON[2]

[1]Associate Professor, P.G. & Research Department of Mathematics,
V. O. Chidambaram College, Thoothukudi, India-628008.

[2]Research Scholar, P.G. & Research Department of Mathematics,
V. O. Chidambaram College, Thoothukudi, India-628008.

(Received On: 20-07-16; Revised & Accepted On: 09-08-16)

ABSTRACT

In this paper a new class of soft generalized closed sets called Soft JP Closed sets in soft topological spaces is introduced and studied. This new class is defined over an initial universe and with a fixed set of parameters. Some basic properties of this new class of soft sets are investigated. Also this new class of sets compared with some of the existing soft sets to prove its own unique identity. This new class of soft JP-Closed sets contributes to widening the scope of Soft Topological Spaces and its applications.

AMS Mathematics subject classification: *Primary: 06D72, 54A05.*

Key Words: *Soft JP closed set, soft closed set, Soft open, Soft JP open set, Soft topological space.*

1. INTRODUCTION

Any Research work should result in addition to the existing knowledge of a particular concept. Such an effort not only widens the scope of the concept but also encourages others to explore new and newer ideas. Molodtsov (1999) [8] initiated the theory of soft sets as a new mathematical tool for dealing uncertainty, which is completely a new approach for modeling vagueness and uncertainties. Soft set theory has a rich potential for application involving practical problems in Economics, Social Sciences, Medical Sciences etc. Applications of Soft set theory in other disciplines and in real life problems are now catching momentum. Molodtsov [8] successfully applied Soft set theory into several directions, such as Smoothness of Functions, Game theory, Operations Research, Riemann Integration, Perron Integration, Theory of Probability, Theory of Measurement and so on. Shabir and Naz (2011) [10] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. They studied some basic concepts of soft topological spaces also some related concepts such as soft interior, soft closure, soft subspace and soft separation axioms.

Chen.B, (2013) [3] has contributed to soft semi open sets in soft topological spaces. K. Kannan (2012) [6] studied soft generalized closed sets in soft topological spaces along with its properties. A.Kalai Selvi and T.Nandhini (2014) [9] paved a new path way by introducing soft $\hat{g}$ -closed sets in soft topological spaces.

Modern topology depends strongly on the ideas of set theory. Therefore, in this work, we introduce a new soft generalized set called Soft JP Set, and its related properties. This may be another starting point for the new soft mathematical concepts and structures that are based on soft set-theoretic operations.

2. PRELIMINARIES

In this section, we present the basic definitions and results of soft set theory which may be found in earlier studies [7, 8, 10, 12]. Throughout this work, U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A \subseteq E$.

Corresponding Author: S. Pious Missier[1]*
[1]Associate Professor, P.G. & Research Department of Mathematics,
V. O. Chidambaram College, Thoothukudi, India-628008.

Definition 2.1 [8]: A soft set F_A on the universe U is defined by the set of ordered pairs $F_A=\{(x, f_A(x)): x \in E, f_A(x) \in P(U)\}$, where $f_A: E \to P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Here f_A is called an approximate function of the soft set F_A. The value of f_A may be arbitrary, some of them may be empty, and some may have non empty intersection.

Example 2.2 [10]: Suppose there are five cars in the universe. Let U = $\{c_1, c_2, c_3, c_4, c_5\}$ under consideration and that E=$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ stand for the parameters expensive, beautiful, manual gear, cheap, automatic gear, in good repair, in bad repair and costly respectively. In this case to define a soft set means to point out expensive cars, beautiful cars and so on. It means that in the mapping f_A given by "cars, (.)" where (.) to be filled in by one of the given parameters $x_i \in E$.

Let $A \subseteq E$, the soft set F_A that describes the "attractiveness in cars" in the opinion of a buyer may be defined like A=$\{x_2,x_3,x_4,x_5,x_7\}$, $f_A(x_2)=\{c_2,c_3,c_5\}$ $f_A(x_3)=\{c_2,c_4\}$ $f_A(x_4)=\{c_1\}$, $f_A(x_5)=\{U\}$ $f_A(x_7)=\{c_3,c_5\}$. Then collection of the above approximations is called as soft set $F_A=\{(x_2,\{c_2,c_3,c_5\})\ (x_3,\{c_2,c_4\}),(x_4,\{c_1\}),\ (x_5,\{U\})\ (x_7,\{c_3,c_5\})\}$

Definition 2.3 [7]: A soft set (F, A) over X is said to be **Null Soft Set** denoted by F_ϕ if for all e ∈ A, F(e) = φ. A soft set (F,E) over X is said to be an **Absolute Soft Set** denoted by F_X if for all e ∈ A,F(e) = X.

Definition 2.4 [4]: The **Union** of two soft sets(F, A) and (G, B)over X is the soft set (H, C), where C = A ∪B, and for all e ∈ C, H(e) = F(e), if e ∈ A\B, H(e) = G(e) if e ∈ B\A and H(e) = F(e) ∪G(e)if e ∈A∩B and is denoted as (F, A) $\widetilde{U}$ (G, B) = (H, C).

