# EDGE TRIMAGIC AND SUPER EDGE TRIMAGIC TOTAL LABELING OF SOME GRAPHS

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### ABSTRACT

*$A$n Edge trimagic total labeling of a graph $G(V, E)$ with $p$ vertices and $q$ edges is a bijection $f : V \cup E \to \{1, 2, 3 \ldots p + q\}$ such that for every edge $uv$ in $E$, $f(u) + f(uv) + f(v)$ is either $\lambda_1$ or $\lambda_2$ or $\lambda_3$. An edge trimagic total graph is called a super edge trimagic total if $f(v) = \{1, 2, 3, \ldots p\}$. In this paper, we prove that the graph Double fan $P_n + 2K_1$ is edge trimagic total labeling and the graphs $Z\text{-}P_n$, $B_{n,n}^2$ are super edge trimagic total labeling.*

***Keywords:*** *Function, Edge trimagic, Super edge trimagic.*

## 1. INTRODUCTION

A graph labeling is an assignment of integer to the vertices or edges or both subject to certain conditions. All graphs considered here are finite, simple and undirected. The useful survey on graph labelings by J. A. Gallian (2015) can be found in [1].In 2013 C.Jayasekaran, M.Ragees and C.Devaraj [2] introduces the edge trimagic total labeling of graphs and also C.Jayasekaran and M.Ragees proved edge trimagic and super edge trimagic total labeling[3][5][6]. An edge trimagic total labeling of a (p, q) graph G is a bijection f : V(G) ∪ E(G) ⟶ {1, 2, 3,…, p+q} such that for each edge uv ϵ E, f(u)+f(uv)+f(v) is equal to any of the distinct constants $k_1$ or $k_2$ or $k_3$. A graph G is said to be edge trimagic total if it admits an edge trimagic total labeling. An edge trimagic total labeling is called super edge trimagic total labeling if G has additional property that the vertices are labeled with the smallest positive integers. A.Nellaimurugan and G.Esther [4] proved that double fan $P_n+2K_1$ and Z-$P_n$ are Mean cordial labeling. S.K.Vaidya and N.H shah proved that splitting graph of star $K_{1,n}$ and $B_{n,n}^2$ are graceful and odd graceful labeling [7]. In this paper, we prove that the graph double fan $P_n+2K_1$ is edge trimagic total labeling and the graph Z-$P_n$, $B_{n,n}^2$ are super edge trimagic total labeling.

**Definition 1.1:** An edge trimagic total labeling of a (p, q) graph G is a bijectionf : V ∪ E⟶ {1,2,3,…,p+q} such that for each edge xy ∈ E(G), the value of f(x)+f(xy)+f(y) ) is equal to any of the distinct costants $k_1$ or $k_2$ or $k_3$. A graph G is said to be an edge trimagic total if it admits an edge trimagic total labeling.An edge trimagic total labeling of a graph is called super edge trimagic if $f(v) = \{1, 2,…,p\}$. An edge trimagic total labeling of graph is called a superior edge trimagic total labeling if $f(E) = \{1, 2,…q\}$.

**Definition 1.2:** The join $G_1 + G_2$ of $G_1$ and $G_2$ consist of $G_1 \cup G_2$ and all the lines joining $v_1$ with $v_2$ as vertex set $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and edge set $E(G_1 \cup G_2) = E(G_1) \cup E(G_2) \cup [uv; u \in V(G_1)$ and $v \in V(G_2)]$. The fan $P_n+2K_1$ is called the double fan.

**Definition 1.3:** For a simple connected graph $G$ the square of graph $G$ is denoted by $G^2$ and defined as the graph with the same vertex set as of $G$ and two vertices are adjacent in $G^2$ if they are at a distance 1 or 2 apart in $G$.

**Definition 1.4:** In a pair of path $P_n$ is a $i^{th}$ vertex of a path $P_1$ is joined with $(i + 1)^{th}$ vertex of a path $P_2$. It is denoted by Z-$P_n$.

