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ON ABSORPTIVE CI-ALGEBRAS

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ABSTRACT

In this paper we introduce the concept of absorptive CI-algebras and investigate some of its properties in details.

Keywords: *CI-algebra, BE-algebra, self-distributive, transitive, absorptive.*

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1. INTRODUCTION

In 1966, Y. Imai and K. Iseki ([2, 3]) introduced the notion of BCK/BCI-algebras. There exist several generalizations of BCK/BCI-algebras, such as BCH-algebras ([1]), BH-algebras ([4]), d-algebras ([8]), etc. As a dualization of a generalization of BCK-algebra ([5]), H.S. Kim and Y. H. Kim introduced the notion of BE-algebra ([6]). In 2010, B. L. Meng ([7]) introduced the notion of CI-algebras as a generalization of BE-algebras. In this paper we introduce the concept of absorptive CI-algebras and investigate some of its properties in details.

2. PRELIMINARIES

Definition 2.1 ([6]): A system $(X; *, 1)$ of type $(2, 0)$ consisting of a non-empty set X, a binary operation $*$ and a fixed element 1 is called a BE–algebra if the following conditions are satisfied:

1. (BE 1) $x * x = 1$
2. (BE 2) $x * 1 = 1$
3. (BE 3) $1 * x = 1$
4. (BE 4) $x * (y * z) = y * (x * z)$ for all $x, y, z \in X$.

Definition 2.2 ([7]): A system $(X; *, 1)$ consisting of a non–empty set X, a binary operation $*$ and a fixed element 1, is called a CI–algebra if the following conditions are satisfied:

1. (CI 1) $x * x = 1$
2. (CI 2) $1 * x = x$
3. (CI 3) $x * (y * z) = y * (x * z)$ for all $x, y, z \in X$

Example 2.3: Let $X = R^+ = \{x \in R: x > o\}$

For $x, y \in X$, we define

$$x * y = y \cdot \frac{1}{x}$$

Then $(X; *, 1)$ is a CI–algebra

Example 2.4: The simplest example of a BE–algebra and a CI –algebra are the following.

Let $X = \{0, 1\}$. We consider binary operations $*$ and o given by the Cayley tables

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$*$	0	1
0	1	1
1	0	1

Table (2.4(a))

o	0	1
0	1	0
1	0	1

Table (2.4(b))

Then (i) (X; $*$, 1) is a BE–algebra,
(ii) (X; o, 1) is a CI–algebra but not a BE–algebra.

Example 2.5: (a) Let X be a non-empty set and let F(X) be the set of all function f: X $\to$ (0, ∞). For f, g $\in$ F(X), we define

$$(f * g)(x) = \frac{g(x)}{f(x)}, x \in X.$$

If we put 1(x) = 1for all x $\in$ X, then 1 $\in$ F(X) and simple computation proves that (F(X); $*$, 1) is a CI–algebra.

(b) For a non-empty set X, let G(X) be the set of all functions f: X $\to$ R. For f, g $\in$ G(X), we define
(f o g)(x) = (1- f(x)) + g(x).

Then simple computation shows that (G (X); o, 1) is a CI–algebra.

Lemma 2.6 ([7]): In a CI–algebra (X; $*$, 1) following results are true:
(1) $x * ((x * y) * y) = 1$
(2) $(x * y) * 1 = (x * 1) * (y * 1)$ for all x, y $\in$ X.

Definition 2.7 ([7]): A CI–algebra (X; $*$, 1) is said to be
(a) self distributive if for any x, y, z $\in$ X, we have
$x * (y * z) = (x * y) * (x * z)$,
(b) transitive if for all x, y, z $\in$ X, we have
$(y * z) * ((x * y) * (x * z)) = 1$

Theorem 2.8 ([9]): Let (X; $*$, 1) be a system consisting of a non-empty set X, a binary operation $*$ and a fixed element 1. Let Y = X x X. For $u = (x_1, x_2)$, $v = (y_1, y_2)$ a binary operation $\otimes$ is defined in Y as
$u \otimes v = (x_1 * y_1, x_2 * y_2)$
Then (Y; $\otimes$, (1, 1)) is a CI- algebra iff (X; $*$, 1) is a CI-algebra .

