On αg*p-Connectedness and αg*p-Compactness in Topological Spaces

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ABSTRACT

In this paper, we introduce the concept of αg*p-connectedness and αg*p-compactness in topological spaces. We investigate and study their basic properties. We also discuss their relationship with already existing concepts.

Keywords: αg*-connected space, gp*-connected space, g*p-connected space, αg*p-connected space, αg*p-compact space.

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1. INTRODUCTION

The notion of connectedness and compactness are useful and fundamental notions of not only general topology but also of other advanced branches of mathematics in topological spaces. In 1974, Das defined the concept of semi-connectedness in topology and investigated its properties. Compactness is one of the most important, useful and fundamental concepts in topology. In 1981, Dorsett introduced and studied the concept of semi-compact spaces. In 2011, P.G.Patil, T.D.Rayanagoudar and Mahesh K.Bhat defined the concept of g*p-compactness and g*p-connectedness in topological spaces. Recently, S.Sekar and P.Jayakumar introduced and studied the concept of gp*-compact and gp*-connected spaces.

The authors[12] introduced αg*p-closed sets and αg*p-open sets in topological spaces and established their relationships with some generalized sets in topological spaces. The aim of this paper is to introduce the concept of αg*p-connected and αg*p-compactness in topological spaces. We investigate their basic properties. We also discuss their relationship with already existing concepts.

Throughout this paper (X,τ) and (Y,σ) represents topological spaces on which no separation axioms are assumed, unless otherwise mentioned. The union of all αg*p-open sets of X contained in A is called αg*p-interior of A and it is denoted by αg*p-int (A). The intersection of all αg*p-closed sets of X containing A is called αg*p-closure of A and it is denoted by αg*p-cl(A).

2. PRELIMINARIES

We recall the following definitions which are useful in the sequel.

Definition 2.1: A subset A of a topological space (X, τ) is called

(i) preopen [7] if A ⊆ int (cl (A)) and preclosed if cl (int(A)) ⊆ A.
(ii) α-open [8] if A ⊆ int (cl (int (A))) and α-closed if cl(int(cl(A))) ⊆ A.

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Definition 2.2: A subset A of a topological space \((X, \tau)\) is called
(i) generalized closed (briefly, g-closed) \([4]\) if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(X\).
(ii) \(\alpha\)-generalized closed (briefly, \(\alpha\)g-closed) \([5]\) if \(\alpha\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(X\).
(iii) generalized pre-closed (briefly, gp-closed) \([6]\) if \(\text{pcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(X\).
(iv) generalized star pre-closed (briefly, g*p-closed set) \([14]\) if \(\text{pcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is g-open in \(X\).
(v) \(\alpha\)-generalized pre-star closed (briefly, \(\alpha\)gp-closed set) \([3]\) if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\alpha\)g-open in \(X\).
(vi) \(\alpha\)-generalized star closed (briefly, \(\alpha\)g*-closed set) if \(\alpha\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\alpha\)g-open in \(X\).
(vii) \(\alpha\)-generalized star pre-closed (briefly, \(\alpha\)g*p-closed) \([12]\) if \(\text{pcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\alpha\)g-open in \(X\).

Definition 2.3: A subset A of a topological space \((X, \tau)\) is called
(i) \(\alpha\)g*p-continuous \([10]\) if \(f^{-1}(V)\) is \(\alpha\)g*p-closed set in \((X, \tau)\) for every closed set \(V\) in \((Y, \sigma)\).
(ii) \(\alpha\)g*p-irresolute \([10]\) if \(f^{-1}(V)\) is \(\alpha\)g*p-closed set in \((X, \tau)\) for every \(\alpha\)g*p-closed set \(V\) in \((Y, \sigma)\).
(iii) contra \(\alpha\)g*p-continuous \([11]\) if \(f^{-1}(V)\) is \(\alpha\)g*p-closed set in \(X\) for every open set \(V\) in \(Y\).

Definition 2.4 \([15]\): A topological space \(X\) is said to be connected if \(X\) cannot be written as the disjoint union of two non empty open sets in \(X\).

Definition 2.5: A topological space \(X\) is said to be \(\alpha\)g*-connected if \(X\) cannot be written as the disjoint union of two non empty \(\alpha\)g*-open sets in \(X\).

