

## MEAN SQUARE SUM LABELING OF PATH RELATED GRAPHS

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### ABSTRACT

Let  $G = (V, E)$  be a graph. A bijection  $f: V(G) \rightarrow \{0, 1, \dots, p-1\}$  is said to be a mean square sum labeling if the induced function  $f^*: E(G) \rightarrow N$  given by  $f^*(uv) = \left\lfloor \frac{[f(u)]^2 + [f(v)]^2}{2} \right\rfloor$  or  $\left\lceil \frac{[f(u)]^2 + [f(v)]^2}{2} \right\rceil$  for every  $uv \in E(G)$  is injective. A graph which admits a mean square sum labeling is called a mean square sum graph. The concept of mean square sum labeling was introduced by C. Jayasekaran, S. Robinson Chellathurai and M. Jaslin Melbha and they investigated the mean square sum labeling of several standard graphs and some cycle related graphs. In this paper, we prove that composition of paths  $P_m$  and  $P_2$ ,  $P_n^2$ ,  $P_n^3$ ,  $D_2(P_n)$ , Splitting graph of path  $P_m$ , Switching of a pendant vertex in path  $P_m$ ,  $T_m$ ,  $DT_m$ , Coconut tree and the graph obtained from  $P_n$  by attaching  $C_3$  in both the end edges of  $P_n$  are mean square sum graph.

**Key words:** labeling, mean square sum labeling, mean square sum graph.

### 1. INTRODUCTION

We begin with simple, finite, connected and undirected graph. For standard terminology and notations we follow Harary [1]. A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges) then the labeling is called a *vertex labeling* (an *edge labeling*). Several types of graph labeling and a detailed survey is available in [2].

S. Somasundaram and R. Ponraj [5] have introduced the notion of mean labeling of graphs. A graph  $G$  with  $p$  vertices and  $q$  edges is called *mean graph* if there is an injective function  $f$  from the vertices of  $G$  to  $\{0, 1, \dots, q\}$  such that when each edge  $uv$  is labeled with  $\frac{f(u)+f(v)}{2}$  if  $f(u)+f(v)$  is even and with  $\frac{f(u)+f(v)+1}{2}$  if  $f(u)+f(v)$  is odd, then the resulting edge labels are distinct.

V. Ajitha, S. Arumugam and K. A. Germina [6] have introduced the notion of square sum labeling. A  $(p, q)$  graph  $G$  is said to be square sum, if there exists a bijection  $f: V(G) \rightarrow \{0, 1, \dots, p-1\}$  such that the induced function  $f^*: E(G) \rightarrow N$  defined by  $f^*(uv) = [f(u)]^2 + [f(v)]^2$  for every  $uv \in E(G)$  is injective.

The concept of mean square sum labeling was introduced by C. Jayasekaran, S. Robinson Chellathurai and M. Jaslin Melbha [3] and they investigated the mean square sum labeling of several standard graphs. Not every graph is mean square sum. For example, any complete graph  $K_n$ , where  $n \geq 6$  is not mean square sum. Also they investigated the mean square sum labeling of some cycle related graphs [4]. We are interested to study different classes of graphs, which are mean square sum.

A brief summary of definitions and other information which are necessary for the present investigation are given below.

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**Definition 1.1:** Let  $G = (V(G), E(G))$  be a graph. A bijection  $f: V(G) \rightarrow \{0, 1, \dots, p-1\}$   $G$  is said to be a *mean square sum labeling* if the induced function  $f^*: E(G) \rightarrow \mathbb{N}$  given by  $f^*(uv) = \left\lfloor \frac{[f(u)]^2 + [f(v)]^2}{2} \right\rfloor$  or  $\left\lceil \frac{[f(u)]^2 + [f(v)]^2}{2} \right\rceil$  for every  $uv \in E(G)$  is injective.

**Definition 1.2:** A graph which admits a mean square sum labeling is called a *mean square sum graph*.

**Definition 1.3:** The *composition* of two graphs  $G_1$  and  $G_2$  denoted by  $G = G_1[G_2]$  has vertex set  $V(G_1[G_2]) = V(G_1) \times V(G_2)$  and edge set  $E(G_1[G_2]) = \{(u_1, v_1)(u_2, v_2) / u_1u_2 \in E(G_1) \text{ or } u_1 = u_2 \text{ and } v_1v_2 \in E(G_2)\}$ .

