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ON TRI STAR TOPOLOGICAL SPACES INDUCED BY BITOPOLOGICAL SPACES

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ABSTRACT

In this paper, a new topological space called Tri star topological space denoted by T^*_{123}-space is introduced. Consequently, various concepts such as T^*_{123}- open, T^*_{123} -pre open, T^*_{123}-semi open sets and T^*_{123}-continuous functions are defined and their properties are investigated.

Keywords: *T^*_{123}- open, T^*_{123} -pre open, T^*_{123}-semi open sets and T^*_{123}-continuous functions.*

AMS Subject Classification: *54A05, 54A10, 54C05, 54E55.*

1. INTRODUCTION

The concept of a bitopological space was first introduced by Kelly [7] in 1963. A nonempty set X with two topologies T_1, T_2 is called a bitopological space, where the topology is defined as $T_1 \cup T_2$ and denoted by T_1T_2. Many research papers on bitopological spaces were then published [1] [2] [3] [4] [5]. As an extension of bitopological space, tri topological space was first initiated by Kovar[8] in 2000, where a nonempty set X with three topology is called a tri topological space. In 2014 Palaniammal and Somasundaram introduced a topology $T_1 \cap T_2 \cap T_3$ in the tri topological space (X, T_1, T_2, T_3) and studied several properties of this topology [9].

In this paper, we introduce a new topology called Tri star topology induced by two bitopology and is denoted by T^*_{123}. The various concepts such as pre open sets, semi open sets and continuous functions in a T^*_{123}- topological space are analyzed.

2. PRELIMINARIES:

Definition 2.1.1: [6] A topology on a non empty set X is a collection T of subsets of X having the following the properties:

1) X and Φ are in T.
2) The union of the elements of any sub collection of T is in T.
3) The intersection of the elements of any finite sub collection of T is in T.

A set X for which a topology T has been specified is called a **Topological space.**

Definition 2.1.2:[9] Let (X, T) be a topological space. $A \subset X$ is called

1. Semi-open if $A \subseteq cl(int(A))$ and Semi-closed set if $int(cl(A)) \subseteq A$.
2. Pre-open if $A \subseteq int(cl(A))$ and Pre-closed set if $cl(int(A)) \subseteq A$.

3. TRI STAR TOPOLOGICAL SPACE

In this section we introduce a new topology in (X, T_1, T_2, T_3)

3.1. T^*_{123}-OPEN SETS

Throughout this article we consider bitopological spaces (X, T_1, T_3) and (X, T_2, T_3) for which the bitopology elements form a topology.

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Definition 3.1.1: Let (X, T_1, T_2, T_3) be a tri topological space. We define a new topology $\mathbf{T^*_{123}}$-called **Tri star topology** induced by two bitopology, as follows $T^*_{123} = (T_1 \cup T_3) \cap (T_2 \cup T_3)$ where $T_1 \cup T_3$ and $T_2 \cup T_3$ are bitopology defined on the bitopological spaces (X, T_1, T_3) and (X, T_2, T_3) respectively.

Definition 3.1.2: $A \subset (X, T_1, T_2, T_3)$ is called T^*_{123}-open in X, if $A \in (T_1 \cup T_3) \cap (T_2 \cup T_3)$. The union of all T^*_{123}-open sets contained in A is called the T^*_{123}-interior of A and denoted by T^*_{123}-int A. We say A is T^*_{123}-closed in X if A^c is T^*_{123}-open, and the intersection of T^*_{123}-closed sets containing A is called T^*_{123}-closure of A and it is denoted by T^*_{123}-cl(A) .

Example 3.1.3: Let X = {a, b, c} with $T_1=\{X, \Phi, \{a\},\{a, b\}\}$, $T_2= \{X, \Phi, \{b\}\}$, $T_3=\{X, \Phi, \{c\},\{a, c\},\{b, c\}\}$. Let $T^*_{123}= (T_1 \cup T_3) \cap (T_2 \cup T_3)$, then {a, c} is T^*_{123}-open and {a, b} is T^*_{123}-closed.

Remark 3.1.4:
1) A is T^*_{123}-open if and only if A is open with respect to T_1T_3 and T_2T_3.
2) A is T^*_{123}-closed if and only if A is closed with respect to T_1T_3 and T_2T_3.
3) X and Φ are both T^*_{123}-open and T^*_{123}-closed.