Definition 2.6 \([9][13]\): A topological space \(X\) is said to be \(g\)p*-connected (resp. gp*-connected) if \(X\) cannot be written as the disjoint union of two non empty \(g\)p-open (resp. gp*-open) sets in \(X\).

Definition 2.7: \([10]\) A space \((X, \tau)\) is called if \(\alpha\)g*p-space if every \(\alpha\)g*p-closed set is closed.

Lemma 2.8 \([12]\): (i) Every closed set is \(\alpha\)g*p-closed.
(ii) Every \(\alpha\)g*-closed set is \(\alpha\)g*p-closed.
(iii) Every gp*-closed set is \(\alpha\)g*p-closed.
(iv) Every \(\alpha\)g*p-closed set is gp*-closed.

Lemma 2.9 \([10]\): Let \(f: (X, \tau) \rightarrow (Y, \sigma)\) be a function. Then the following are equivalent.
(i) \(f\) is \(\alpha\)g*p-continuous.
(ii) The inverse image of each closed set in \(Y\) is \(\alpha\)g*p-closed in \(X\).
(iii) The inverse image of each open set in \(Y\) is \(\alpha\)g*p-open in \(X\).

Lemma 2.10 \([10]\): A function \(f: X \rightarrow Y\) is \(\alpha\)g*p-irresolute if and only if the inverse image of every \(\alpha\)g*p-open set in \(Y\) is \(\alpha\)g*p-open in \(X\).

3. \(\alpha\)g*p -CONNECTED SPACES

In this section we introduce \(\alpha\)g*p-connected spaces and investigate their basic properties.

Definition 3.1: A topological space \(X\) is said to be \(\alpha\)g*p-connected if \(X\) cannot be written as the disjoint union of two non empty \(\alpha\)g*p-open sets in \(X\).

Example 3.2: Let \(X = \{a, b, c, d\}\) be given the topology \(\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}\).
\(\alpha\)g*p-O(X) = \(\{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c, \{a, b, d\}, X\}\) Then the space \(X\) is \(\alpha\)g*p-connected.

Definition 3.3: A subset \(S\) of a topological space \(X\) is said to be \(\alpha\)g*p-connected relative to \(X\) if \(S\) cannot be written as the disjoint union of two non empty \(\alpha\)g*p-open sets in \(X\).

Theorem 3.4: For a topological space \(X\), the following are equivalent.
(i) \(X\) is \(\alpha\)g*p-connected
(ii) The only subsets of \(X\) which are both \(\alpha\)g*p-open and \(\alpha\)g*p-closed are the empty set and \(X\).
(iii) Each \(\alpha\)g*p-continuous function of \(X\) into a discrete space \(Y\) with at least two points is a constant map.
**Example 3.7:** Let \( X = \{a, b, c, d\} \) be given the topology \( \tau = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}, X\} \).

\( \alpha g^p\)-O(X) = \( \{\emptyset, \{c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\} \)

The space \( X \) is connected but not \( \alpha g^p\)-connected.

**Example 3.8:** Let \( X = \{a, b, c, d\} \) be given the topology \( \tau = \{\emptyset, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\} \).

\( \alpha g^p\)-O(X) = \( \{\emptyset, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\} \)

The converse of Theorem 3.6 is not true as shown in the following example.
The space $X$ is $\alpha g^*$-connected but not $\alpha g^p$-connected.

**Theorem 3.11:** Every $\alpha g^p$-connected space is $gp^*$-connected.

**Proof:** Let $X$ be an $\alpha g^p$-connected space. Suppose $X$ is not $gp^*$-connected. Then by using Lemma 3.8, there exists a proper non empty subset $B$ of $X$ which is both $gp^*$-open and $gp^*$-closed in $X$. Since every $gp^*$-closed (open) is $\alpha g^p$-closed (open) then $X$ is not $\alpha g^p$-connected. This proves the theorem.

The converse of Theorem 3.11 is not true as shown in the following example.

**Example 3.12:** Let $X = \{a, b, c, d\}$ be given the topology $\tau = \{\emptyset, \{a, b\}, X\}$.

$gp^*$-O$(X) = \{\emptyset, \{a, b\}, X\}$.

$\alpha g^p$-O$(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$.

The space $X$ is $gp^*$-connected but not $\alpha g^p$-connected.