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## 2. EDGE TRIMAGIC TOTAL LABELING

**Theorem 2.1:** The double fan $P_n + 2K_1$ has edge trimagic total labeling.

**Proof:** Let $w_1, w_2, \ldots, w_n$ be the vertices of path $P_n$. Let G be a graph $P_n + 2K_1$.

Let $V(G) = \{u, v, w_i / 1 \leq i \leq n\}$ and $E(G) = \{w_i w_{i+1}, uw_i, vw_i / 1 \leq i \leq n\}$.

The order of $G$ is $p = n + 2$ and size $q = 3n - 1$

Let us define the function $f: V \cup E \rightarrow \{1, 2, 3, \ldots, 3n + 2\}$ by

**Case-1: n is odd**

$$f(u) = 2n+1;$$

$$f(v) = 2n+2;$$

$$f(w_i) = \begin{cases} \frac{i+1}{2}, & i \text{ is odd} \\ \frac{n+i+1}{2}, & i \text{ is even} \end{cases}$$

$$f(w_i u) = \begin{cases} 2n - \frac{i+1}{2} + 1, & i \text{ is odd} \\ 2n - \frac{n+i+1}{2} + 1, & i \text{ is even} \end{cases}$$

$$f(w_i w_{i+1}) = 4n+2-i \text{ for all } 1 \leq i \leq n$$

$$f(w_i v) = \begin{cases} 3n + 3 - \frac{i+1}{2}, & i \text{ is odd} \\ 3n + 3 - \frac{n+i+1}{2}, & i \text{ is even} \end{cases}$$

Now we prove that the double fan $P_n + 2K_1$ has edge trimagic total labeling.

For the edges $w_i w_{i+1}, 1 \leq i \leq n$

For odd $i$,
$$f(w_i) + f(w_i w_{i+1}) + f(w_{i+1}) = \frac{i+1}{2} + 4n+2-i + \frac{n+i+1+1}{2} = \frac{9n+7}{2} = \lambda_1$$

For even $i$,
$$f(w_i) + f(w_i w_{i+1}) + f(w_{i+1}) = \frac{n+i+1}{2} + 4n+2-i + \frac{i+1+1}{2} = \frac{9n+7}{2} = \lambda_1.$$

For the edges $w_i u, 1 \leq i \leq n$

For odd $i$,
$$f(w_i) + f(w_i u) + f(u) = \frac{i+1}{2} + 2n - \frac{i+1}{2} + 1 + 2n+1 = 4n+2 = \lambda_2$$

For even $i$,
$$f(w_i) + f(w_i u) + f(u) = \frac{n+i+1}{2} + 2n - \frac{n+i+1}{2} + 1 + 2n+1 = 4n+2 = \lambda_2$$

For the edges $w_i v, 1 \leq i \leq n$

For odd $i$,
$$f(w_i) + f(w_i v) + f(v) = \frac{i+1}{2} + 3n+3 - \frac{i+1}{2} + 2n+2 = 5n+5 = \lambda_3$$

For even $i$,
$$f(w_i) + f(w_i v) + f(v) = \frac{n+i+1}{2} + 3n+3 - \frac{n+i+1}{2} + 2n+2 = 5n+5 = \lambda_3$$

Hence for each $uv \in E$, f(u) + f(uv) + f(v) has any one of the magic constants $\lambda_1 = \frac{9n+7}{2}$, $\lambda_2 = 4n+2$, $\lambda_3 = 5n+5$

Hence the double fan $P_n + 2K_1$ has edge trimagic total labeling.

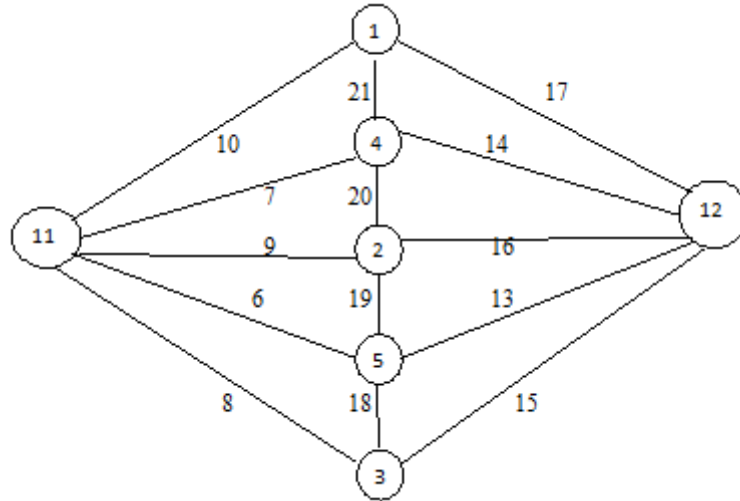**Example 2.2:** The double fan $P_5 + 2K_1$ given in figure is edge trimagic total labeling.