Theorem 3.1.5: Let (X, T_1, T_2, T_3) be a T^*_{123}-topological space. A is T^*_{123}-open if and only if $A \subseteq T_1T_3$-int(T_2T_3-int A) = T_2T_3-int(T_1T_3-int A).

Proof: If A is T^*_{123}-open, then by Remark 3.1.4, A is open with respect to T_1T_3 and T_2T_3. Hence A= T_iT_3-int A, i = 1, 2. Then T_1T_3-int(T_2T_3-int A)=T_1T_3-int A = A= T_1T_3-int A= T_2T_3-int(T_1T_3-int A). Hence $A \subseteq T_1T_3$-int(T_2T_3-int A) = T_2T_3-int(T_1T_3-int A).

Conversely, $A \subseteq T_1T_3$-int(T_2T_3-int A) = T_2T_3-int(T_1T_3-int A). Since T_iT_3-int $A \subseteq A$, $A \subseteq T_iT_3$-int $A \subseteq A$, i =1, 2. It follows that A = T_iT_3-int A. Hence A is T^*_{123}-open.

Theorem 3.1.6: Let (X, T_1, T_2, T_3) be a T^*_{123}-topological space then A is T^*_{123}-closed if and only if $A \supseteq T_1T_3$-cl(T_2T_3-cl A).

Proof: If A is T^*_{123}-closed then A^c is T^*_{123}-open. By Theorem 3.1.5, $A^c \subseteq T_1T_3$-int(T_2T_3-int (A^c)). Since T_2T_3-int (A^c) = $(T_2T_3\text{-cl } A)^c$, $A^c \subseteq T_1T_3$-int$(T_2T_3\text{-cl } A)^c$. Also, T_1T_3-int$(T_2T_3\text{-cl } A)^c \subseteq (T_1T_3\text{-cl}(T_2T_3\text{-cl } A))^c$, implies $A^c \subseteq (T_1T_3\text{-cl}(T_2T_3\text{-cl } A))^c$. Hence $A \supseteq T_1T_3$-cl(T_2T_3-cl A).

Retracing the above steps, we get the converse.

Theorem 3.1.7:
i) Arbitrary union of T^*_{123}-open set is T^*_{123}-open.
ii) Finite intersection of T^*_{123}-open set is T^*_{123}-open.

Proof:
i) Let $\{A_\alpha \mid \alpha \in I\}$ be the family of T^*_{123}-open sets. By Theorem 3.1.5, for each α, $A_\alpha \subseteq T_1T_3$- int($T_2T_3$-int($A_\alpha$)), this implies $\cup A_\alpha \subseteq \cup (T_1T_3\text{-int}(T_2T_3\text{-int}(A_\alpha))$. Since $\cup (T_1T_3\text{-int}(T_2T_3\text{-int}(A_\alpha)) \subseteq T_1T_3\text{-int } (\cup T_2T_3\text{-int}(A_\alpha)) \subseteq T_1T_3\text{-int } (T_2T_3\text{-int}(\cup A_\alpha))$, this implies $\cup A_\alpha \subseteq T_1T_3\text{-int } (T_2T_3\text{-int}(\cup A_\alpha))$. Hence union of T^*_{123}-open set is T^*_{123}-open.
ii) Let $\{A_i, i=1,2,\ldots n\}$ be the family of T^*_{123}-open sets, then by Theorem 3.1.5, for each i, $A_i \subseteq T_1T_3$-int(T_2T_3-int A_i). This implies that $\cap A_i \subseteq \cap (T_1T_3\text{-int}(T_2T_3\text{-int } A_i))$. Since $\cap (T_1T_3\text{-int}(T_2T_3\text{-int } A_i)) = T_1T_3\text{-int } (\cap T_2T_3\text{-int } A_i)$ and $T_1T_3\text{-int } (\cap T_2T_3\text{-int } A_i) = T_1T_3\text{-int } (T_2T_3\text{-int } \cap A_i)$, we have $\cap A_i \subseteq T_1T_3\text{-int } (T_2T_3\text{-int } \cap A_i)$. Thus by Theorem3.1.5, $\bigcap_{i=1}^{n} A_i$ is T^*_{123}-open.

Remark 3.1.8: T^*_{123} defined in Definition 3.1.1, forms a topology.