**Theorem 3.13:** Every $gp^*$-connected space is $\alpha g^p$-connected.

**Proof:** Let $X$ be an $gp^*$-connected space. Suppose $X$ is not $\alpha g^p$-connected. Then by using Lemma 3.8, there exists a proper non empty subset $B$ of $X$ which is both $\alpha g^p$-open and $\alpha g^p$-closed in $X$. Since every $\alpha g^p$-closed (open) is $gp^*$-closed (open) then $X$ is not $\alpha g^p$-connected. This proves the theorem.

The converse of Theorem 3.13 is not true as shown in the following example.

**Example 3.14:** Let $X = \{a, b, c, d\}$ be given the topology $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$.

$g^*$-O$(X) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}, X\}$.

The space $X$ is $\alpha g^p$-connected but not $\alpha g^p$-connected but not $\alpha g^p$-connected.

**Remark 3.15:** If $A$ is $\alpha g^p$-closed in $(X, \tau)$, then $A$ is closed in $(X, \tau_{\alpha g^p})$ provided $\tau_{\alpha g^p}$ is a topology.

**Theorem 3.16:** Suppose $X$ is a topological space with $\tau_{\alpha g^p} = \tau$. Then $X$ is connected if and only if $X$ is $\alpha g^p$-connected.

**Proof:** Suppose $X$ is not $\alpha g^p$-connected. Then there exists a proper non empty subset $B$ of $X$ which is both $\alpha g^p$-open and $\alpha g^p$-closed in $X$. Since $\tau_{\alpha g^p} = \tau$, every $\alpha g^p$-closed set is closed. Therefore $B$ is both open and closed in $X$ that implies $X$ is not connected. This proves that connectedness implies $\alpha g^p$-connectedness.

**Theorem 3.17:** Suppose $X$ is an $\alpha g^p$-space. Then $X$ is $\alpha g^p$-connected if and only if $X$ is $\alpha g^*$-connected.

**Proof:** Suppose $X$ is $\alpha g^p$-connected. Then by using Theorem 3.9, $X$ is $\alpha g^*$-connected.

Conversely we assume that $X$ is $\alpha g^*$-connected. Suppose $X$ is not $\alpha g^p$-connected. Then there exists a proper non empty subset $B$ of $X$ which is both $\alpha g^p$-open and $\alpha g^p$-closed in $X$. Since $\tau_{\alpha g^p} = \tau$, every $\alpha g^p$-closed set is closed. Therefore $B$ is both open and closed in $X$ that shows $X$ is not $\alpha g^*$-connected, a contradiction. Therefore $X$ is $\alpha g^p$-connected.

**Theorem 3.18:** A contra $\alpha g^p$-continuous image of an $\alpha g^p$-connected space is connected.

**Proof:** Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a contra $\alpha g^p$-continuous function from an $\alpha g^p$-connected space $X$ on to a space $Y$. Assume that $Y$ is disconnected. Then $Y = A \cup B$ where $A$ and $B$ are non empty clopen sets in $Y$ with $A \cap B = \emptyset$. Since $f$ is contra $\alpha g^p$-continuous, we have that $f^{-1}(A)$ and $f^{-1}(B)$ are non empty $\alpha g^p$-open sets in $X$ with $f^{-1}(A) \cup f^{-1}(B) = f^{-1}(A \cup B) = f^{-1}(Y) = X$ and $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = f^{-1}(\emptyset) = \emptyset$. This means that $X$ is not $\alpha g^p$-connected, which is a contradiction. This proves the theorem.

**Definition 3.19:** Let $X$ be a topological space. Two non-empty subsets $A$ and $B$ of $X$ are called $\alpha g^p$-separated iff $\alpha g^p$-cl$(A) \cap B = A \cap \alpha g^p$-cl$(B) = \emptyset$.

**Theorem 3.20:** Let $X$ be a topological space, then the following statements are equivalent:

(i) $X$ is a $\alpha g^p$-connected space.

(ii) $X$ is not the union of any two $\alpha g^p$-separated sets.
Proof: (⇒) Let A and B be a two αg*p-separated sets such that X = A ∪ B. Then αg*p-cl(A) ∩ B = A ∩ αg*p-cl(B) = ∅. Since A ⊆ αg*p-cl(A) and B ⊆ αg*p-cl(B), then A ∩ B = ∅. Now αg*p-cl(A) ⊆ X \ B = A. Hence A = αg*p-cl(A). Then A is αg*p-closed set. By the same way we can show that B is αg*p-closed set which is a contradiction. Hence X cannot be written as the union of two αg*p-separated sets.

