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UNCOUNTABLY MANY POSITIVE SOLUTIONS OF FIRST ORDER NONLINEAR NEUTRAL DELAY DIFFERENCE EQUATIONS

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ABSTRACT

We consider the difference equation

$$\Delta (x_n - P_n x_{n-\tau}) + q_{(n)} f(x_{n-\sigma}) = 0 \qquad (1)$$

Where $n \geq n_0$, $\tau > 0$ & $\sigma \geq 0$ are integers.

Also a ϵ C ([t_0, ∞), (0, ∞)), p_n, $q_n \epsilon$ C(R, (0, ∞)) and f ϵ C(R, R), where f is non decreasing function for $f(x) > 0, x > 0$.

INTRODUCTION

We are concerned with the first order neutral Delay nonlinear Difference equation

$$\Delta(x_n - P_n x_{n-\tau}) + q_{(n)} f(x_{n-\sigma}) = 0 \qquad (1)$$

(H1) $r_n \epsilon$ c' [(n_0, ∞), (0, ∞)), $\sum_{s=n}^{\infty} \frac{1}{rs} = \infty$
(H2) $p_n \epsilon$ c ((n_0, ∞) (0, ∞)) p≡ 0
(H3) Ø(λ) ϵ c' ((-∞, ∞), (0, ∞)) Ø (λ) ≠0 | x≠0)
(H4) f(x) ϵ c' ((-∞, ∞), (-∞, ∞)) λf(x) > 0 | x≠0)
(H5) G(x) = $\frac{\Delta f(x)}{\emptyset(\lambda)}$ > 0 (x ≠0): G(x) is non decreasing (0, ∞) and non increasing is (-∞, 0)
(H6) g(n) ∈ c [(n_0,∞) → (0,∞)) g(n) ≥ n

A non trivial solution {x_n} is said to be oscillatory if it has arbitrarily large Zeros otherwise {x_n} is said to be non oscillatory The proof is an adaptation of that given (1) where the special case g(n) = n was consider

Lemma 1.1: (Krasnoselskii's fixed point theorem)

Let X be a Banach space, Let Ω be a bounded close convex subset of x and let s_1 s_2 be maps ofΩ into x such that $s_1x + s_2y$ ϵ Ω for every x, y ϵ Ω.

If s_1 is contractive and s_2 is completely continuous. Then the equation $s_1x+s_2x=x$ solution in Ω

Theorem: Suppose that there exist bounded from below and from above by the function $u_n, v_n \in c'$ ([n_0, ∞), (0, ∞)) constant c > 0, $k_2 > k_1 \geq 0$ & $n_1 \geq n_0$+ m such that

$$u_n \leq v_n, \quad n \geq n_0 \qquad (2)$$

$$v_n - v_{n1} - u_n + u_{n1} \geq 0, \quad n_0 \leq n \leq n_1 \qquad (3)$$

$$\frac{1}{u(n-\tau)} \left(u_n - k_1 + \sum_{s=n}^{\infty} p_s f(v_s - \sigma)\right) \leq a_n < \frac{1}{v(n-\tau)} \left(v_n - k_2 + \sum p_s f(u_s - \sigma)\right) \leq c \leq 1 \; n \geq n_1 \qquad (4)$$

Then eq. (1) has uncountable many positive solution which are bounded by the Functions u, v.

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Proof: Let c $((k_0, \infty)$, R) be the set of all continuous bounded functions with The norm $||x|| = \sup_{n\geq n0} |x_n|$. Then $c([n_{0,}\infty),R)$ is Banach space.

We define a close bounded an convex subset of c $((n_0, \infty)$, R) as

$$\Omega = \{x = x_n \epsilon\ c\ ((n_0, \infty), R): u_n \leq x_n \leq v_{n,}\ n \geq n_0\}$$

For kϵ $[k_1, k_2]$ we define two maps s_1 & s_2: $\Omega \to c\ ((n_0, \infty))$ as follows

$$s_1\, x_n = \begin{cases} k + a_n x_{n-\tau} & n \geq n_1 \\ s_1 x_{n_1} & n_0 \leq t \leq n_1 \end{cases} \tag{5}$$

$$s_2\, x_n = \begin{cases} -\sum_{s=n}^{\infty} p_s f(x_{s-\sigma}) & n \geq n_1 \\ s_2 x_{n_1} + v_n - v_{n_1} & n_0 \leq t \leq n_1 \end{cases}$$

We will show that for any x, y $\epsilon\Omega$ we have $s_1x+s_2y\epsilon\Omega$ for every x, y ϵ Ω and $t \geq t_1$ with regard to (4) we obtain

$$\begin{aligned} s_1x_n + s_2y_n &= k + a_n x_{n-\tau} - \textstyle\sum_{s=n}^{\infty} p_s f(y_{s-\sigma}) \\ &\leq k + a_n v_{n-\tau} - \textstyle\sum_{s=n}^{\infty} p_s f(y_{s-\sigma}) \\ &\leq k + v_n - k_2 \leq v_n \end{aligned}$$