Definition 2.5 [10]: The **Intersection** of two soft sets (F, A) and (G, B) over X is the soft set (H, C), where C = A ∩ B and H(e) = F(e) ∩ G(e)for all e ∈ C and is denoted as (F, A) $\widetilde{\bigcap}$ (G,B) = (H, C).

Definition 2.6 [10]: The **Relative Complement** of (F, A) is denoted by $(F, A)^c$ and is defined by $(F,A)^c = (F^c, A)$ where F^c: A →P(X) is a mapping given by $F^c(e) = X - F(e)$ for all e ∈ A.

Definition 2.7 [10]: The **Difference** (H, E) of two soft sets (F, E) and (G, E) over X, denoted by (F, E) \ (G, E) is defined as H(e) = F(e)\G(e) for all e ∈ E.

Definition 2.8 [8]: Let (F, A) and (G, B) be soft sets over X, we say that (F, A) is a **Soft Subset** of (G, B) if A ⊆B and for all e ∈ A, F(e)and G(e) are identical approximations. We write (F, A) $\widetilde{\subseteq}$ (G, B).

Definition 2.9 [10]: Let τ be a collection of soft sets over X with the fixed set E of parameters. Then τ is called a **Soft Topology** on X if

i. $\widetilde{\Phi}$, $\widetilde{X}$ belongs to τ.
ii. The union of any number of soft sets in τ belongs to τ.
iii. The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called **Soft Topological Spaces** over X.

The members of τ are called **Soft Open** sets in X and complements of them are called **Soft Closed** sets in X.

Definition 2.10[10]: Let (X, τ, E) be a Soft Topological Spaces over X. The **Soft Interior** of (F, E) denoted by Int(F, E) is the union of all soft open subsets of (F, E). Clearly Int(F, E) is the largest soft open set over X which is contained in (F, E).

The **Soft Closure** of (F, E) denoted by Cl (F, E) is the intersection of soft closed sets containing (F, E). Clearly (F, E) is the smallest soft closed set containing (F, E).

i) Int (F, E) = $\widetilde{\bigcup}$ {(O.E): (O, E) is soft open and (O, E) $\widetilde{\subseteq}$ (F, E)}.
ii) Cl (F, E) = $\widetilde{\bigcap}$ {(O. E): (O, E) is soft closed and (F, E) $\widetilde{\subseteq}$ (O, E)}.

Definition 2.11: A Subset of a soft topological space (X, τ, E) is said to be

(1) a **soft Semi-Open** set [3] if (A,E) $\widetilde{\subseteq}$ Cl(int(A,E)) and a **Soft Semi-Closed** set if int(Cl(A,E)) $\widetilde{\subseteq}$ (A,E)).
(2) a **soft Pre-Open** set [1] if (A,E) $\widetilde{\subseteq}$ Int(Cl(A,E)) and a **Soft Pre-Closed** set if Cl (int(A,E) $\widetilde{\subseteq}$ (A,E).
(3) a **soft α-Open** set [1] if (A,E) $\widetilde{\subseteq}$ Int(Cl(int(A,E))) and a **Soft α-Closed** set if Cl(int(Cl(A,E))) $\widetilde{\subseteq}$ (A,E)).
(4) a **soft β-Open** set [2] if (A,E) $\widetilde{\subseteq}$ Cl(Int(Cl(A,E))) and a **Soft β-Closed** set if Int(Cl (int(A,E)) $\widetilde{\subseteq}$ (A,E).

(5) a **Soft generalized Closed** set(briefly **Soft g-Closed**)[6] if Cl(A,E) $\widetilde{\subseteq}$(U,E) whenever (A,E)$\widetilde{\subseteq}$(U,E) and (U,E) is soft Open in (X, τ, E). The complement of a Soft g-Closed set is called a **Soft g-Open** set.

(6) a **Soft Semi-generalized Closed** set(briefly S**oft Sg-Closed**) if SCl(A,E)$\widetilde{\subseteq}$(U,E) whenever (A,E)$\widetilde{\subseteq}$(U,E) and (U,E) is soft semi Open in (X, τ, E). The complement of a Soft Sg-Closed set is called a **Soft** S**g-Open** set.

(7) a **generalized Soft Semi-Closed** set(briefly **gs-Closed**) if SCl(A,E)$\widetilde{\subseteq}$(U,E) whenever (A,E)$\widetilde{\subseteq}$(U,E) and (U,E) is soft Open in (X, τ, E). The complement of a Soft gs-Closed set is called a **Soft gs-Open** set.

(8) a **Soft $\hat{g}$ -Closed**[9] if Cl(A,E)$\widetilde{\subseteq}$(U,E) whenever (A,E)$\widetilde{\subseteq}$(U,E) and (U,E) is soft semi Open in (X, τ, E). The complement of a Soft $\hat{g}$ -Closed is called a **Soft $\hat{g}$ -open** set.