(⇐) Let A and B be a two disjoint non-empty and αg*p-closed sets such that X = A ∪ B. Then αg*p-cl(A) ∩ B = A ∩ αg*p-cl(B) = A ∩ B = ∅ which is a contradiction with the hypothesis. Therefore X is a αg*p-connected space.

**Theorem 3.21:** Let A be a αg*p-connected set and H, K be an αg*p-separated sets. If A ⊆ H ∪ K then either A ⊆ H or A ⊆ K.


**Theorem 3.22:** If H is αg*p-connected set and H ⊆ E ⊆ αg*p-cl(H) then E is αg*p-connected.

Proof: If E is not αg*p-connected, then there exists two sets A, B such that αg*p-cl(A) ∩ B = A ∩ αg*p-cl(B) = ∅ and E = A ∪ B. Since H ⊆ E, either H ⊆ A or H ⊆ B. Suppose H ⊆ A then αg*p-cl(H) ⊆ αg*p-cl(A). Thus αg*p-cl(H) ∩ B = αg*p-cl(A) ∩ B = φ. But B ⊆ E ⊆ αg*p-cl(H) then αg*p-cl(H) ∩ B = φ. Therefore B = φ which is a contradiction.

Thus E is αg*p-connected set. If H ⊆ E, then by the same way we can prove that A = φ which is a contradiction. Then E is αg*p-connected.

**Corollary 3.23:** If a space X contains a αg*p-connected subspace A such that αg*p-cl(A) = X then X is αg*p-connected.

Proof: Suppose A is a αg*p-connected subspace of X such that αg*p-cl(A) = X. Since A ⊆ X = αg*p-cl(A) then by Theorem 3.21, X is αg*p-connected.

**Theorem 3.24:** If A is αg*p-connected set then αg*p-cl(A) is αg*p-connected.

Proof: Suppose A is αg*p-connected set and αg*p-cl(A) is not. Then there exist two αg*p-separated sets H, K such that αg*p-cl(A) = H ∪ K. But A ⊆ αg*p-cl(A), then A ⊆ H ∪ K and since A is αg*p-connected set then either A ⊆ H or A ⊆ K (by theorem 3.21).

Case-(i): If A ⊆ H, then αg*p-cl(A) ⊆ H. But αg*p-cl(H) ∩ K = φ, hence αg*p-cl(A) ∩ K = φ.

Since K ⊆ αg*p-cl(A) then K = φ which is a contradiction.

Case-(ii): If A ⊆ K, then the same way we can prove that H = φ which is a contradiction.

Therefore αg*p-cl(A) is αg*p-connected set.

**Theorem 3.25:** Let X be a topological space such that any two elements a and b of X are contained in some αg*p-connected subspace of X. Then X is αg*p-connected.

Proof: Suppose X is not αg*p-connected space. Then X is the union of two αg*p-separated sets A, B. Since A, B are non-empty sets, thus there exist a, b such that a ∈ A, b ∈ B. Let H be a αg*p-connected subspace of X which contains a and b. Therefore by theorem 3.21 either H ⊆ A or H ⊆ B which is a contradiction since A ∩ B = φ. Then X is αg*p-connected space.

**Theorem 3.26:** If A and B are αg*p-connected subspace of a space X such that A ∩ B ≠ φ, then A ∪ B is αg*p-connected subspace.

Proof: Suppose that A ∪ B is not αg*p-connected. Then there exist two αg*p-separated sets H and K such that A ∪ B = H ∪ K. Since A ⊆ A ∪ B = H ∪ K and A is αg*p-connected then either A ⊆ H or A ⊆ K.
Since \( B \subseteq \Lambda \cap B = H \cup K \) and \( B \) is \( \alpha g^*p \)-connected, either \( B \subseteq H \) or \( B \subseteq K \).

1. If \( A \subseteq H \) and \( B \subseteq H \), then \( A \cup B \subseteq H \). Hence \( K = \emptyset \) which is a contradiction.
2. If \( A \subseteq H \) and \( B \subseteq K \), then \( A \cap B \subseteq H \cap K = \emptyset \). Therefore \( A \cap B = \emptyset \) which is a contradiction.