For n $\epsilon[n_0,n_1]$ we have

$$\begin{aligned} s_1x_n + s_2y_n &= s_1x_{n_1} + s_2y_{n_1} + v_n - v_{n_1} \\ &\leq v_{n_1} + v_n - v_{n_1} = v_n \end{aligned}$$

Further more for $n \geq n_1$ we get

$$\begin{aligned} s_1x_n + s_2y_n &\geq k + a_n u_{n-\tau} - \textstyle\sum_{s=n}^{\infty} p_s\, f\,(v_{s-\sigma}) \\ &\geq k + u\,(t) - k \geq u_n \end{aligned}$$

Let n ϵ (n_0, n_1) with regards to (3) we get

$$v_n - v_{n_1} + u_{n_1} \geq u_n, \quad n_0 \leq t \leq n_1$$

Then n ϵ $[n_0, n_1]$ and any x, y ϵ Ω we obtain

$$\begin{aligned} s_1x_n + s_2y_n &= s_1x_{n_1} + s_2y_{n_1}1 + u_t - u_{t_1} \\ &= u_{n_1} + u_n - u_{n_1} \geq u_n \end{aligned}$$

Then we have prove that s_1x+s_2y ϵ Ω for any x, y ϵ Ω

We will show that s_1 is a contraction mapping on Ω for x, y ϵ Ω & $n \geq n_1$ we have

$$|s_1x_n - s_1y_n| = |a_n||x_{n-\tau} + y_{n-\tau}| \leq c||x-y||$$

This implies

$$||s_1x - s_2y|| \leq c\ ||x-y||$$

Also for n ϵ (n_0, n_1) the above inequalities is valid.

We conclude that S_1 is a contraction mapping on Ω

We now show that s_2 is completely continuous. First we show that s_2 is continuous. Let $x^i = \{x_n^{(i)}\}$ ϵ Ω be such that $x_n^{(i)} \to x_n$ as $n \to \infty$ Because x is close x $=(x_n)$ ϵ Ω for $n \geq n_1$ we have

$$\begin{aligned} |(s_2\, x_n^{\,i} - s_2x_n) &\leq |\textstyle\sum_{s=n}^{\infty} p_s[fx^i_{s-\sigma} - f(x_{s-\sigma})] \\ &\leq \textstyle\sum_{s=n1}^{\infty} p_s\, |fx^i_{s-\sigma} - f(x_{s-\sigma})| \end{aligned}$$

Since $|f\ (x^i_{s-\sigma}) - f\ (x_{s-\sigma})| \to 0$ as $i \to \infty$ be applying the lebesgue dominant Convergence their we obtain $\lim_{i \to \infty} |s_2\, x^i - s_2\, x\,|| = 0$ This means the s_2 is continuous.

we now show that s_2 is relatively compact in Ω, it is sufficient to share

By Arzela ascolic theorem that the family of functions $\{s_2x: x\ \epsilon\ \Omega\}$ is uniformly

$$\textstyle\sum_{s=n}^{\infty} p_s\ f\ (x_{s-\sigma}) < \epsilon/2$$

The x ϵ Ω, $N_2 > N_1 \geq n$
where

$$\begin{aligned} |(s_2\,x)\,(N_2) - (s_2\,x)\,(N_1)| &\leq \textstyle\sum_{s=N_2}^{\infty} p_s\ f\ (x_{s-\sigma}) + \textstyle\sum_{s=N_1}^{\infty} p_s\ f\ (x_{s-\sigma}) \\ &\leq \varepsilon/2 + \varepsilon/2 = \varepsilon \end{aligned}$$

$|(s_2 x)(N_2) - (s_2 x)(N_1)| \leq \sum_{s=n1}^{N2\infty} p_s \, f(x_{s-\sigma})$
$\leq \text{Max}\{p_s \, f(x_{s-\tau})\}(N_2 - N_1), \; n_1 \leq \& \leq n$

Then there exist $s_1 = \epsilon/M$ when $M = \max p_x f(x_{s-\sigma})$ there exist $n_1 \leq \& \leq n$
$|(s_2x)(N_2) - (s_2x)(n_1)| < \epsilon$ if $0 < N_2 - N_1 < s_1$

Next we show that equation (1) has uncountable many bounded positive solution Ω.

Let $\bar{k} \in [k_1, k_2]$ be such that $\bar{k} \neq k$.

We assume that x, y $\epsilon\ \Omega$
$s_1x + s_2x = x, \; \bar{s_1}y + \bar{s_2}y = y$
$x_n = k + a_n \, x_{(n-\sigma)} - \sum_{s=n}^{\infty} p_s \, f(x_{s-\sigma}), \quad n \geq n_1$
$y_n = \bar{k} + an \; y_{(n-\sigma)} - \sum_{s=n}^{\infty} p \, s \, f(y_{s-\sigma}), \quad n \geq n_1$

Is follow that there exist a $n_2 > n_1$ satisfy
$\sum_{s=n2}^{\infty} p_s[f(x_{s-\sigma}) + f(y_{s-\sigma})] \leq |k - \bar{k}|$

In order to prove that the set of bounded positive solution of equation (1) is Constant it is sufficient to very that $x \neq y$ for $n \geq n_{2.}$