Corollary 2.9 ([9]): If (X; $*$, 1) and (Y; o, e) are two CI–algebras, then Z = X x Y is also a CI–algebra under the binary operation defined as follows:

For $u = (x_1, y_1)$ and $v = (x_2, y_2)$ in Z,
$u \otimes v = (x_1 * x_2, y_1 \text{ o } y_2)$

Here the distinct element of Z is (1, e).

Theorem 2.10 ([10]): Let (X; $*$, 1) be a CI–algebra and let F(X) be the class of all functions f: X $\to$ X. Let a binary operation o be defined in F(X) as follows:

For f, g $\in$ F(X) and x $\in$ X,
(f o g)(x) = f(x) $*$ g(x).

Then (F(X); o, $1^{\sim}$) is a CI–algebra where $1^{\sim}$ is defined as $1^{\sim}(x) = 1$ for all x $\in$ X.

Notation 2.11 ([7]): Let (X; $*$, 1) is a CI–algebra. Let B(X) = {x $\in$ X: x $*$ 1 = 1}. B(X) is called the BE–part of X. Clearly B(X) is non–empty, since 1 $\in$ B(X).

3. ABSORPTIVE CI-ALGEBRA

Definition 3.1: A CI–algebra (X; $*$, 1) is said to be absorptive if for any x, y, z $\in$ X
$(x * y) * (x * z) = (y * z)$

Example 3.2: We may consider example 2.4.

Algebra given by table (2.4 (a)) is self distributive but not absorptive. For,

$(1 * 0) * (1 * 1) = 0 * 1 = 1,$
$1 * (0 * 1) = 1 * 1 = 1,$
$(0 * 1) * (0 * 0) = 1 * 1 = 1,$
$0 * (1 * 0) = 0 * 0 = 1,$
but $1 * 0 = 0$.

Again CI–algebra given by table (2.4 (b)) is not self–distributive, because

$0 \circ (1 \circ 0) = 0 \circ 0 = 1$
and $(0 \circ 1) \circ (0 \circ 0) = 0 \circ 1 = 0.$
But it is absorptive. For,
$(1 \circ 0) \circ (1 \circ 1) = 0 \circ 1 = 0 = 0 \circ 1,$
$(1 \circ 1) \circ (1 \circ 0) = 1 \circ 0 = 0 = 1 \circ 0,$
$(0 \circ 1) \circ (0 \circ 0) = 0 \circ 1 = 0 = 1 \circ 0,$
and $(0 \circ 0) \circ (0 \circ 1) = 1 \circ 0 = 0 = 0 \circ 1,$

Example 3.3: We may consider example (2.5) (a) and (b).

If f, g, h $\in$ F(X). Then

$$\big((f*g)*(f*h)\big)(x) = \frac{(f*h)(x)}{(f*g)(x)} = \frac{h(x)}{f(x)}\frac{f(x)}{g(x)} = \frac{h(x)}{g(x)} = (g*h)(x) \text{ for all } x \in X.$$

So $(f * g) * (f * h) = (g * h)$.

Hence (F(X); $*$, 1) is an absorptive CI–algebra.

Again if f, g, h $\in$ G(X), then

$$\begin{aligned}((f \circ g) \circ (f \circ h)\ (x) &= (1 - (f \circ g)(x)) + (f \circ h)(x)\\ &= 1 - [(1 - f(x)) + g(x)] + (1- f(x)) + h(x)\\ &= 1 - g(x) + h(x)\\ &= (g \circ h)(x) \text{ for all } x \in X.\end{aligned}$$

So $(f \circ g) \circ (f \circ h) = (g \circ h)$.