(9) a **Soft alpha-generalized Closed** set (briefly S**oft αg-Closed**) if αCl(A,E)$\widetilde{\subseteq}$(U,E) whenever (A,E)$\widetilde{\subseteq}$(U,E) and (U,E) is soft α Open in (X, τ, E). The complement of a Soft αg-Closed set is called a **Soft** α**g-Open** set.

(10) a **Soft generalized alpha Closed** set (briefly S**oft gα-Closed**) if αCl(A,E)$\widetilde{\subseteq}$(U ,E) whenever (A,E)$\widetilde{\subseteq}$(U,E) and (U,E) is soft Open in (X, τ, E). The complement of a Soft gα-Closed set is called a **Soft g**α**-Open** set.

(11) a **Soft generalized pre Closed** set (briefly S**oft gp-Closed**) [1] if pCl(A,E) $\widetilde{\subseteq}$(U,E) whenever (A,E)$\widetilde{\subseteq}$(U,E) and (U,E) is soft Open in (X, τ, E). The complement of a Soft gp-Closed set is called a **Soft g**p**-Open** set.

(12) a **Soft generalized pre regular Closed** set (briefly S**oft gpr-Closed**) [5] if pCl(A,E)$\widetilde{\subseteq}$(U,E) whenever (A,E) $\widetilde{\subseteq}$(U,E) and (U,E) is soft regular Open in (X, τ, E). The complement of a Soft **gpr**-Closed set is called a **Soft gpr-Open** set.

III: SOFT JP CLOSED SET

Definition 3.1: Let (X, τ, E) be a soft topological space. A Soft set(F,E) is called **soft JP closed set** if Scl(F,E)$\widetilde{\subseteq}$ Int(U,E) whenever (F,E)$\widetilde{\subseteq}$(U,E) and (U,E) is soft $\hat{g}$ - open. The set of all soft JP closed sets is denoted by SJPC(X).

Example 3.2: Let X={x_1,x_2,x_3} E={e_1,e_2} and τ={$\widetilde{\Phi}$, $\widetilde{X}$,(F_1, E),(F_2,E), (F_3,E), (F_4,E), (F_5,E), (F_6,E)} where (F_1, E), (F_2,E), (F_3,E), (F_4,E), (F_5,E), (F_6,E) defined as follows

$F_1(e_1)$ = {x_2}, $F_1(e_2)$ = {x_1}, $F_2(e_1)$ = {x_2}, $F_2(e_2)$ = {x_1, x_3},$F_3(e_1)$ = {x_1, x_2}, $F_3(e_2)$=X, $F_4(e_1)$ = {x_2, x_3},$F_4(e_2)$= {x_1}, $F_5(e_1)$ = {x_2, x_3}, $F_5(e_2)$ = {x_1, x_3}, $F_6(e_1)$ =X, $F_6(e_2)$ = {x_1, x_2}. Then τ defines a soft topology on X. Define a soft set (G, E) such that G(e_1) = {x_1, x_3} G(e_2)= {x_2}.Then (G,E) is a soft JP closed set in (X, τ, E).

Lemma 3.3[11]: A Set is $\hat{g}$ open if and only if F$\subseteq$Int(A) when ever F is semi closed and F$\subseteq$A.

Proposition 3.4: Every soft semi closed set is soft JP closed set but not conversely.

Proof: let (F,E) be a soft semi closed set in the soft topological space (X, τ, E) and (U,E) be a soft $\hat{g}$ open set such that (F,E)$\widetilde{\subseteq}$(U,E). Then by Lemma 3.3, (F, E) =Scl(F,E)$\widetilde{\subseteq}$Int(U,E).Then (F,E) is soft JP closed set.

Example 3.5: In the soft topological space(X, τ, E), X={x_1, x_2} E={e_1,e_2} and τ={$\widetilde{\Phi}$, $\widetilde{X}$, (F_1, E),(F_2,E)} where (F_1, E), (F_2,E) defined as follows

$F_1(e_1)$ = ϕ $F_1(e_2)$ = {x_1}

$F_2(e_1)$ = {x_1} $F_2(e_2)$ = {x_1}.

Then the soft set (H, E) defined by H(e_1) = ϕ, H(e_2) = X is a soft JP closed set but not soft semi closed in (X, τ, E).

Proposition 3.6: Every soft closed set is soft JP closed set but not conversely.

Proof: let (F, E) be a soft closed set in the soft topological space (X, τ, E). Then it is a soft semi closed set. Then by Proposition 3.4, (F, E) is soft JP closed set.

Example 3.7: In the soft topological space (X, τ, E) defined in example 3.5, the soft set (G, E) such that G(e_1) = {x_1} G(e_2)= X is a soft JP closed set but not soft closed in (X, τ, E).

Proposition 3.8: Every soft α closed set is soft JP closed set but not conversely.