**Figure 1:** $P_5 + 2K_1$ with $\lambda_1 = 26$, $\lambda_2 = 22$, $\lambda_3 = 30$

**Case-1: n is even**

$f(u) = 2n+1;$

$f(v) = 3n+2;$

$$f(w_i) = \begin{cases} \frac{i+1}{2}, & i \text{ is odd} \\ \frac{n+i}{2}, & i \text{ is even} \end{cases}$$

$$f(w_i u) = \begin{cases} 2n - \frac{i+1}{2} + 1, & i \text{ is odd} \\ 2n - \frac{n+i}{2} + 1, & i \text{ is even} \end{cases}$$

$f(w_i w_{i+1}) = 4n+2-i$ for all $1 \le i \le n$

$$f(w_i v) = \begin{cases} 3n + 2 - \frac{i+1}{2}, & i \text{ is odd} \\ 3n + 2 - \frac{n+i}{2}, & i \text{ is even} \end{cases}$$

Now we prove that the double fan $P_n + 2K_1$ yields edge trimagic total labeling.

For the edges $w_i w_{i+1}, 1 \le i \le n$

For odd $i$,
$$f(w_i) + f(w_i w_{i+1}) + f(w_{i+1}) = \frac{i+1}{2} + 4n+2-i + \frac{n+i+1}{2} = \frac{9n+6}{2} = \lambda_1$$

For even $i$,
$$f(w_i) + f(w_i w_{i+1}) + f(w_{i+1}) = \frac{n+i}{2} + 4n+2-i + \frac{i+1+1}{2} = \frac{9n+6}{2} = \lambda_1$$

For the edges $w_i u, 1 \le i \le n$

For odd $i$,
$$f(w_i) + f(w_i u) + f(u) = \frac{i+1}{2} + 2n - \frac{i+1}{2} + 1 + 2n+1 = 4n+2 = \lambda_2$$

For even $i$,

$$f(w_i)+f(w_iu)+f(u) = \frac{n+i}{2}+2n-\frac{n+i}{2}+1+2n+1 = 4n+2 = \lambda_2$$

For the edges $w_iv$, $1 \le i \le n$

For odd $i$,

$$f(w_i)+f(w_iv)+f(v) = \frac{i+1}{2}+3n+2-\frac{i+1}{2}+3n+2 = 6n+4 = \lambda_3$$

For even $i$,

$$f(w_i)+f(w_iv)+f(v) = \frac{n+i}{2}+3n+2-\frac{n+i}{2}+3n+2 = 6n+4 = \lambda_3$$

Hence for each $uv \in E$, f(u) + f(uv) + f(v) has any one of the magic constants $\lambda_1 = \frac{9n+6}{2}$, $\lambda_2 = 4n+2$, $\lambda_3 = 6n+4$

Hence the double fan $P_n+2K_1$ has edge trimagic total labeling

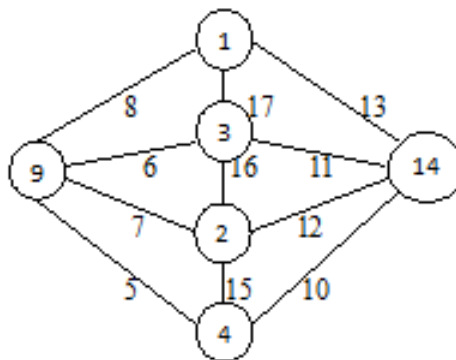**Example 2.3:** The double fan $P_4+2K_1$ given in figure is edge trimagic total labeling