**Definition 1.4:** *Square of a graph*  $G$  denoted by  $G^2$  has the same vertex set as of  $G$  and two vertices are adjacent in  $G^2$  if they are at a distance of 1 or 2 apart in  $G$ .

**Definition 1.5:** *Cube of a graph*  $G$  denoted by  $G^3$  has the same vertex set as of  $G$  and two vertices are adjacent in  $G^3$  if they are at a distance at most 3 apart in  $G$ .

**Definition 1.6:** For a connected graph  $G$ , let  $G'$  be the copy of  $G$ . The *shadow graph*  $D_2(G)$  is obtained by joining each vertex  $u$  in  $G$  to the neighbours of the corresponding vertex  $u'$  in  $G'$ .

**Definition 1.7:** Let  $G$  be a graph. For each point  $v$  of a graph  $G$ , take a new point  $v'$ . Join  $v'$  to those points of  $G$  adjacent to  $v$ . The graph thus obtained is called the *splitting graph* of  $G$ . We denote it by  $S'(G)$ .

**Definition 1.8:** A *vertex switching* of a graph  $G$  by a vertex of  $G$  is the graph  $G^v$  which is obtained by removing all the edges incident with  $v$  and adding all non adjacent edges as edges incident with  $v$ .

**Definition 1.9:** A *triangular snake*  $T_n$  is obtained from a path  $v_1v_2\dots v_n$  by joining  $v_i$  and  $v_{i+1}$  to a new vertex  $w_i$  for  $1 \leq i \leq n-1$ . That is, every edge of the path is replaced by a triangle  $C_3$ .

**Definition 1.10:** A double triangular snake consists of two triangular snakes that have a common path. That is, a *double triangular snake*  $DT_n$  is a graph obtained from a path  $u_1u_2\dots u_n$  by joining  $u_i$  and  $u_{i+1}$  to two new vertices  $v_i$  and  $w_i$  for  $1 \leq i \leq n-1$ .

## 2. MAIN RESULTS

**Theorem 2.1:** The composition of paths  $P_m$  and  $P_2$  denoted as  $P_m[P_2]$  is a mean square sum graph.

**Proof:** Let  $u_1, u_2, \dots, u_m$  be the vertices of path  $P_m$  and  $v_1, v_2$  be the vertices of path  $P_2$ . The composition  $P_m[P_2]$  consists of  $2m$  vertices, can be partitioned into two sets  $V_1 = \{(u_i, v_1) / i = 1, 2, \dots, m\}$  and  $V_2 = \{(u_i, v_2) / i = 1, 2, \dots, m\}$ . The edge set is  $E = \{[(u_i, v_1)(u_{i+1}, v_1)], [(u_i, v_2)(u_{i+1}, v_2)], [(u_i, v_1)(u_{i+1}, v_2)], [(u_i, v_2)(u_{i+1}, v_1)], [(u_j, v_1)(u_j, v_2)] / 1 \leq j \leq m, 1 \leq i \leq m-1\}$ . Let  $G = P_m[P_2]$ . Define  $f: V(G) \rightarrow \{0, 1, \dots, 2m-1\}$  by  $f(u_i, v_1) = 2i-2$ ,  $f(u_i, v_2) = 2i-1$ , for  $1 \leq i \leq m$ . The induced function  $f^*: E(G) \rightarrow \mathbb{N}$  is defined by  $f^*[(u_i, v_1)(u_{i+1}, v_1)] = 4i^2-4i+2$ ,  $f^*[(u_i, v_2)(u_{i+1}, v_2)] = 4i^2+1$ ,  $f^*[(u_i, v_1)(u_{i+1}, v_2)] = 4i^2-2i+2$ ,  $f^*[(u_i, v_2)(u_{i+1}, v_1)] = 4i^2-2i+1$ , for  $1 \leq i \leq m-1$  and  $f^*[(u_i, v_1)(u_i, v_2)] = 4i^2-6i+3$ ,  $1 \leq i \leq m$  is injective. Hence  $P_m[P_2]$  is a mean square sum graph.