By the same way we can get a contradiction if \( A \subseteq K \) and \( B \subseteq H \) or if \( A \subseteq K \) and \( B \subseteq K \). Therefore \( A \cup B \) is \( \alpha g^*p \)-connected subspace of a space \( X \).

**Theorem 3.27:** If \( X \) and \( Y \) are \( \alpha g^*p \)-connected spaces, then \( X \times Y \) is \( \alpha g^*p \)-connected space.

**Proof:** For any points \( (x_1,y_1) \) and \( (x_2,y_2) \) of the space \( X \times Y \), the subspace \( X \times \{y_1\} \cup \{x_1\} \times Y \) contains the two points and this subspace is \( \alpha g^*p \)-connected since it is the union of two \( \alpha g^*p \)-connected subspaces with a point in common. Thus \( X \times Y \) is \( \alpha g^*p \)-connected.

### 4. \( \alpha g^*p \)-COMPACTNESS

In this section we introduce the concept of \( \alpha g^*p \)-compactness and studied some of their properties.

**Definition 4.1:** A collection \( \{A_i: i \in \Lambda\} \) of \( \alpha g^*p \)-open sets in a topological space \( X \) is called a \( \alpha g^*p \)-open cover of a subset \( S \) if \( S \subseteq \bigcup \{A_i: i \in \Lambda\} \).

**Definition 4.2:** A topological space \( (X, \tau) \) is called \( \alpha g^*p \)-compact if every \( \alpha g^*p \)-open cover of \( X \) has a finite subcover.

**Definition 4.3:** A subset \( S \) of a topological space \( X \) is said to be \( \alpha g^*p \)-compact relative to \( X \) if for every collection \( \{A_i: i \in \Lambda\} \) of \( \alpha g^*p \)-open subsets of \( X \) such that \( S \subseteq \bigcup \{A_i: i \in \Lambda\} \) there exists a finite subset \( \Lambda_0 \) of \( \Lambda \) such that \( S \subseteq \bigcup \{A_i: i \in \Lambda_0\} \).

**Definition 4.4:** A subset \( S \) of a topological space \( X \) is said to be \( \alpha g^*p \)-compact if \( S \) is \( \alpha g^*p \)-compact as a subspace of \( X \).

**Definition 4.5:** A space \( X \) is said to be \( \alpha g^*p \)-lindelof if cover of \( X \) by \( \alpha g^*p \)-open sets contains a countable subcover.

**Theorem 4.6:**

i) Every \( \alpha g^*p \)-compact space is compact.

ii) Every \( gp^* \)-compact space is \( \alpha g^*p \)-compact.

iii) Every \( \alpha g^*p \)-compact space is \( \alpha g^*p \)-lindelof.

**Proof:**

(i) Let \( (X, \tau) \) be a \( \alpha g^*p \)-compact space. Let \( \{A_i: i \in \Lambda\} \) be an open cover of \( (X, \tau) \). By lemma 2.8, \( \{A_i: i \in \Lambda\} \) is a \( \alpha g^*p \)-open cover of \( (X, \tau) \). Since \( (X, \tau) \) is \( \alpha g^*p \)-compact, \( \alpha g^*p \)-open cover \( \{A_i: i \in \Lambda\} \) of \( (X, \tau) \) has a finite subcover say \( \{A_i: i = 1, 2, \ldots, n\} \) for \( X \). Hence \( (X, \tau) \) is compact.

(ii) and (iii) follows from definitions (4.2, 4.5) and lemma (2.8).

**Theorem 4.7:** \( \alpha g^*p \)-closed subset of \( \alpha g^*p \)-compact space is \( \alpha g^*p \)-compact relative to \( X \).