**Figure 2:** $P_4+2K_1$ with $\lambda_1 = 21$, $\lambda_2 = 18$, $\lambda_3 = 28$

## 3. SUPER EDGE TRIMAGIC TOTAL LABELING

**Theorem 3.1:** The graph $B_{n,n}^2$ has super edge trimagic total labeling.

**Proof:** Let G be a restricted $B_{n,n}^2$ graph and let V(G) = {v,u,$v_iu_i$ /1 $\le i \le n$ }, where $v_iu_i$ are pendant vertices with $V(G) = V(B_{n,n}) = V(B_{n,n}^2)$ and $E(B_{n,n}^2) = E(B_{n,n}) \cup \{uv_i, vu_i/1 \le i \le n\}$. The order of $B_{n,n}^2$ is p = 2n + 2 and size is q = 4n + 1

Let us define the function $f : V \cup E \longrightarrow \{1, 2, 3\ldots,6n+3\}$ by

$$f(u) = 1; f(v) = 2;$$

$$f(u_i) = i+2 \text{ for all } 1 \le i \le n;$$

$$f(v_i) = 2n-i+3 \text{ for all } 1 \le i \le n;$$

$$f(uv_i) = 2n+i+2 \text{ for all } 1 \le i \le n;$$

$$f(uu_i) = 4n-i+3 \text{ for all } 1 \le i \le n;$$

$$f(uv) = 4n+3;$$

$$f(uu_i) = 5n-i+4 \text{ for all } 1 \le i \le n;$$

$$f(vv_i) = 5n+i+3 \text{ for all } 1 \le i \le n;$$

Now we prove that the graph $B_{n,n}^2$ admits super edge trimagic total labeling.

For the edge,
$$f(u) + f(uv) + f(v) = 1+4n+3+2 = 4n+6 = \lambda_1$$

For the edges $uu_i$, $1 \leq i \leq n$
$$f(u) + f(uu_i) + f(u_i) = 1+4n-i+3+i+2 = 4n+6 = \lambda_1$$

For the edges $vu_i$, $1 \leq i \leq n$
$$f(v) + f(vu_i) + f(u_i) = 2+5n-i+4+i+2 = 5n+8 = \lambda_2$$

For the edges $vv_i$, $1 \leq i \leq n$
$$f(v) + f(vv_i) + f(v_i) = 2+5n+i+3+2n-i+3 = 7n+8 = \lambda_3$$

For the edges $uv_i$, $1 \leq i \leq n$
$$f(u) + f(uv_i) + f(v_i) = 1+2n+i+2+2n-i+3 = 4n+6 = \lambda_1$$

Hence for each $uv \in E$, $f(u) + f(uv) + f(v)$ has any one of the magic constants $\lambda_1 = 4n+6$, $\lambda_2 = 5n+8$, $\lambda_3 = 7n+8$.

Hence the graph $B_{n,n}^2$ is a super edge trimagic total labeling.

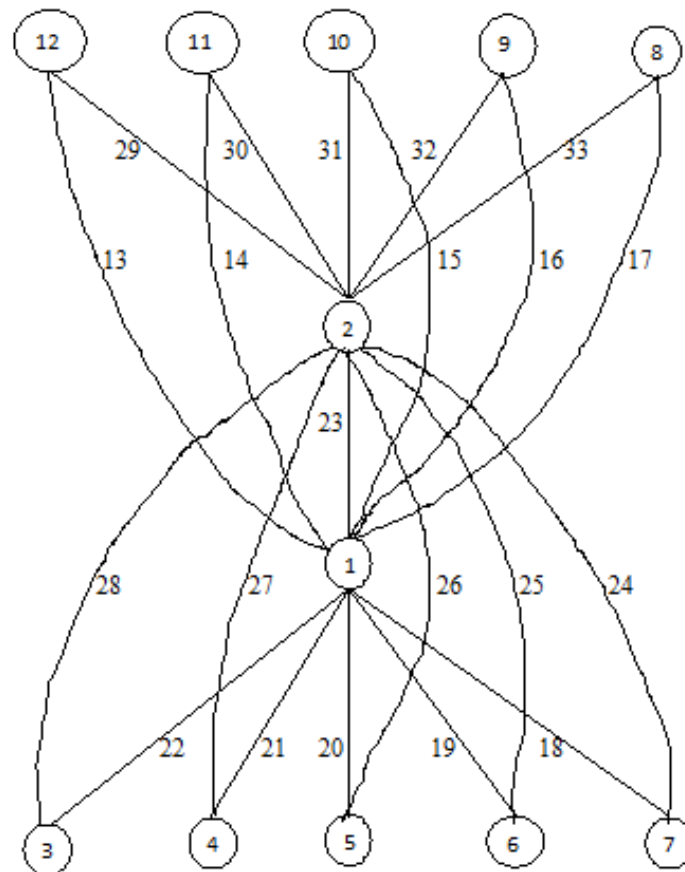**Example 3.2:** The graph $B_{5,5}^2$ is given in figure super edge trimagic total labeling