3.2. $\mathbf{T^*_{123}}$ PRE OPEN SETS:

Definition 3.2.1: Let (X, T_1, T_2, T_3) be a T^*_{123} -topological space. A subset A of (X, T_1, T_2, T_3) is called T^*_{123}-pre open in X, if $A \subseteq T_1T_3$-int(T_2T_3-cl A). The complement of T^*_{123}-pre open set is called T^*_{123}-pre closed.
i.e., T_1T_3-cl(T_2T_3-int A) $\subseteq$ A.

Example 3.2.2: Let X = {a, b, c} with T_1={X, Φ, {a},{a, b}}, T_2= {X, Φ, {b}},T_3={X, Φ,{c},{a, c},{b, c}}. Clearly A = {a} is T^*_{123}-pre open.

Theorem 3.2.3: Every T^*_{123}-open set is T^*_{123}-pre open.

Proof: Let A be T^*_{123}-open. Then by Theorem 3.1.5, $A \subseteq T_1T_3$-int(T_2T_3-int A). SinceT_1T_3-int(T_2T_3-int A) $\subseteq T_1T_3$-int(T_2T_3-cl A), it follows that $A \subseteq T_1T_3$-int(T_2T_3-cl A). Hence A is T^*_{123}-pre open.

Remark 3.2.4: Converse of the above Theorem need not be true.

Example 3.2.5: Let X = {a, b, c} with T_1 = {X, Φ, {a},{a, b}}, T_2 = {X, Φ, {b}},T_3={X, Φ,{c},{a, c},{b, c}}. Then A = {a, b} is T^*_{123}-pre open but not T^*_{123}-open.

Theorem 3.2.6:
i) Arbitrary union of T^*_{123}- pre open sets is T^*_{123}-pre open.
ii) Arbitrary intersection of T^*_{123}- pre closed sets is T^*_{123}-pre closed.

Proof:
i) Let $\{A_\alpha \mid \alpha \in I\}$ be the family of T^*_{123}-pre open sets in X. By Definition 3.2.1, for each α, $A_\alpha \subseteq T_1T_3$-int($T_2T_3$-cl($A_\alpha$)), this implies that $\cup A_\alpha \subseteq \cup$($T_1T_3$-int($T_2T_3$-cl($A_\alpha$)). Since $\cup$(T_1T_3-int(T_2T_3-cl(A_α)) $\subseteq T_1T_3$-int ($\cup T_2T_3$-cl(A_α)) and T_1T_3-int ($\cup T_2T_3$-cl(A_α)) = T_1T_3-int (T_2T_3-cl ($\cup A_\alpha$)), this implies that $\cup A_\alpha \subseteq$ T_1T_3-int (T_2T_3-cl($\cup A_\alpha$)). Hence $\cup A_\alpha$ is T^*_{123}-pre open.
ii) Let $\{B_\alpha \mid \alpha \in I\}$ be a family of T^*_{123}-pre closed sets in X. Let $A_\alpha = B_\alpha^c$, then $\{A_\alpha / \alpha \in I\}$ is a family of T^*_{123}-pre open sets. By (i), $\cup A_\alpha = \cup B_\alpha^c$ is T^*_{123}-pre open. Consequently $(\cap B_\alpha)^c$ is T^*_{123}-pre open. Hence $(\cap B_\alpha)$ is T^*_{123}-pre closed.

Remark 3.2.7: Finite intersection of T^*_{123}- pre open sets need not be T^*_{123}-pre open.

Example 3.2.8: Let X = {a, b, c} with T_1={X, Φ, {a},{a, b}}, T_2= {X, Φ, {b}},T_3={X, Φ,{c},{a, c},{b, c}}. {a, b} and {b, c} are T^*_{123}- pre open sets, but {a, b} $\cap$ {b, c} ={b} is not T^*_{123}- pre open.

Theorem 3.2.9: In a T^*_{123} topological space (X, T_1, T_2, T_3) the set of all T^*_{123}- pre open sets form a generalized topology.

Proof: Proof follows from Remark 3.1.4, Theorem 3.2.3, Theorem 3.2.6 (i) and Remark 3.2.7.

Definition 3.2.10: Let (X, T_1, T_2, T_3) be a T^*_{123} -topological space. An element $x \in A$ is called T^*_{123}- pre interior point of A, if there exist a T^*_{123}- pre open set V such that $x \in V \subset A$.