**Example 2.2:** A mean square sum labeling of  $P_7[P_2]$  is given in figure 1.

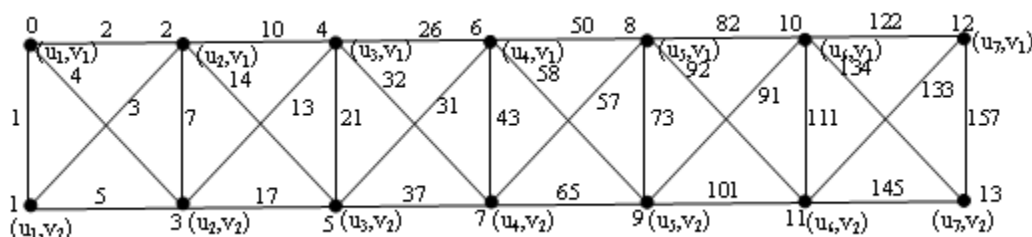


Figure-1:  $P_7[P_2]$

**Theorem 2.3:**  $P_n^2$  is a mean square sum graph.

**Proof:** Let  $u_1u_2\dots u_n$  be the path  $P_n$ . Let  $G = P_n^2$ . Then  $V(G) = \{u_1, u_2, \dots, u_n\}$  and  $E(G) = \{u_iu_{i+1}, u_ju_{j+2} / 1 \leq i \leq n-1, 1 \leq j \leq n-2\}$ . Clearly,  $G$  has  $n$  vertices and  $2n-3$  edges. Define  $f: V(G) \rightarrow \{0, 1, \dots, n-1\}$  as follows  $f(u_i) = i-1$ ,  $1 \leq i \leq n$ . The induced function  $f^*: E(G) \rightarrow \mathbb{N}$  is defined by  $f^*(u_iu_{i+1}) = i^2-i+1$ ,  $1 \leq i \leq n-1$  and  $f^*(u_iu_{i+2}) = i^2+1$ ,  $1 \leq i \leq n-2$  is injective. Hence  $P_n^2$  is a mean square sum graph.

**Example 2.4:** A mean square sum labeling of  $P_7^2$  is given in figure 2.

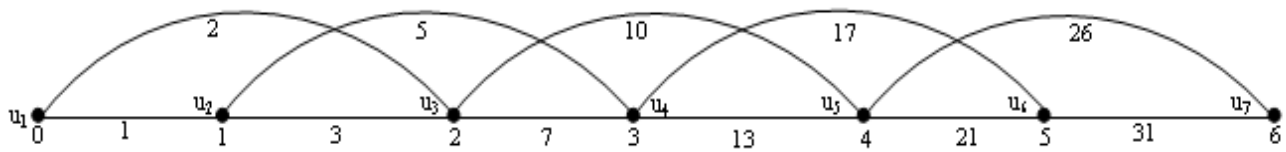


Figure-2:  $P_7^2$

**Theorem 2.5:**  $P_n^3$  is a mean square sum graph.

**Proof:** Let  $u_1 u_2 \dots u_n$  be the path  $P_n$ . Let  $G = P_n^3$ . Then  $V(G) = \{u_1, u_2, \dots, u_n\}$  and  $E(G) = \{u_i u_{i+1}, u_j u_{j+2}, u_k u_{k+3} / 1 \leq i \leq n-1, 1 \leq j \leq n-2, 1 \leq k \leq n-3\}$ . Clearly,  $G$  has  $n$  vertices and  $3n-6$  edges. Define  $f: V(G) \rightarrow \{0, 1, \dots, n-1\}$  as follows  $f(u_i) = i-1$ ,  $1 \leq i \leq n$ . The induced function  $f^*: E(G) \rightarrow \mathbb{N}$  is defined by  $f^*(u_i u_{i+1}) = i^2 - i + 1$ ,  $1 \leq i \leq n-1$ ;  $f^*(u_i u_{i+2}) = i^2 + 1$ ,  $1 \leq i \leq n-2$ ;  $f^*(u_i u_{i+3}) = i^2 + i + 2$ ,  $1 \leq i \leq n-3$  is injective. Hence  $P_n^3$  is a mean square sum graph.