Proof: let (F, E) be a soft α closed set in the soft topological space (X, τ, E). Then it is a soft semi closed set. Then by Proposition 3.4, (F, E) is soft JP closed set.

Example 3.9: In the soft topological space (X, τ, E) defined in example 3.5, the soft set (G, E) such that $G(e_1) = \{x_2\}$ $G(e_2)= X$ is a soft JP closed set but not soft α closed in (X, τ, E).

Proposition 3.10: Every soft JP closed set is soft gs closed set but not conversely.

Proof: Let (F, E) be a soft JP closed set in the soft topological space (X, τ, E). We know that every soft open is soft $\hat{g}$ open. Therefore (F, E) is soft gs closed set.

Example 3.11: In the soft topological space (X, τ, E) defined in example 3.5, the soft set (G, E) such that $G(e_1) = \{x_2\}$, $G(e_2) = \{x_1\}$ is a soft gs closed set but not soft JP closed in (X, τ, E).

Example 3.12: The following examples show that soft g closedness and soft JP closedness are independent.

In the soft topological space (X, τ, E) defined in example 3.2, the soft set (G, E) such that $G(e_1) = \{x_1, x_3\}$ $G(e_2)=\phi$ is a soft JP closed set but not soft g closed in (X, τ, E). Also the soft set (H, E) such that $H(e_1) = \{x_3\}$ $H(e_2) = X$ is a soft g closed set but not soft JP closed in (X, τ, E).

Example 3.13: The following examples show that soft $\hat{g}$ closedness and soft JP closedness are independent.

In the soft topological space (X, τ, E), $X=\{a, b\}$ $E=\{e_1,e_2\}$ and $\tau=\{\tilde{\Phi}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ where (F_1, E), (F_2, E), (F_3, E) defined as follows$F_1(e_1) = \phi$, $F_1(e_2) = \{a\}$, $F_2(e_1) = \{b\}$, $F_2(e_2) = \phi$, $F_3(e_1) = \{b\}$, $F_3(e_2) = \{a\}$.

The soft set (G, E) such that $G(e_1) = \{b\}$, $G(e_2) = \phi$ is soft JP closed but not soft $\hat{g}$ closed in (X, τ, E). Also the soft set (H, E) such that $H(e_1) = \{b\}$, $H(e_2) = \{a\}$ is soft $\hat{g}$ closed but not soft JP closed in (X, τ, E).

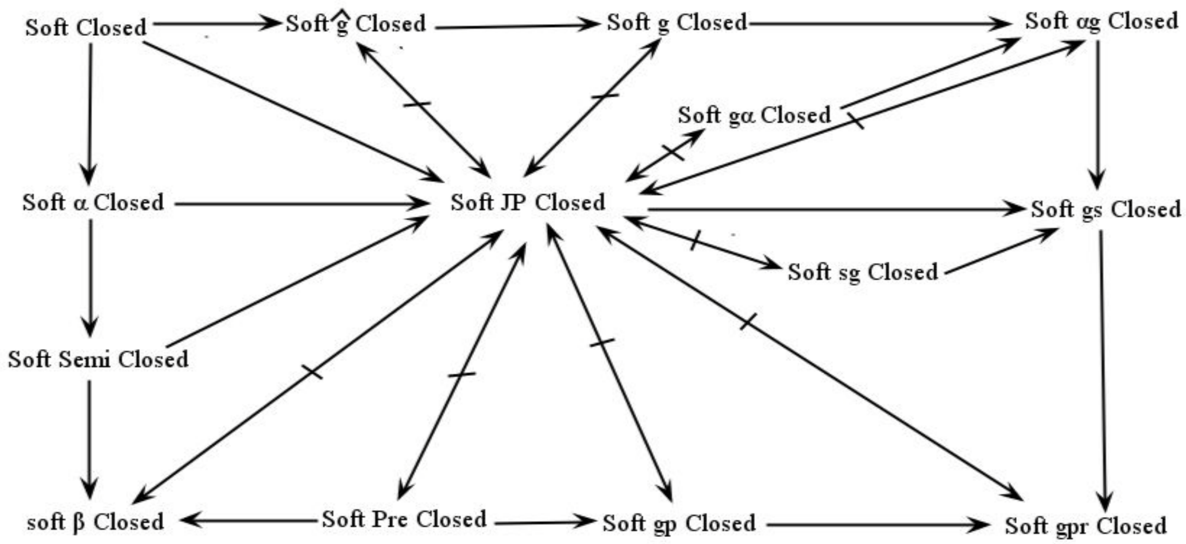
Example 3.14: The following examples show that soft sg closedness and soft JP closedness are independent.

In the soft topological space (X, τ, E) defined in example 3.5, the soft set (G, E) such that $G(e_1) = \phi$, $G(e_2) = \{x_1, x_2\}$ is a soft JP closed set but not soft sg closed in (X, τ, E). Also in the soft topological space (X, τ, E) defined in example 3.2, the soft set (H, E) such that $H(e_1) = \{x_3\}$ $H(e_2) = \{x_1\}$ is a soft g closed set but not soft JP closed in (X, τ, E).