Hence (G(X); o, 1) is an absorptive CI-algebra.

Now we prove the following results:

Proposition 3.4: If (X; $*$, 1) is an absorptive CI–algebra then B(X) = {1}.

Proof: Let (X; $*$, 1) is an absorptive CI–algebra. Then

$(x * y) * (x * z) = y * z$ for all x, y, z $\in$ X.

If possible, let $1 \neq x \in B(X)$. This means that $x * 1 = 1$.

Now putting y = 1 and z = x we see that above equality is not satisfied. For,

$(x * 1) * (x * x) = 1 * 1 = 1$
and $1 * x = x$.

This proves that B(X) = {1}.

Corollary 3.5: A BE–algebra containing more than 1 element cannot be absorptive.

Theorem 3.6: Let (X; $\ast$, 1) be a CI–algebra and let (F(X); o, $\tilde{1}$) be function CI–algebra discussed in theorem (2.10). Then F(X) is absorptive iff X is absorptive.

Proof: Let X be an absorptive CI–algebra. For f, g, h $\in$ F(X) and x $\in$ X, we have

$$((f \circ g) \circ (f \circ h))(x) = (f(x) \ast g(x)) \ast (f(x) \ast h(x)) = g(x) \ast h(x) = (g \circ h)(x).$$

This gives (f o g) o (f o h) = (g o h) for all f, g, h $\in$ F(X).

Hence F (X) is absorptive.

Conversely, suppose that F(X) is absorptive.

Then for all f, g, h $\in$ F(X), we have

$$(f \circ g) \circ (f \circ h) = (g \circ h). \quad (1)$$

Let x, y, z $\in$ X, we consider f_x, f_y, f_z $\in$ F(X) defined as

$$f_x(t) = x,\ f_y(t) = y,\ f_z(t) = z \quad \text{for all } t \in X.$$

Using (1) we get

$$((f_x \circ f_y) \circ (f_x \circ f_z))(t) = (f_y \circ f_z)(t) \text{ for all } t \in X.$$

This gives

$$(x \ast y) \ast (x \ast z) = (y \ast z)$$

Hence X is absorptive.

Theorem 3.7: Let X, Y and Z be CI-algebras as considered in corollary (2.9). Then Z is absorptive iff X and Y are absorptive.

Proof: First suppose that (Z; $\otimes$, (1, e)) is absorptive. Let x, y, z $\in$ X. We choose u = (x, e), v= (y, e) and w = (z, e) of Z.

Since Z is absorptive, we have

$$(u \otimes v) \otimes (u \otimes w) = v \otimes w \quad (2)$$

This gives

$$((x \ast y) \ast (x \ast z), e) = (y \ast z, e)$$
$$\Rightarrow (x \ast y) \ast (x \ast z) = y \ast z.$$

Hence X is absorptive. Similarly if x, y, z $\in$ Y then taking u = (1, x), v = (1, y) and w = (1, z) in (2) we see that Y is absorptive.

Conversely, suppose that X and Y are absorptive CI-algebras. Let $u = (x_1, y_1)$, $v = (x_2, y_2)$ and $w = (x_3, y_3)$ where $x_1, x_2, x_3 \in X$ and $y_1, y_2, y_3 \in Y$. Then

$$\begin{aligned}(u \otimes v) \otimes (u \otimes w) &= ((x_1, y_1) \otimes (x_2, y_2)) \otimes ((x_1, y_1) \otimes (x_3, y_3)) \\ &= (x_1 \ast x_2, y_1 \circ y_2) \otimes (x_1 \ast x_3, y_1 \circ y_3) \\ &= ((x_1 \ast x_2) \ast (x_1 \ast x_3), (y_1 \circ y_2) \circ (y_1 \circ y_3)) \\ &= (x_2 \ast x_3, y_2 \circ y_3) \\ &= (x_2, y_2) \otimes (x_3, y_3) = v \otimes w.\end{aligned}$$

Hence Z is absorptive.

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