**Example 2.6:** A mean square sum labeling of  $P_5^3$  is given in figure 3.

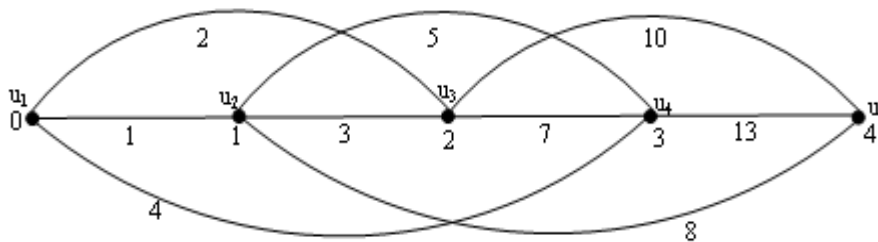


Figure-3:  $P_5^3$

**Theorem 2.7:** The graph  $D_2(P_n)$  is a mean square sum graph.

**Proof:** Let  $u_1 u_2 \dots u_n$  be a path  $P_n$  and  $v_1 v_2 \dots v_n$  be another path  $P'_n$ . Join  $u_i v_{i+1}$ ,  $v_i u_{i+1}$ ;  $1 \leq i \leq n-1$ . The resultant graph is  $D_2(P_n)$ . Let  $G = D_2(P_n)$ . Then  $V(G) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$  and  $E(G) = \{u_i u_{i+1}, v_i v_{i+1}, u_i v_{i+1}, v_i u_{i+1} / 1 \leq i \leq n-1\}$ . Clearly  $G$  has  $2n$  vertices and  $4(n-1)$  edges. Define  $f: V(G) \rightarrow \{0, 1, \dots, 2n-1\}$  as follows  $f(u_i) = 2i-2$ , and  $f(v_i) = 2i-1$ , for  $1 \leq i \leq n$ . The induced function  $f^*: E(G) \rightarrow \mathbb{N}$  is defined by  $f^*(u_i u_{i+1}) = 4i^2 - 4i + 2$ ,  $f^*(v_i v_{i+1}) = 4i^2 + 1$ ,  $f^*(u_i v_{i+1}) = 4i^2 - 2i + 2$ ,  $f^*(v_i u_{i+1}) = 4i^2 - 2i + 1$ , for  $1 \leq i \leq n-1$  is injective. Hence  $D_2(P_n)$  is a mean square sum graph.

**Example 2.8:** A mean square sum labeling of  $D_2(P_5)$  is given in figure 4.

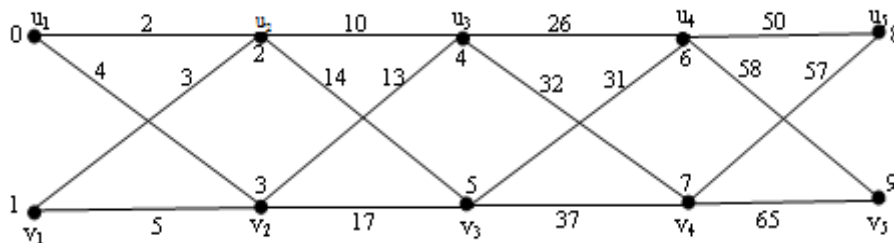


Figure-4:  $D_2(P_5)$

**Theorem 2.9:** The splitting graph of path  $P_n$  is a mean square sum graph.

**Proof:** Let  $u_1 u_2 \dots u_n$  be the vertices of path  $P_n$  and  $v_1 v_2 \dots v_n$  be the newly added vertices to form the splitting graph of  $P_n$ . Let  $G = S'(P_n)$ . Then  $V(G) = \{u_i, v_i / 1 \leq i \leq n\}$  and  $E(G) = \{u_i u_{i+1}, u_i v_{i+1}, v_i u_{i+1} / 1 \leq i \leq n-1\}$ . Also  $G$  has  $2n$  vertices and  $3n-3$  edges. Define  $f: V(G) \rightarrow \{0, 1, \dots, 2n-1\}$  as follows  $f(u_i) = 2i-2$ , and  $f(v_i) = 2i-1$ , for  $1 \leq i \leq n$ . The induced function  $f^*: E(G) \rightarrow \mathbb{N}$  is defined by  $f^*(u_i u_{i+1}) = 4i^2 - 4i + 2$ ,  $f^*(u_i v_{i+1}) = 4i^2 - 2i + 2$ ,  $f^*(v_i u_{i+1}) = 4i^2 - 2i + 1$ , for  $1 \leq i \leq n-1$  is injective. Hence  $S'(P_n)$  is a mean square sum graph.