We get $|x_n - y_n|(1+x) \, \| x-y \| \geq |k - \bar{k}| - \sum_{s=n}^{\infty} p_s \, (f(x_{s-\sigma}) + f(y_{s-\sigma}))$

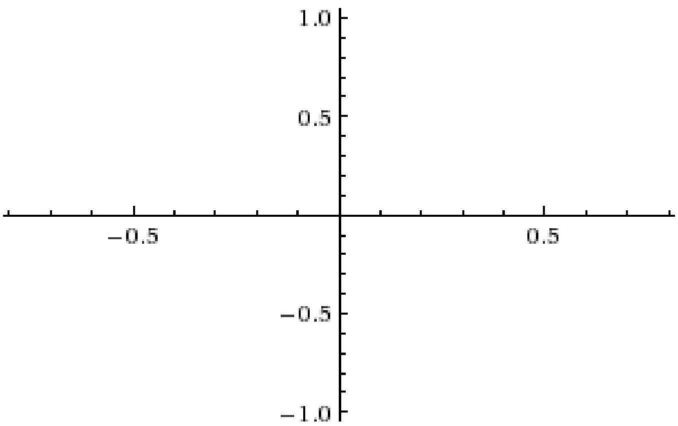
Corollary: Suppose that their exist bounded from below and from above by Function u and vϵ C $[(n_0, \infty)(0, \infty))$ that $c > 0$ $k_2 > k_1 \geq 0$, $n_1 \geq n_0 + m$ such that (2) (4) holds
$\Delta u_n - \Delta u_n \leq 0 \; n_1 \leq n \leq n_1$
$H(t) = v_{n+1} - u_n - u_{n+1} + u_n$
$H^1(t) = \Delta v_n - \Delta u_n \leq 0$
$H_n(t) = 0$

Example: $\Delta(x_n - x_{n-1}) + \frac{1}{n} x_{n-1} = 0$

Input:

$$x(n+1) - 2x(n) + x(n-1) + \frac{1}{n} x(n-1) = 0$$

Plot:



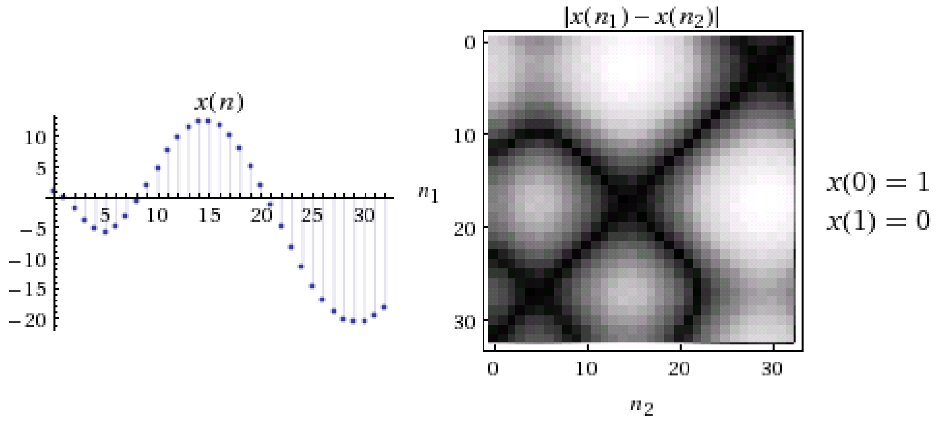
Alternate forms:

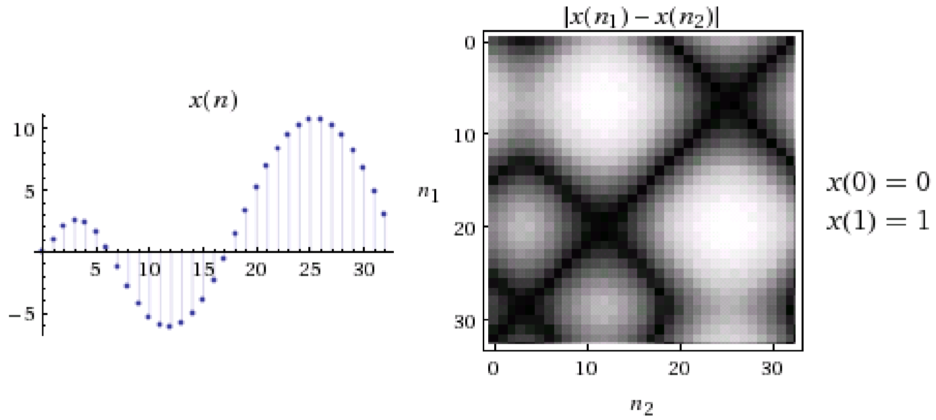
$$\left(\frac{1}{n}+1\right) x(n-1) + x(n+1) = 2x(n)$$

$$nx(n+1) = (-n-1)\; x(n-1) + \; 2nx(n)$$

$$\frac{n \; x(n-1) + x(n-1) - 2n \; x(n) + n \; x(n+1)}{n} = 0$$

Value plot and recurrence plot:





Values:

n	0	1	2	3	4
$x(n)$	0	1	2	2.5	2.33333

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