**Proof:** Let \( A \) be a \( \alpha g^*p \)-closed subset of a \( \alpha g^*p \)-compact space \( X \). Then \( X \setminus A \) is \( \alpha g^*p \)-open. Let \( S = \{A_i: i \in \Lambda\} \) be a \( \alpha g^*p \)-open cover for \( A \) by \( \alpha g^*p \)-open subsets in \( X \). Then \( S^* = S \cup A^c \) is a \( \alpha g^*p \)-open cover for \( X \). By hypothesis, \( X \) is \( \alpha g^*p \)-compact and hence \( S^* \) is reducible to a finite subcover of \( X \), say \( X = A_{i_1} \cup A_{i_2} \cup \ldots \cup A_{i_n} \cup A^c \), \( A_{i_k} \in S \). Hence \( A \subseteq A_{i_1} \cup A_{i_2} \cup \ldots \cup A_{i_n} \cup A^c \). Thus a \( \alpha g^*p \)-open cover \( S \) of \( A \) contains a finite subcover. Hence \( A \) is \( \alpha g^*p \)-compact relative to \( X \).

**Theorem 4.8:** A space \( X \) is \( \alpha g^*p \)-compact if and only if every family of \( \alpha g^*p \)-closed sets in \( X \) with empty intersection has a finite subfamily with empty intersection.

**Proof:** Suppose \( X \) is \( \alpha g^*p \)-compact and \( \{U_\alpha: \alpha \in \Lambda\} \) is a family of \( \alpha g^*p \)-closed sets in \( X \) such that \( \bigcap \{U_\alpha: \alpha \in \Lambda\} = \emptyset \). Then \( \bigcup \{X \setminus U_\alpha: \alpha \in \Lambda\} \) is \( \alpha g^*p \)-open cover for \( X \). Since \( X \) is \( \alpha g^*p \)-compact, this cover has a finite subcover(say) \( \{X \setminus U_{i_1}, X \setminus U_{i_2}, \ldots, X \setminus U_{i_n}\} \) for \( X \). That is \( X = \bigcup \{X \setminus U_{i_\alpha}: i = 1, 2, \ldots, n\} \). This implies that \( \bigcap_{i=1} U_{i_\alpha} = \emptyset \).
Conversely, Suppose that every family of $\alpha g^*p$-closed sets in $X$ which has empty intersection has a finite subfamily with empty intersection. Let $\{V_\alpha : \alpha \in \Lambda\}$ be a $\alpha g^*p$-open cover for $X$. Then $\bigcup \{V_\alpha : \alpha \in \Lambda\} = X$. This implies that $\bigcap \{X \setminus V_\alpha : \alpha \in \Lambda\} = \emptyset$. Since $X \setminus V_\alpha$ is $\alpha g^*p$-closed for each $\alpha \in \Lambda$, there is a finite subfamily $\{X \setminus V_{\alpha_1}, X \setminus V_{\alpha_2}, \ldots\}$, $X \setminus V_{\alpha_n}$ with empty intersection. This implies $\bigcup_{i=1}^n V_{\alpha_i} = X$. Hence $X$ is $\alpha g^*p$-compact.

**Theorem 4.9:** Let $f: (X, \tau) \to (Y, \sigma)$ be a surjective, $\alpha g^*p$-continuous map. If $X$ is $\alpha g^*p$-compact then $Y$ is compact.

**Proof:** Let $\{A_\alpha : \alpha \in \Lambda\}$ be an open cover of $Y$. Then $\{f^{-1}(A_\alpha) : \alpha \in \Lambda\}$ is an $\alpha g^*p$-open cover of $X$. Since $X$ is $\alpha g^*p$-compact, it has a finite subcover, say $\{f^{-1}(A_{\alpha_1}), f^{-1}(A_{\alpha_2}), \ldots, f^{-1}(A_{\alpha_n})\}$. Surjectiveness of $f$ implies $\{A_{\alpha_1}, A_{\alpha_2}, \ldots, A_{\alpha_n}\}$ is an open cover of $Y$ and hence $Y$ is compact.

**Theorem 4.10:** If a map $f: (X, \tau) \to (Y, \sigma)$ is $\alpha g^*p$-irresolute and a subset $S$ of $X$ is $\alpha g^*p$-compact relative to $X$ then the image $f(S)$ is $\alpha g^*p$-compact relative to $Y$.

**Theorem 4.11:** Let $f: (X, \tau) \to (Y, \sigma)$ be a surjective, $\alpha g^*p$-irresolute map. If $X$ is $\alpha g^*p$-compact then $Y$ is $\alpha g^*p$-compact.

**Theorem 4.12:** If $f: (X, \tau) \to (Y, \sigma)$ is a $\alpha g^*p$-open function and $Y$ is $\alpha g^*p$-compact then $X$ is compact.

**REFERENCES**


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