**Example 2.10:** A mean square sum labeling of  $S'(P_6)$  is given in figure 5.

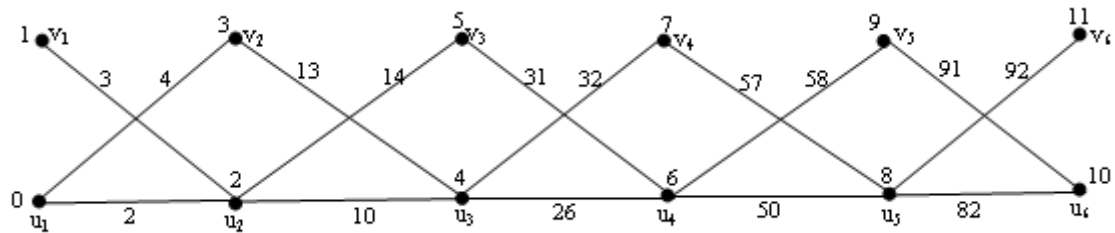


Figure-5:  $S'(P_6)$

**Theorem 2.11:** Switching of a pendant vertex in path  $P_n$  is a mean square sum graph.

**Proof:** Let  $u_1u_2\dots u_n$  be the vertices of path  $P_n$  and  $G^v$  denotes the graph obtained by switching of a pendant vertex  $v$  of  $G = P_n$ . Without loss of generality let the switching vertex be  $v = u_1$ . Clearly  $V(G^v) = \{u_1, u_2, \dots, u_n\}$  and  $E(G^v) = \{u_iu_{i+1}, u_1u_j / 2 \leq i \leq n-1, 3 \leq j \leq n\}$ . Hence  $G$  has  $n$  vertices and  $2n-4$  edges. Define  $f: V(G) \rightarrow \{0, 1, \dots, p-1\}$  as follows  $f(u_1) = 0$  and  $f(u_i) = i-1, 2 \leq i \leq n$ . The induced function  $f^*: E(G) \rightarrow N$  is defined by  $f^*(u_iu_{i+1}) = i^2-i+1, 2 \leq i \leq n-1$ ;  $f^*(u_1u_i) = \left\lfloor \frac{(i-1)^2}{2} \right\rfloor, 3 \leq i \leq n$  is injective. Hence  $G^v$  is a mean square sum graph.

**Example 2.12:** A mean square sum labeling of  $G^v$  when  $n = 5$  is given in figure 6.

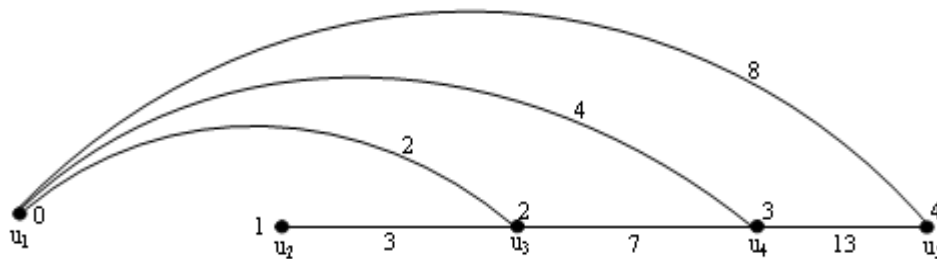


Figure-6

**Theorem 2.13:** Let  $P_n$  be the path and  $G$  be the graph obtained from  $P_n$  by attaching  $C_3$  in both end edges of  $P_n$ . Then  $G$  is a mean square sum graph.

**Proof:** Let  $P_n$  the path  $u_1u_2\dots u_n$ . Add two new vertices  $v_1$  and  $v_2$ . Join  $v_1u_1, v_1u_2, v_2u_{n-1}$ , and  $v_2u_n$ . The resultant graph is  $G$  with  $V(G) = \{u_1, u_2, \dots, u_n, v_1, v_2\}$  and  $E(G) = \{u_iu_{i+1}, u_1v_1, u_2v_1, u_{n-1}v_2, u_nv_2 / 1 \leq i \leq n-1\}$ . Then  $G$  has  $n+2$  vertices and  $n+3$  edges. Define  $f: V(G) \rightarrow \{0, 1, \dots, n+1\}$  as follows  $f(u_i) = i, 1 \leq i \leq n$ ;  $f(v_1) = 0$ ;  $f(v_2) = n+1$ . The induced function  $f^*: E(G) \rightarrow N$  is defined by  $f^*(u_iu_{i+1}) = i^2+i+1, 1 \leq i \leq n-1$ ;  $f^*(u_1v_1) = 1$ ;  $f^*(u_2v_1) = 2$ ;  $f^*(u_{n-1}v_2) = n^2+1$ ;  $f^*(u_nv_2) = n^2+n+1$  is injective. Hence  $G$  is a mean square sum graph.