Example 3.15: The following examples show that soft pre closedness, soft αg closedness, soft β closedness, soft gα closedness, soft gp closedness, soft gpr closedness and soft JP closedness are independent.

In the soft topological space (X, τ, E) defined in example 3.13, the soft set (G, E) such that $G(e_1) = \{a\}$, $G(e_2) = \phi$ is a soft pre closed (resp. soft αg closed, soft β closed, soft gα closed, soft gp closed and soft gpr closed) but not soft JP closed. Also the soft set (H, E) such that $H(e_1) = \{b\}$ $H(e_2) = \phi$ is a soft JP closed set but not soft pre closed (resp. soft αg closed, soft gα closed, soft gp closed and soft gpr closed). Also In the soft topological space (X, τ, E) defined in example 3.5, the soft set (I, E) such that $I(e_1) = \phi$, $I(e_2) = X$ is a soft JP closed set but not soft β closed.

The following diagram shows the above discussions.



Example 3.16: The Soft intersection of two soft JP closed sets need not to be a soft JP closed set.

In the soft topological space (X, τ, E) defined in example 3.13, the soft sets (G, E), (H, E) such that $G(e_1) = \{a\}$, $G(e_2) = \{a\}$ and $H(e_1) = \{a\}$, $H(e_2) = \{b\}$ are soft JP closed sets but their intersection $(G, E) \tilde{\bigcap} (H, E) = (I, E)$ where $I(e_1) = \{a\}$, $I(e_2) = \phi$ is not a soft JP closed set in (X, τ, E).

Example 3.17: The Soft Union of two soft JP closed sets need not to be a soft JP closed set.

In the soft topological space (X, τ, E) defined in example 3.2, the soft sets (G, E), (H, E) such that $G(e_1) = \{x_1, x_3\}$ $G(e_2) = \phi$, $H(e_1) = \phi$ $H(e_2) = \{x_3\}$ are soft JP closed sets but the union $(G, E) \tilde{\bigcup} (H, E) = (A, E)$ where $A(e_1) = \{x_1, x_3\}$, $A(e_2) = \{x_3\}$ is not a soft JP closed set in (X, τ, E).

Definition 3.18[3]: Let (X, τ, E) be a soft topological space and (A, E) be a soft subset over X. Then the set of all soft semi limit points is called soft semi derived set of (A,E) and is denoted by $D_S[A,E]$.

Theorem 3.19: If $D[A,E] \tilde{\subseteq} D_S[A,E]$ for each soft subset(A,E) in the soft topological space (X, τ, E), Then the union of two soft JP closed sets is a soft JP closed set.

Proof: Let (A,E) and (B,E) be two soft JP closed sets in the soft topological space(X, τ, E) and (U,E) be a soft $\hat{g}$ closed set such that $(A,E) \tilde{\bigcup} (B,E) \tilde{\subseteq} (U,E)$.Then $Scl(A,E) \tilde{\subseteq} Int(U,E)$ and $Scl(B,E) \tilde{\subseteq} Int(U,E)$. Since for each subset (A, E), $D[A,E] \tilde{\subseteq} D_S[A,E]$. Then Cl(A,E)=Scl(A,E) and Cl(B,E) = Scl(B,E).Therefore $Scl((A,E) \tilde{\bigcup} (B,E)) \tilde{\subseteq} Cl((A,E) \tilde{\bigcup} (B,E)) = Cl(A,E) \tilde{\bigcup} Cl(B,E) = Scl(A,E) \tilde{\bigcup} Scl(B,E) \tilde{\subseteq} Int(U,E)$.Hence $(A,E) \tilde{\bigcup} (B,E)$ is a soft JP closed set.

Theorem 3.20: Let (F, E) be a soft JP closed subset of the soft topological space (X, τ, E). Then Scl(F,E)\(F,E) contains no non empty soft closed set in (X, τ, E) but not conversely .

Proof: Let (A, E) be a nonempty soft closed subset of SCl(F,E)\ (F,E). Now,$(A,E) \tilde{\subseteq} Scl(F,E)\backslash(F,E)$ and $(F,E) \tilde{\subseteq} (A,E)^c$ where (F,E) be a soft JP closed set and $(A,E)^C$ is soft open .Thus $Scl(F,E) \tilde{\subseteq} Int(A,E)^c = Cl\ (F,E)^c$. then$(A,E) \tilde{\subseteq} Cl(F,E) \tilde{\subseteq} SCl(F,E)^C$. but $(A,E) \tilde{\subseteq} Scl(F,E)$. Therefore $(A, E) \subseteq Scl(F,E)^C \bigcap Scl(F,E)$. i.e) (A, E) = ϕ. Hence the proof.