Definition 3.2.11: The set of all T^*_{123}-pre interior points of A is called the T^*_{123}- pre interior of A, and is denoted by T^*_{123}- pre-int(A).

Theorem 3.2.12:
i) Let A $\subset$(X, T_1, T_2, T_3). Then T^*_{123}- pre int A is equal to the union of all T^*_{123}- pre open set contained in A.
ii) If A is a T^*_{123}- pre open set then A= T^*_{123}- pre int A.

Proof:
i) We need to prove that, T^*_{123}- pre int A= $\cup\{B \mid B \subset A$, B is T^*_{123}- pre open set}. Let $x \in T^*_{123}$- pre int A. Then there exist a T^*_{123}- pre open set B such that $x \in B \subset A$. Hence $x \in \cup\{B \mid B \subset A$, B is T^*_{123}- pre open set}. Conversely, suppose $x \in \cup\{ B \mid B \subset A$, B is T^*_{123}- pre open set}, then there exist a set $B_o \subset A$ such that $x \in B_o$, where B_o is T^*_{123}- pre open set. i.e., $x \in T^*_{123}$- pre int A. Hence $\cup\{B \mid B \subset A$, B is T^*_{123}- pre open set} $\subset T^*_{123}$- pre int A. So T^*_{123}- pre int A= $\cup\{B \mid B \subset A$, B is T^*_{123}- pre open set}.
ii) Assume A is a T^*_{123}- pre open set then $A \in \{B \mid B \subset A$, T^*_{123}- pre open set}, and every other element in this collection is subset of A. Hence by part (i) T^*_{123}- pre int A=A.

Note 3.2.13:
1. T^*_{123}- pre int A is T^*_{123}- pre open.
2. T^*_{123}- pre int A is the largest T^*_{123}- pre open set contained in A.

Theorem 3.2.14:
i) T^*_{123}- pre int $(A \cup B) \supset T^*_{123}$- pre int $A \cup T^*_{123}$- pre int B.
ii) T^*_{123}- pre int $(A \cap B) = T^*_{123}$- pre int $A \cap T^*_{123}$- pre int B.

Proof:

i) The fact that T^*_{123}- pre int $A \subset A$ and T^*_{123}- pre int $B \subset B$ implies T^*_{123}- pre int $A \cup T^*_{123}$- pre int $B \subset A \cup B$. Since pre interior of a set is pre open, T^*_{123}-pre int A and T^*_{123}-pre int B are pre open. Hence by Theorem 3.2.6 of (i), T^*_{123}-pre int $A \cup T^*_{123}$- pre int B is pre open and contained in $A \cup B$. Since T^*_{123}- pre int $(A \cup B)$ is the largest T^*_{123}-pre open set contained in $A \cup B$, it follows that T^*_{123}- pre int $A \cup T^*_{123}$- pre int $B \subset T^*_{123}$- pre int $(A \cup B)$.

ii) Let $x \in T^*_{123}$- pre int $(A \cap B)$. Then there exist a T^*_{123}-pre open set V, such that $x \in V \subset (A \cap B)$. That is there exist a T^*_{123}-pre open set, such that $x \in V \subset A$ and $x \in V \subset B$. Hence $x \in T^*_{123}$- pre int A and $x \in T^*_{123}$- pre int B. That is $x \in T^*_{123}$- pre int $A \cap T^*_{123}$- pre int B. Thus T^*_{123}- pre int $(A \cap B) \subset T^*_{123}$- pre int $A \cap T^*_{123}$- pre int B.

Retracing the above steps, we get the converse.

3.3. T^*_{123} – PRE CLOSED SETS

Definition 3.3.1: Let (X, T_1, T_2, T_3) be a T^*_{123}-topological space. Let $A \subset X$. The intersection of all T^*_{123}- pre closed sets containing A is called T^*_{123}- pre closure of A and it is denoted by T^*_{123}- pre cl(A). T^*_{123}- pre cl(A) = $\cap \{B / B \supset A$, B is T^*_{123}- pre closed set$\}$.

Note 3.3.2:

1. T^*_{123}- pre cl(A) is also a T^*_{123}- pre closed set.
2. T^*_{123}- pre cl(A) is smallest T^*_{123}- pre closed set containing A.

Theorem 3.3.3: Every T^*_{123}-closed set is T^*_{123}-pre closed.