**Example 2.14:** A mean square sum labeling of  $G$  when  $n = 8$  is given in figure 7.

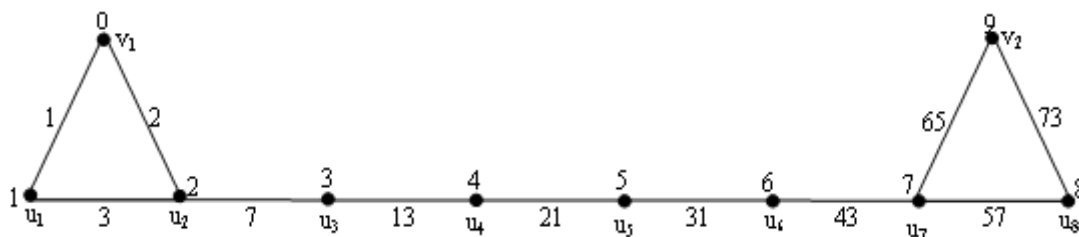


Figure-7

**Theorem 2.15:** A Triangular snake  $T_n$  is a mean square sum graph for  $n \geq 3$ .

**Proof:** Let  $u_1, u_2, \dots, u_n$  be the vertices of  $P_n$ . For  $1 \leq i \leq n-1$ , add new vertex  $v_i$  which is joined to the vertices  $u_i$  and  $u_{i+1}$  of path  $P_n$ . The resultant graph  $G = T_n$  with  $V(G) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_{n-1}\}$  and  $E(G) = \{u_iu_{i+1}, u_iv_i, u_{i+1}v_i / 1 \leq i \leq n-1\}$ . Then  $G$  has  $2n-1$  vertices and  $3n-3$  edges. Define  $f: V(G) \rightarrow \{0, 1, \dots, 2n-2\}$  by  $f(u_i) = 2i-2, 1 \leq i \leq n$ ;  $f(v_i) = 2i-1, 1 \leq i \leq n-1$ . The induced function  $f^*: E(G) \rightarrow N$  is defined by  $f^*(u_iv_i) = 4i^2-6i+3$ ,  $f^*(u_{i+1}v_i) = 4i^2-2i+1$ , and  $f^*(u_iu_{i+1}) = 4i^2-4i+2$ , for  $1 \leq i \leq n-1$  is injective. Hence a triangular snake  $T_n$  is a mean square sum graph.

**Example 2.16:** A mean square sum labeling of  $T_7$  is given in figure 8.

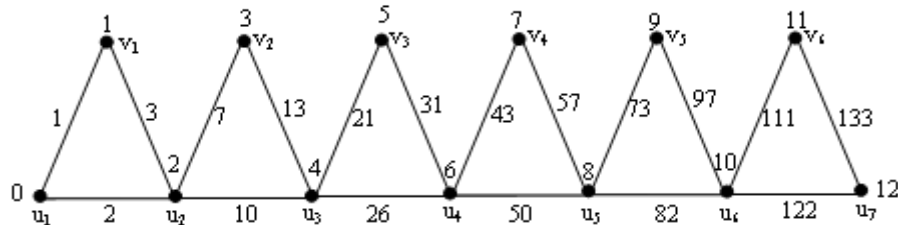


Figure-8:  $T_7$

**Theorem 2.17:** A double triangular snake  $DT_n$  is a mean square sum graph.

**Proof:** Consider a path  $u_1u_2\dots u_n$ . Join  $u_i$  and  $u_{i+1}$  to two new vertices  $v_i$  and  $w_i$ ,  $1 \leq i \leq n-1$ . The resultant graph  $G = DT_n$  with  $V(G) = \{u_i, v_i, w_i / 1 \leq i \leq n\}$  and  $E(G) = \{u_iu_{i+1}, u_iv_i, u_{i+1}v_i, u_iw_i, u_{i+1}w_i, u_iw_i / 1 \leq i \leq n-1\}$ . Then  $G$  has  $3n-2$  vertices and  $5n-5$  edges. Define  $f: V(G) \rightarrow \{0, 1, \dots, 3n-3\}$  by  $f(u_1) = 0$ ;  $f(u_i) = 3i-5$ ,  $2 \leq i \leq n$ ;  $f(v_i) = 3i-1$ ,  $1 \leq i \leq n-1$ ;  $f(w_i) = 3i$ ,  $1 \leq i \leq n-1$ . The induced function  $f^*: E(G) \rightarrow N$  is defined by  $f^*(u_1u_2) = 1$ ;  $f^*(u_iu_{i+1}) = 9i^2-21i+14$ ,  $2 \leq i \leq n-1$ ;  $f^*(u_1v_1) = 2$ ;  $f^*(u_iv_i) = 9i^2-18i+13$ ,  $2 \leq i \leq n-1$ ;  $f^*(u_{i+1}v_i) = 9i^2-9i+3$ ,  $1 \leq i \leq n-1$ ;  $f^*(u_1w_1) = 4$ ;  $f^*(u_iw_i) = 9i^2-15i+12$ ,  $2 \leq i \leq n-1$ ;  $f^*(u_{i+1}w_i) = 9i^2-6i+2$ ,  $1 \leq i \leq n-1$  is injective. Hence the double triangular snake  $DT_n$  is a mean square sum graph.