**Figure 3:** $B_{5,5}^2$ with $\lambda_1 = 26$, $\lambda_2 = 33$, $\lambda_3 = 43$

**Theorem 3.3:** $Z - (P_n)$ has a super edge trimagic total labeling.

**Proof:** Let G be a $Z - (P_n)$. $P_n$ is the $i^{th}$ vertex of a path $P_1$ is joined with $(i + 1)^{th}$ vertex of a path $P_2$.

Let $V(G) = \{u_i, v_i / 1 \leq i \leq n\}$ and $E(G) = \{u_i u_{i+1}, v_i v_{i+1}, u_{i+1} v_i / 1 \leq i \leq n\}$.

The order of $Z - (P_n)$ is $p = 2n$ and size is $q = 3n - 3$. Let us define the function $f: V \cup E \rightarrow \{1, 2, 3, \ldots 5n-3\}$ by

**Case-1: n is odd**

$$f(u_i) = \begin{cases} \frac{i+1}{2}, i \text{ is odd} \\ \frac{n+i+1}{2}, i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} \frac{2n+i+1}{2}, i \text{ is odd} \\ \frac{3n+i+1}{2}, i \text{ is even} \end{cases}$$

$f(u_i u_{i+1}) = 3n\text{-}i$ for all $1 \leq i \leq n$

$f(v_i v_{i+1}) = 4n\text{-}1\text{-}i$ for all $1 \leq i \leq n$

$f(u_{i+1} v_i) = 5n\text{-}3\text{-}i+1$ for all $1 \leq i \leq n$

Now we prove that the graph $Z - (P_n)$ has edge trimagic total labeling.

For the edges $u_i u_{i+1}, 1 \leq i \leq n$

For odd $i$,
$$f(u_i) + f(u_i u_{i+1}) + f(u_{i+1}) = \frac{i+1}{2} + 3n\text{-}i + \frac{n+i+1+1}{2} = \frac{7n+3}{2} = \lambda_1$$

For even $i$,
$$f(u_i) + f(u_i u_{i+1}) + f(u_{i+1}) = \frac{n+i+1}{2} + 3n\text{-}i + \frac{i+1+1}{2} = \frac{7n+3}{2} = \lambda_1$$

For the edges $v_i v_{i+1}, 1 \leq i \leq n$

For odd $i$,
$$f(v_i) + f(v_i v_{i+1}) + f(v_{i+1}) = \frac{2n+i+1}{2} + 4n\text{-}1\text{-}i + \frac{3n+i+1+1}{2} = \frac{13n+1}{2} = \lambda_2$$

For even $i$,
$$f(v_i) + f(v_i v_{i+1}) + f(v_{i+1}) = \frac{3n+i+1}{2} + 4n\text{-}1\text{-}i + \frac{2n+i+1+1}{2} = \frac{13n+1}{2} = \lambda_2$$

For the edges $u_{i+1} v_i, 1 \leq i \leq n$

For odd $i$,
$$f(u_{i+1}) + f(u_{i+1} v_i) + f(v_i) = \frac{n+i+1+1}{2} + 5n\text{-}3\text{-}i+1 + \frac{2n+i+1}{2} = \frac{13n-1}{2} = \lambda_3$$

For even $i$,
$$f(u_{i+1}) + f(u_{i+1} v_i) + f(v_i) = \frac{i+1+1}{2} + 5n\text{-}3\text{-}i+1 + \frac{3n+i+1}{2} = \frac{13n-1}{2} = \lambda_3$$

Hence for each $uv \in E$, $f(u) + f(uv) + f(v)$ has any one of the magic constants $\lambda_1 = \frac{7n+3}{2}$, $\lambda_2 = \frac{13n+1}{2}$, $\lambda_3 = \frac{13n-1}{2}$.