Proof: Let A be T^*_{123}-closed, then by Theorem 3.1.6**,** we haveT_1T_3-cl(T_2T_3-cl A) $\subseteq$ A. Since T_1T_3-cl(T_2T_3-int A) $\subseteq$ T_1T_3-cl(T_2T_3-cl A) $\subseteq$ A, A is T^*_{123}-pre closed.

Remark 3.3.4: Converse of the above Theorem need not be true.

Example 3.3.5: Let X = {a, b, c} with T_1 = {X, Φ , {a},{a, b}}, T_2 = {X, Φ , {b}},T_3 = {X, Φ ,{c},{a, c},{b, c}}. Then A = {c, b} is T^*_{123}-pre closed but not T^*_{123}-closed.

Theorem 3.3.6: A is T^*_{123}- pre closed if and only if A = T^*_{123}- pre cl(A).

Proof: T^*_{123}- pre cl(A) = $\cap \{B/B \supset A$, B is T^*_{123}- pre closed set$\}$. If A is a T^*_{123}- pre closed set then A is a member of the above collection and each member contains A. Hence their intersection is A and T^*_{123}- pre cl(A) = A. Conversely, if A= T^*_{123}- pre cl(A), then A is T^*_{123}- pre closed by Note 3.3.2.

3.4. T^*_{123}-SEMI OPEN SETS

Definition 3.4.1: Let (X, T_1, T_2, T_3) be a T^*_{123}- topological space. A subset A of (X, T_1, T_2, T_3) is called T^*_{123}-semi open in X, if A $\subseteq T_1T_3$-cl(T_2T_3-int A). The complement of T^*_{123}-semi open set is called T^*_{123}-semi closed.

i.e., T_1T_3-int(T_2T_3-cl A) $\subseteq$ A.

Example 3.4.2: Let X = {a, b, c} with T_1 = {X, Φ , {a},{a, b}}, T_2= {X, Φ , {b}},T_3={X, Φ ,{c},{a, c},{b, c}}. Clearly A = {b} is T^*_{123}-semi open.

Theorem 3.4.3:

i) Every T^*_{123}-open set is T^*_{123}-semi open.
ii) Every T^*_{123}-closed set is T^*_{123}-semi closed.

Proof:

i) If A is T^*_{123}-open set then by Theorem 3.1.5, A $\subseteq T_1T_3$-int(T_2T_3-int A). Since T_1T_3-int(T_2T_3-int A) $\subseteq T_1T_3$-cl(T_2T_3-int A), A $\subseteq$ T_1T_3-cl(T_2T_3-int A)**.** Hence A is T^*_{123}-semi open.

ii) If A is T^*_{123}-closed set then by Theorem 3.1.6**,** we have T_1T_3-cl(T_2T_3-cl A) $\subseteq$ A. Since T_1T_3-int(T_2T_3-cl A) $\subseteq$ T_1T_3-cl(T_2T_3-clA), T_1T_3-int(T_2T_3-cl A) $\subseteq$ A. Hence A is T^*_{123}-semi closed.

Remark 3.4.4: Converse of the above Theorem need not be true.

Example 3.4.5:

i) Let X = {a, b, c} with T_1 = {X, Φ, {a},{a, b}}, T_2= {X, Φ, {b}},T_3={X, Φ, {c},{a, c},{b, c}}. Clearly A = {b} is T^*_{123}-semi open, but not T^*_{123}-open.
ii) Let X = {a, b, c} with T_1 ={X, Φ ,{a},{a, b}}, T_2 = {X, Φ , {b}},T_3 = {X, Φ ,{c},{a, c}, {b, c}}. Clearly A = {a, c} is T^*_{123}-semi closed, but not T^*_{123}-closed.

3.5. CONTINUOUS FUNCTIONS IN T^*_{123}-TOPOLOGICAL SPACES

Definition 3.5.1: Let (X, T_1, T_2, T_3) and (Y, σ_1, σ_2, σ_3) be two T^*_{123}-topological spaces. A function f: $X \to Y$ is called T^*_{123}-continuous function, if $f^{-1}(V)$ is T^*_{123}-open in X for every T^*_{123}- open set V in Y.