**Example 2.18:** A mean square sum labeling of  $DT_6$  is given in figure 9.

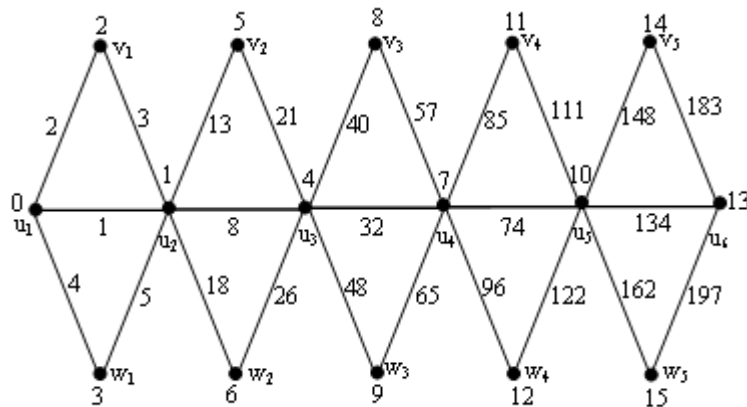


Figure-9:  $DT_6$

**Theorem 2.19:** The coconut tree is a mean square sum graph.

**Proof:** Let  $u_1u_2\dots u_n$  be the vertices of path  $P_n$  and  $v_1v_2\dots v_m$  be the pendent vertices being adjacent with the vertex  $u_1$ . The resultant graph  $G$  is a coconut tree with  $V(G) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_m\}$  and  $E(G) = \{u_iu_{i+1}, u_1v_j / 1 \leq i \leq n-1, 1 \leq j \leq m\}$ . Then  $G$  has  $n+m$  vertices and  $m+n-1$  edges. Define  $f: V(G) \rightarrow \{0, 1, \dots, m+n-1\}$  as follows  $f(u_i) = i-1$ ,  $1 \leq i \leq n$  and  $f(v_i) = n-1+i$ ,  $1 \leq i \leq m$ . The induced function  $f^*: E(G) \rightarrow N$  is defined by  $f^*(u_iu_{i+1}) = i^2-i+1$ ,  $1 \leq i \leq n-1$ ;  $f^*(u_1v_i) = \left\lfloor \frac{(n-1+i)^2}{2} \right\rfloor$ ,  $1 \leq i \leq m$  is injective. Hence the coconut tree is a mean square sum graph.

**Example 2.20:** A mean square sum labeling of  $G$  when  $n = 5$ ,  $m = 6$  is given in figure 10.

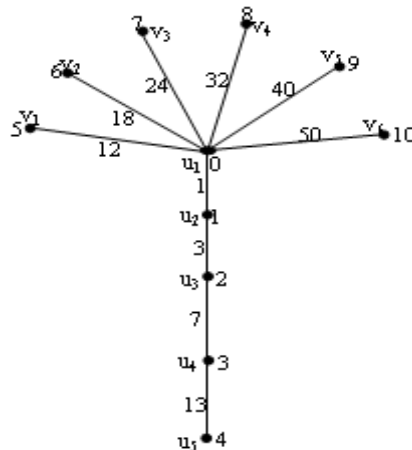


Figure-10

**Theorem 2.21:** Let  $G$  be a graph obtained by attaching each vertex of  $P_n$  to the central vertex of  $K_{1,2}$ . Then  $G$  is a mean square sum graph.