Example 3.21: In the soft topological space (X, τ, E) defined in example 3.13, the soft set (G,E) such that $G(e_1) = \{a\}$, $G(e_2) = \phi$, then Scl(G,E)\(G,E) does not contain any non empty closed set but it is not soft JP closed.

Theorem 3.22: Let (F, E) be a soft JP closed subset of the soft topological space (X, τ, E). Then Scl(F,E)\(F,E) contains no non empty soft $\hat{g}$ closed set in (X, τ, E) but not conversely.

Proof: Let (A, E) be a nonempty soft $\hat{g}$ closed subset of Scl(F,E)\ (F,E). Now, $(A,E) \tilde{\subseteq} Scl(F,E)\backslash(F,E)$ and $(F,E) \tilde{\subseteq} (A,E)^c$ where (F,E) be a soft JP closed set and $(A,E)^C$ is soft $\hat{g}$ open. Thus $Scl(F,E) \tilde{\subseteq} Int(A,E)^c = Cl\ (F,E)^c$. then$(A,E) \tilde{\subseteq} Cl(F,E) \tilde{\subseteq} Scl(F,E)^C$. but $(A,E) \tilde{\subseteq} Scl(F,E)$. Therefore $(A, E) \subseteq Scl(F,E)^C \bigcap Scl(F,E)$. i.e) (A, E) = ϕ. Hence the proof.

Example 3.23: In the soft topological space (X, τ, E) defined in example 3.13 , the soft set (G,E) such that $G(e_1) = \phi$, $G(e_2) = \{a\}$, then Scl(G,E)\(G,E) does not contain any nonempty $\hat{g}$ closed set but it is not soft JP closed.

Theorem 3.24: In a soft topological space (X, τ, E) each singleton set (A, E) is either soft $\hat{g}$ closed in X or it's complement is soft JP closed set.

Proof: If (A, E) is not soft $\hat{g}$ closed then the only soft $\hat{g}$ open set containing $(A, E)^C$ is X. Then $Scl((A,E)^C) \tilde{\subseteq} X$. therefore $Scl((A,E)^C)$ is soft JP closed .Hence the proof.

Theorem 3.25: In a soft topological space (X, τ, E), if (A, E) is both soft $\hat{g}$ open and soft JP closed then it is soft semi closed.

Proof: Let $(A, E) \tilde{\subseteq} (A, E)$ and (A,E) is soft $\hat{g}$ open. (A, E) is soft JP closed then $Scl(A,E) \tilde{\subseteq} int\ (A,E) \tilde{\subseteq} (A,E)$. Then Scl(A, E) = (A, E).Then (A,E) is soft semi closed.

Theorem 3.26: Let (A,E) and (B,E) be two soft subsets of a soft topological space (X, τ, E), If (A,E) is soft JP closed set and (A,E) $\tilde{\subseteq}$ (B,E) $\tilde{\subseteq}$ Scl((A,E) Then (B,E) is Soft JP Closed set.

Proof: Let (U, E) be a soft $\hat{g}$ open set in (X, τ, E) and (B, E) $\tilde{\subseteq}$ (U,E). Since (A,E) $\tilde{\subseteq}$ (U,E) and (A,E) is soft JP closed set Scl(A,E) $\tilde{\subseteq}$ Int(U,E).Now, Scl(B,E) $\tilde{\subseteq}$ Scl(Scl(A,E))= Scl(A,E) $\tilde{\subseteq}$ Int(U,E). Hence (B, E) is Soft JP Closed set.

IV. SOFT JP OPEN SET

Definition 4.1: Let (X, τ, E) be a soft topological space. A Soft set (F, E) is called **soft JP Open** set if it's complement is Soft JP closed set. The collection of all soft JP open sets is denoted by SJPO(X).

Example 4.2: In the soft topological space (X, τ, E), X={x_1,x_2} E={e_1,e_2} and τ={ $\tilde{\Phi}$, $\tilde{X}$, (F_1, E), (F_2, E)} where (F_1, E), (F_2,E) defined as follows F_1 (e_1) = {x_1}, F_1 (e_2) = {x_1}, F_2 (e_1) = ϕ, F_2 (e_2) = {x_1}. Then the soft set (H, E) defined by H (e_1) = ϕ, H (e_2) ={x_3} is a soft JP open set.

Proposition 4.3:
1. Every soft Open set is soft JP Open set.
2. Every soft semi Open set is soft JP Open set.
3. Every soft α Open set is soft JP Open set.
4. Every soft JP Open set is soft gs Open set.

Proof: These are obvious from propositions 3.4, 3.6, 3.8, 3.10.

Example 4.4: The converse of the above proposition 4.3 is not true and it can be seen from the following examples.