Example 3.5.2: Let X = {a, b, c} with topologies T_1 = {X, Φ , {a}}, T_2 = {X, Φ , {b, c}}, T_3 = {X, Φ , {c},{a,},{b, c}} and Y={1,2,3} with topologies σ_1={Y, Φ , {1}}, σ_2= {X, Φ , {2,3}}, σ_3={X, Φ , {1},{2,3}} and f: $X \to Y$ be a function defined as f(a) = 1, f(b) = 2, f(c) = 3. T^*_{123}-open sets in X are {a},{b, c} and T^*_{123}-open sets in Y are {1},{2,3}. Therefore for every T^*_{123}-open set V in Y, $f^{-1}(V)$ is T^*_{123}-open set in X. Then f is T^*_{123}-continuous function.

Definition 3.5.3: Let X and Y be the two T^*_{123}-topological space. A function f: $X \to Y$ is called T^*_{123}-continuous at a point a ϵ X if for every T^*_{123}-open set V containing f(a) in Y, there exist a T^*_{123}-open set U containing a in X, such that f(U) $\subset$ V.

Theorem 3.5.4: f: $X \to Y$ is T^*_{123}-continuous if and only if f is T^*_{123}-continuous at each point of X.

Proof: Let f: $X \to Y$ be T^*_{123}-continuous. Let a ϵ X, and V be a T^*_{123}-open set in Y containing f(a). Since f is T^*_{123}-continuous, $f^{-1}(V)$ is T^*_{123}-open in X containing a. Let U=$f^{-1}(V)$, then f(U) $\subset$ V, and f(a)ϵU. Hence f is continuous at a.

Conversely, suppose f is T^*_{123}-continuous at each point of X. Let V be T^*_{123}-open set in Y. If $f^{-1}(V) = \Phi$ then it is T^*_{123}-open. So let $f^{-1}(V) \neq \Phi$. Take any a ϵ f^{-1} (V), then f(a) ϵ V. Since f is T^*_{123}-continuous at each point there exist a T^*_{123}-open set U_a containing a such that f (U_a) $\subset$ V. Let Let U = $\bigcup$ (U_a |a ϵ $f^{-1}(V)$).

Claim: U = $f^{-1}(V)$
If x ϵ $f^{-1}(V)$ then x ϵ $U_x \subset$ U. Hence $f^{-1}(V) \subset$ U. On the other hand, suppose yϵU then y ϵ U_x for some x and y ϵ f^{-1} (V). Hence U = f^{-1} (V).

Since U_x is T^*_{123}-open, by Theorem 3.1.7 (i) U is T^*_{123}-open and hence U = f^{-1} (V) is T^*_{123}-open for every T^*_{123}-open set V in Y. Hence f is T^*_{123}-continuous.

Theorem 3.5.5: Let (X, T_1, T_2, T_3) and (Y, σ_1, σ_2, σ_3) be two T^*_{123}-topological spaces. Then f: $X \to Y$ is T^*_{123}-continuous function if and only if f^{-1} (V) is T^*_{123}- closed in X, whenever V is T^*_{123}- closed in Y.

Proof: Let f: $X \to Y$ is T^*_{123}-continuous function and V be T^*_{123}- closed in Y. Then V^c is T^*_{123}-open in Y. By hypothesis $f^{-1}(V^c)$ is T^*_{123}-open in X, i.e., $[f^{-1}(V)]^c$ is T^*_{123}-open in X. Hence $f^{-1}(V)$ is T^*_{123}-closed in X whenever V-is T^*_{123}- closed in Y. Conversely, suppose f^{-1} (V) is T^*_{123}- closed in X whenever V is T^*_{123}-closed in Y. Let U is T^*_{123}-open in Y then U^c is T^*_{123}-closed in Y. By assumption $f^{-1}(U^c)$ is T^*_{123}-closed in X. i.e., $[f^{-1}(U)]^c$ is T^*_{123}-closed in X. Then $f^{-1}(U)$ is T^*_{123}-open in X. Hence f is T^*_{123}-continuous.

Theorem3.5.6: Let (X, T_1, T_2, T_3) and (Y, σ_1, σ_2, σ_3) be two T^*_{123}-topological space. Then f: $X \to Y$ is T^*_{123}-continuous function if and only if f(T^*_{123}-cl A) $\subset T^*_{123}$-cl [f(A)].