**Proof:** Let  $u_1, u_2, \dots, u_n$  be the path  $P_n$  and  $v_i, w_i, x_i$  be the vertices of  $K_{1,2}$  such that the central vertex  $x_i$  is considered with  $u_i, 1 \leq i \leq n$ . The resultant graph is  $G$  with  $V(G) = \{u_i, v_i, w_i / 1 \leq i \leq n\}$  and  $E(G) = \{u_i v_i, v_i w_i, u_j u_{j+1} / 1 \leq i \leq n, 1 \leq j \leq n-1\}$ . Then  $G$  has  $3n$  vertices and  $3n-1$  edges. Define  $f: V(G) \rightarrow \{0, 1, \dots, 3n-2\}$  by  $f(u_i) = 3i-3, 1 \leq i \leq n$ ;  $f(v_i) = 3i-2, 1 \leq i \leq n$ ;  $f(w_i) = 3i-1, 1 \leq i \leq n$ . The induced function  $f^*: E(G) \rightarrow N$  is defined by  $f^*(u_i w_i) = 9i^2 - 12i + 5, 1 \leq i \leq n$ ;  $f^*(u_i v_i) = 9i^2 - 15i + 7, 1 \leq i \leq n$ ;  $f^*(u_i u_{i+1}) = 9i^2 - 9i + 5, 1 \leq i \leq n-1$  is injective. Hence  $G$  is a mean square sum graph.

**Example 2.22:** A mean square sum labeling of  $G$  when  $n = 4$  is given in figure 11.

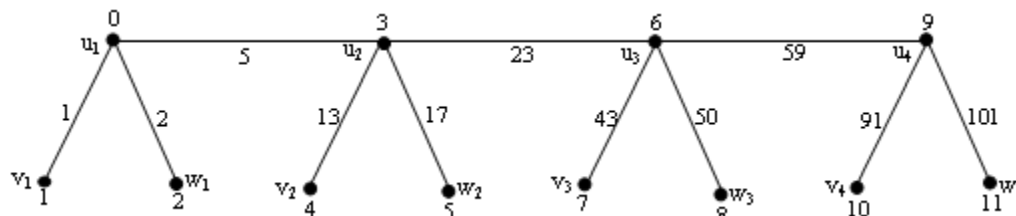


Figure-11

**Theorem 2.23:** Let  $G$  be a graph obtained by attaching each vertex of  $P_n$  to the central vertex of  $K_{1,3}$ . Then  $G$  is a mean square sum graph.

**Proof:** Let  $u_1, u_2, \dots, u_n$  be the path  $P_n$  and  $v_i, w_i, y_i, x_i$  be the vertices of  $K_{1,3}$  such that the central vertex  $x_i$  is considered with  $u_i, 1 \leq i \leq n$ . The resultant graph is  $G$  with  $V(G) = \{u_i, v_i, w_i, y_i / 1 \leq i \leq n\}$  and  $E(G) = \{u_i v_i, v_i w_i, u_i y_i, u_j u_{j+1} / 1 \leq i \leq n, 1 \leq j \leq n-1\}$ . Then  $G$  has  $4n$  vertices and  $4n-1$  edges. Define  $f: V(G) \rightarrow \{0, 1, \dots, 4n-1\}$  by  $f(u_i) = 4i-4, 1 \leq i \leq n$ ;  $f(v_i) = 4i-3, 1 \leq i \leq n$ ;  $f(w_i) = 4i-2, 1 \leq i \leq n$ ;  $f(y_i) = 4i-1, 1 \leq i \leq n$ . The induced function  $f^*: E(G) \rightarrow N$  is defined by  $f^*(u_i v_i) = 16i^2 - 28i + 13, 1 \leq i \leq n$ ;  $f^*(u_i w_i) = 6i^2 - 24i + 10, 1 \leq i \leq n$ ;  $f^*(u_i y_i) = 16i^2 - 20i + 9, 1 \leq i \leq n$ ;  $f^*(u_i u_{i+1}) = 16i^2 - 16i + 8, 1 \leq i \leq n-1$  is injective. Hence  $G$  is a mean square sum graph.

**Example 2.24:** A mean square sum labeling of  $G$  when  $n = 4$  is given in figure 12.

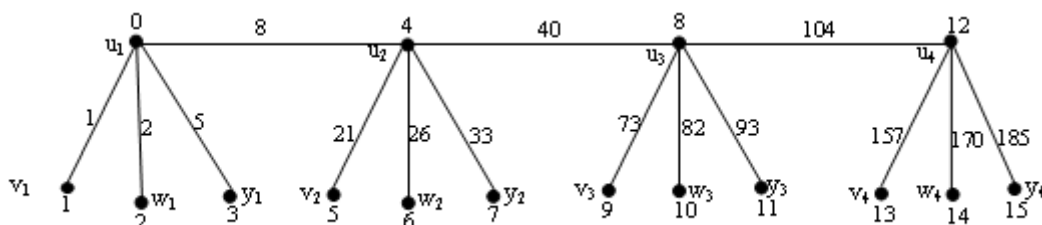


Figure-12

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