Let X={x_1,x_2,x_3} E={e_1,e_2} and τ={ $\tilde{\Phi}$, $\tilde{X}$, (F_1, E), (F_2, E),(F_3, E), (F_4, E), (F_5, E), (F_6, E)} where (F_1, E),(F_2, E), (F_3,E), (F_4,E), (F_5,E), (F_6, E) defined as follows, F_1(e_1) = X, F_1 (e_2) = {x_1, x_2}, F_2 (e_1) = {x_2}, F_2 (e_2) = {x_1, x_3}, F_3 (e_1) = {x_2}, F_3 (e_2) = {x_1}, F_4 (e_1) = {x_1, x_2}, F_4 (e_2) = X, F_5 (e_1) = {x_2, x_3}, F_5 (e_2) = {x_1}, F_6 (e_1) = {x_2, x_3}, F_6 (e_2) = {x_1, x_3}.Then τ defines a soft topology on X.
1. The soft set (G, E) defined by that G (e_1) = {x_2, x_3}, G(e_2) = X is a soft JP open set but not soft open.
2. The soft set (H, E) defined by that H (e_1) = X, H(e_2) = {x_1,x_3} is a soft JP open set but not soft semi open (resp. soft α open).
3. The soft set (I, E) defined by that I (e_1) = {x_1, x_2}, I (e_2) = ϕ is a soft gs open set but not soft JP open.

Example 4.5: The following examples show that soft g Openness and soft JP Openness are independent.

In the soft topological space (X, τ, E) defined in the example, the soft set (G, E) such that G (e_1) = {x_2} G(e_2)=X is a soft JP open set but not soft g open in (X, τ, E). Also the soft set (H, E) such that H(e_1)= {x_1, x_2} H(e_2) = ϕ is a soft g open set but not soft JP open in (X, τ, E).

Example 4.6: The following examples show that soft sg Openness and soft JP Openness are independent.

In the soft topological space (X, τ, E) defined in example 4.2, the soft set (G, E) such that G (e_1) =X, G(e_2)= {x_3} is a soft JP open set but not soft sg open in (X, τ, E). Also in the soft topological space (X, τ, E) defined in example 4.4, the soft set (H, E) such that H(e_1) {x_1,x_2} H(e_2) = {x_2, x_3} is a soft g open set but not soft JP open in (X, τ, E). The following diagram shows the above discussions.

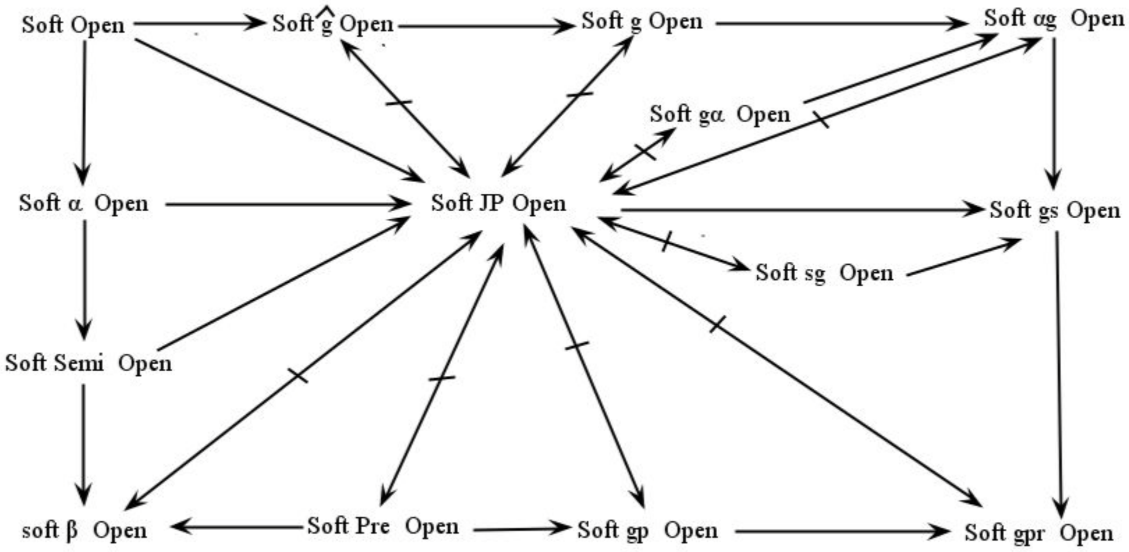
Example 4.7: The following examples show that soft $\hat{g}$ Openness and soft JP Openness are independent.

In the soft topological space (X, τ, E) defined in example 3.10, The soft set (G, E) such that G(e_1) = {a},G(e_2) = X is soft JP open but not soft $\hat{g}$ open in (X, τ, E). Also the soft set (H, E) such that H(e_1) = {a}, H(e_2) = {b} is soft $\hat{g}$ open but not soft JP open in (X, τ, E).

Example 4.8: The following examples show that soft pre openness, soft αg openness, soft β openness, soft gα openness, soft gp openness, soft gpr openness and soft JP openness are independent.