Proof: Suppose f: $X \to Y$ is T^*_{123}-continuous and T^*_{123}- cl [f(A)] is T^*_{123}-closed in Y. Then by Theorem 3.5.5, $f^{-1}(T^*_{123}$- cl [f(A)]) is T^*_{123}-closed in X. Consequently, T^*_{123}- cl[$f^{-1}(T^*_{123}$- cl [f(A)])] = $f^{-1}(T^*_{123}$- cl [f(A)]). Since f(A) $\subset T^*_{123}$- cl [f(A)], A $\subset$ $f^{-1}(T^*_{123}$- cl [f(A)]) and T^*_{123}- cl(A) $\subset T^*_{123}$- cl ($f^{-1}(T^*_{123}$- cl [f(A)])) = $f^{-1}(T^*_{123}$- cl [f(A)]) Hence f (T^*_{123}- cl (A)) $\subset T^*_{123}$- cl [f(A)].

Conversely, if f(T^*_{123}- cl (A)) $\subset T^*_{123}$- cl [f(A)] for all A $\subset$ X. Let F be T^*_{123}-closed set in Y, so that

$$T^*_{123}\text{- cl}(F)=F \tag{1}$$

By hypothesis, f(T^*_{123}- cl (f^{-1}(F)) $\subset T^*_{123}$- cl [f(f^{-1}(F))] $\subset T^*_{123}$- cl (F), then by (1), T^*_{123}- cl (f^{-1}(F))$\subset$F. It follows that T^*_{123}- cl (f^{-1}(F)) $\subset$ f^{-1}(F). But always f^{-1}(F) $\subset T^*_{123}$- cl (f^{-1}(F)], so that T^*_{123}- cl (f^{-1}(F)) = f^{-1}(F). Hence f^{-1}(F) is T^*_{123}-closed in X and f is continuous by Theorem 3.5.5.

Theorem 3.5.7: Let (X, T_1, T_2, T_3), $(Y, \sigma_1, \sigma_2, \sigma_3)$ and $(Z, \theta_1, \theta_2, \theta_3)$ be three T^*_{123}-topological spaces. If $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ are T^*_{123}-continuous mappings then $g \circ f: X \rightarrow Z$ is also T^*_{123}- continuous.

Proof: Let G be a T^*_{123}-open set in Z. Since by g is T^*_{123}-continuous, $g^{-1}(G)$ is T^*_{123}-open set in Y. Now, $(g \circ f)^{-1}G = (f^{-1} \circ g^{-1})G = f^{-1}(g^{-1}(G))$. Take $g^{-1}(G) = H$ which is T^*_{123}-open in Y, then $f^{-1}(H)$ is T^*_{123}-open in X, since by f is T^*_{123}-continuous. Hence $g \circ f: X \rightarrow Z$ is T^*_{123}- continuous function.

3.6. T^*_{123}-PRE CONTINUOUS AND T^*_{123}-SEMI CONTINUOUS FUNCTIONS

Definition 3.6.1: Let (X, T_1, T_2, T_3) and $(Y, \sigma_1, \sigma_2, \sigma_3)$ be two T^*_{123}-topological spaces, then $f: X \rightarrow Y$ is T^*_{123}-pre continuous if $f^{-1}(V)$ is T^*_{123}- pre closed in X whenever V is T^*_{123}-closed in Y.

Example 3.6.2: Let X = {a, b, c} with topologies $T_1 = \{X, \Phi, \{a\}, \{a, b\}\}$, $T_2 = \{X, \Phi, \{b\}, \{c\}, \{b, c\}\}$, $T_3 = \{X, \Phi, \{c\}, \{a, c\}\}$ and Y = {1,2,3} with topologies $\sigma_1 = \{Y, \Phi, \{1\}, \{1,2\}\}$, $\sigma_2 = \{X, \Phi, \{2\}, \{3\}, \{2,3\}\}$, $\sigma_3 = \{X, \Phi, \{3\}, \{2,3\}\}$ and let $f: X \rightarrow Y$ be a function defined as f(a) = 1, f(b) = 2, f(c) = 3. Here T^*_{123}-closed sets in Y are {2} and {1, 2}. Then the inverse images of these sets are {b}, {a, b} and they are T^*_{123}- pre closed in X. Hence f is T^*_{123}-pre continuous.

Theorem 3.6.3: Every T^*_{123}-continuous function is T^*_{123}-pre continuous.