**Case-2: n is even**

$$f(u_i) = \begin{cases} \frac{i+1}{2}, i \text{ is odd} \\ \frac{n+i}{2}, i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} \frac{2n+i+1}{2}, i \text{ is odd} \\ \frac{3n+i}{2}, i \text{ is even} \end{cases}$$

$f(u_i u_{i+1}) = 3n\text{-}i$ for all $1 \leq i \leq n$

$f(v_i v_{i+1}) = 4n\text{-}1\text{-}i$ for all $1 \leq i \leq n$

$f(u_{i+1} v_i) = 5n\text{-}3\text{-}i+1$ for all $1 \leq i \leq n$

Now we prove that the graph $Z - (P_n)$ has edge trimagic total labeling.

For the edges $u_i u_{i+1}, 1 \le i \le n$

For odd $i$,
$$f(u_i) + f(u_i u_{i+1}) + f(u_{i+1}) = \frac{i+1}{2} + 3n - i + \frac{n+i+1}{2} = \frac{7n+2}{2} = \lambda_1$$

For even $i$,
$$f(u_i) + f(u_i u_{i+1}) + f(u_{i+1}) = \frac{n+i}{2} + 3n - i + \frac{i+1+1}{2} = \frac{7n+2}{2} = \lambda_1$$

For the edges $v_i v_{i+1}, 1 \le i \le n$

For odd $i$,
$$f(v_i) + f(v_i v_{i+1}) + f(v_{i+1}) = \frac{2n+i+1}{2} + 4n - 1 - i + \frac{3n+i+1}{2} = \frac{13n}{2} = \lambda_2$$

For even $i$,
$$f(v_i) + f(v_i v_{i+1}) + f(v_{i+1}) = \frac{3n+i}{2} + 4n - 1 - i + \frac{2n+i+1+1}{2} = \frac{13n}{2} = \lambda_2$$

For the edges $u_{i+1} v_i, 1 \le i \le n$

For odd $i$,
$$f(u_{i+1}) + f(u_{i+1} v_i) + f(v_i) = \frac{n+i+1}{2} + 5n - 3 - i + 1 + \frac{2n+i+1}{2} = \frac{13n-2}{2} = \lambda_3$$

For even $i$,
$$f(u_{i+1}) + f(u_{i+1} v_i) + f(v_i) = \frac{i+1+1}{2} + 5n - 3 - i + 1 + \frac{3n+i}{2} = \frac{13n-2}{2} = \lambda_3$$

Hence for each $uv \in E, f(u) + f(uv) + f(v)$ has any one of the magic constants $\lambda_1 = \frac{7n+2}{2}, \lambda_2 = \frac{13n}{2}, \lambda_3 = \frac{13n-2}{2}$.

Hence the graph $Z - (P_n)$ admits super edge trimagic total labeling

**Example 3.4:** The graphs $Z - (P_3), Z - (P_4)$ given in figure is super edge trimagic total labeling
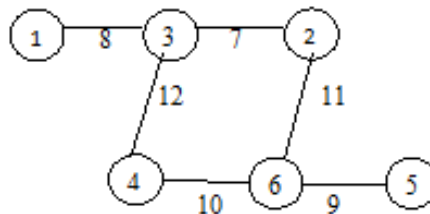


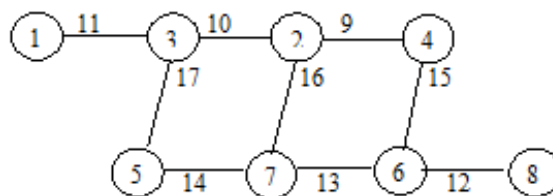**Figure 4:** $Z - (P_3)$ with $\lambda_1 = 12, \lambda_2 = 20, \lambda_3 = 19$



**Figure 5:** $Z - (P_4)$ with $\lambda_1 = 15, \lambda_2 = 26, \lambda_3 = 25$

**CONCLUSION**

In this paper we proved that the Double fan $P_n + 2K_1$ is edge trimagic total labeling and the graphs $Z$-$P_n$, $B_{n,n}^2$ are super edge trimagic total labeling.

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