In the soft topological space (X, τ, E) defined in example 3.13, the soft set (G, E) such that G (e_1) = {x_2, x_3}, G(e_2) = X is a soft pre open (resp. soft αg open, soft β open, soft gα open, soft gp open and soft gpr open) but not soft JP open. Also the soft set (H, E) such that H(e_1)= {x_1, x_3}, H(e_2)= X is a soft JP open set but not soft pre open (resp. soft αg open, soft gα open, soft gp open and soft gpr open). In the soft topological space (X, τ, E) defined in example 3.5, the soft set (J, E) such that J(e_1)= X, J(e_2)= ϕ is a soft JP open set but not soft β open.

The following diagram shows the above discussions.



Example 4.9: The soft Union of two soft JP open sets is need not to be a soft JP open set.

In the soft topological space (X, τ, E) defined in example 3.10, the soft sets (G, E), (H, E) such that G (e_1) = {b}, G(e_2)={b} and H (e_1) = {b}, H(e_2) ={a} are soft JP closed sets but their union (G, E) $\tilde{\bigcup}$ (H, E) = (I, E) where I (e_1) = {b}, I(e_2) =X is not a soft JP open set in (X, τ, E).

Example 4.10: The soft intersection of two soft JP open sets is need not to be a soft JP open set.

In the soft topological space (X, τ, E) defined in example 3.2, the soft sets (G, E), (H, E) such that G (e_1) = {x_2} G(e_2) = X, H(e_1) =X, H(e_2) = {x_1, x_2} are soft JP open sets but the intersection (G,E) $\tilde{\bigcap}$ (H, E) = (J, E) where J(e_1) = {x_2}, A(e_2) = {x_1, x_2} is not a soft JP open set in (X, τ, E).

Theorem 4.11: Let (A, E) and (B, E) be two soft subsets of a soft topological space (X, τ, E), If (A, E) is soft JP Open set and SInt(A, E) $\tilde{\subseteq}$ (B, E) $\tilde{\subseteq}$ (A, E) Then (B, E) is Soft JP Open set.

Proof: Suppose SInt(A, E) $\tilde{\subseteq}$ (B, E) $\tilde{\subseteq}$ (A, E) and (A, E) is soft JP open set then $(A, E)^C \tilde{\subseteq} (B, E)^C \tilde{\subseteq}$ Scl($(A,E)^C$. Since $(A, E)^C$ is soft JP closed by theorem 3.26, $(B, E)^C$ is soft JP closed. Hence (B, E) is soft JP open.

V. ACKNOWLEDGEMENT

The first author is thankful to University Grants Commission (UGC) New Delhi, for sponsoring this under grants of Major research project-MRP-Math-Major-2013-30929.F.NO 43-433/2014(SR) dt 11.09.2015.

The second author is thankful to University Grants Commission (UGC) New Delhi, India for sponsoring this under Maulana Azad National Fellowship MANF-2013-14-CHR-TAM-25711 dated 06.02.2014.

VI. REFERENCES

1. Arokia rani.I and Albinaa T., A Soft generalized pre closed sets and space proceedings of ICMSCA (2014) 138-187.
2. Arokia rani.I and Lancy A.A., soft gβ closed sets and soft gsβ closed sets in soft topological spaces, International journal of mathematical archive vol4, no 2(2013), 17-23.
3. Bin chen., “Soft Semi-Open Sets And Related Properties In Soft Topological Spaces”, Appl. math. Inf. Sci., Vol. 7, pp. 287-294, 2013.
4. Cagman, N., Karatas, S., and Enginoglu, S., “Soft Topology”, Comput. Math. Appl., Vol. 62, pp. 351-358, 2011.
5. Guzel Z.E., Yskel.S., Tozlu.N., “On Soft generalized preregular closed and open sets in soft topological spaces” Applied Mathemaitical Sciences, 8(2014), 7875-7884.
6. Kannan, K., “Soft Generalized closed sets in Soft Topological Spaces”, Journal of theoretical and applied information technology, vol 37, pp.17-20, 2012.
7. Maji, P.K., Biwas, R., and Roy, R., “Soft Set Theory.” Comput. Math. Appl., Vol. 45, pp. 555-562, 2003.

8. Molodtsov, D., "Soft Set Theory First Results." Compu. Math. Appl., Vol. 37, pp. 19-31, 1999.
9. Nandhini.T., Kalaiselvi.A., "Soft $\hat{g}$ closed sets in Soft Topological Spaces", International Journal of Innovative Research in Science, Engineering and Technology, Vol 3, issue 7,pp.14595-14600, 2014.
10. Shabir, M., and Naz, M., "On Soft topological spaces." Comput. Math. Appl., Vol. 61, pp. 1786-1799, 2011.
11. Veerakumar M.K.R.S., "On $\hat{g}$ closed sets and $\hat{g}$ LC functions", Indian.J.Math 43(2) (2001), 231-247.
12. Zorlutuna, I., Akdag, M., Min, W.K., and Atmaca, S., "Examples On Soft Topological Spaces." Annals of Fuzzy Mathematics and Informatics, 2012.

Source of support: University Grants Commission (UGC) New Delhi, India.
Conflict of interest: None Declared