Proof: Let $f: X \rightarrow Y$ be T^*_{123}-continuous. i.e., $f^{-1}(V)$ is T^*_{123}- closed in X, whenever V is T^*_{123}-closed in Y. By Theorem 3.3.3, every T^*_{123}-closed set is T^*_{123}-pre closed, and hence $f^{-1}(V)$ is T^*_{123}- pre closed in X whenever V is closed in Y. Hence $f: X \rightarrow Y$ be T^*_{123}-pre continuous.

Remark 3.6.4: Converse of above Theorem need not be true.

Example 3.6.5: Let X ={a, b, c} with topologies $T_1=\{X, \Phi, \{a\}, \{a, b\}\}$, $T_2=\{X, \Phi, \{b\}, \{c\}, \{b, c\}\}$, $T_3=\{X, \Phi, \{c\}, \{a, c\}\}$ and Y={1,2,3} with topologies $\sigma_1=\{Y, \Phi, \{1\}, \{1,2\}\}$, $\sigma_2= \{X, \Phi, \{2\}, \{3\}, \{2,3\}\}$, $\sigma_3=\{X, \Phi, \{3\}, \{2,3\}\}$ and let $f: X \rightarrow Y$ be a function defined as f(a) = 2, f(b) = 1, f(c) = 1. Here f is T^*_{123}-pre continuous but not T^*_{123}-continuous. For {2} is T^*_{123}-closed in Y, $f^{-1}(\{2\}) = \{a\}$ is T^*_{123}-pre closed in X, but not T^*_{123}-closed in X.

Definition 3.6.6: Let (X, T_1, T_2, T_3) and $(Y, \sigma_1, \sigma_2, \sigma_3)$ be two T^*_{123}-topological space, then $f: X \rightarrow Y$ is T^*_{123}-semi continuous if $f^{-1}(V)$ is T^*_{123}- semi closed in X whenever V is closed in Y.

Example 3.6.7: Let X = {a, b, c} with topologies $T_1 = \{X, \Phi, \{a\}, \{a, b\}\}$, $T_2 = \{X, \Phi, \{b\}, \{c\}, \{b, c\}\}$, $T_3=\{X, \Phi, \{c\}, \{a, c\}\}$ and Y={1,2,3} with topologies $\sigma_1=\{Y, \Phi, \{1\}, \{1,2\}\}$, $\sigma_2= \{X, \Phi, \{2\}, \{3\}, \{2,3\}\}$, $\sigma_3=\{X, \Phi, \{3\}, \{2,3\}\}$ and let $f: X \rightarrow Y$ be a function defined as f(a) = 1, f(b) = 2, f(c) = 3. Here T^*_{123}-closed sets in Y are {2} and {1,2}. Then the inverse images of these sets are {b}, {a, b} and they are T^*_{123}- semi closed in X. Hence f is T^*_{123}-semi continuous.

Theorem 3.6.8: Every T^*_{123}-continuous function is T^*_{123}-semi continuous.

Proof: Let $f: X \rightarrow Y$ be T^*_{123}-continuous. i.e.,$f^{-1}(V)$ is T^*_{123}- closed in X, whenever V is T^*_{123}-closed in Y. By Theorem 3.4.3 (ii), every T^*_{123}-closed set is T^*_{123}-semi closed. This implies that $f^{-1}(V)$ is T^*_{123}- semi closed in X whenever V is closed in Y. Hence $f: X \rightarrow Y$ be T^*_{123}-semi continuous.

Remark 3.6.9: Converse of above Theorem need not be true.

Example 3.6.10: Let X ={a, b, c} with topologies $T_1=\{X, \Phi, \{a\}, \{a, b\}\}$, $T_2= \{X, \Phi, \{b\}, \{c\}, \{b, c\}\}$, $T_3=\{X, \Phi, \{c\}, \{a, c\}\}$and Y={1,2,3} with topologies $\sigma_1=\{Y, \Phi, \{1\}, \{1,2\}\}$, $\sigma_2= \{X, \Phi, \{2\}, \{3\}, \{2,3\}\}$, $\sigma_3=\{X, \Phi, \{3\}, \{2,3\}\}$ and let $f: X \rightarrow Y$ be a function defined as f(a) = 2, f(b) = 1, f(c) = 1. Here f is T^*_{123}-semi continuous but not T^*_{123}-continuous, since {2} is T^*_{123}-closed in Y, $f^{-1}(\{2\}) = \{a\}$ is T^*_{123}-semi closed in X, but not T^*_{123}-closed in X.

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