ON $W_I^g$-CONTINUOUS AND $W_{I^*g}$-CONTINUOUS FUNCTIONS IN IDEAL TOPOLOGICAL SPACES

B. MAHESWARI*, A. REVATHI

Assistant Professor,
Department of Mathematics,
SVS College of Engineering, Coimbatore, Tamil Nadu, India.

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ABSTRACT

In this paper we introduce and study the notions of $W_I^g$-continuous and $W_{I^*g}$-continuous, $W_I^g$-irresolute and $W_{I^*g}$-irresolute in ideal topological spaces, and also we studied their properties.

Keywords: $W_I^g$-closed, $W_{I^*g}$-closed, $W_I^g$-continuous, $W_{I^*g}$-continuous, $W_I^g$-irresolute, $W_{I^*g}$-irresolute.

1. INTRODUCTION AND PRELIMINARIES

Ideals in topological spaces have been considered since 1930. In 1990, Jankovic and Hamlett [2] once again investigated applications of topological ideals. The notion of $I^g$-closed sets was first by Dontchev et.al [1] in 1999. Navaneethakrishnan and Joseph [3] further investigated and characterized $I^g$-closed sets and $I^g$-open sets by the use of local functions. The notion of $I^g_{*}$-closed sets was introduced by Ravi. et.al [4] in 2013. Recently the notion of $W_I^g$-closed sets and $W_{I^*g}$-closed sets was introduced and investigated by Maragathamvali.et.al [5]. In this paper, we introduce the notions of $W_I^g$-continuous and $W_{I^*g}$-continuous functions in ideal topological spaces.

An ideal $I$ on a topological space $(X, \tau)$ is a non-empty collection of subsets of $X$ which satisfies the following properties. (1) $A \in I$ and $B \subseteq A$ implies $B \in I$, (2) $A \in I$ and $B \in I$ implies $A \cup B \in I$. An ideal topological space is a topological space $(X, \tau)$ with an ideal $I$ on $X$ and is denoted by $(X, \tau, I)$. For a subset $A \subseteq X$, $A^*(I, \tau) = \{x \in X: A \cap U \not\in I \text{ for every } U \in \tau (X, x)\}$ is called the local function of $A$ with respect to $I$ and $\tau$ [6]. We simply write $A^*$ in case there is no chance for confusion. A Kuratowski closure operator $cl^*(\cdot)$ for a topology $\tau^*(I, \tau)$ called the $^*$-topology, finer than $\tau$ is defined $cl^*(A) = A \cup A^*$ [7]. If $A \subseteq X$, $cl(A)$ and int$(A)$ will respectively, denote the closure and interior of $A$ in $(X, \tau)$.

Definition 1.1: A subset $A$ of a topological space $(X, \tau)$ is called
1. $g$-closed [8], if $cl (A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.
2. $g^*$-closed [9], if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi open in $(X, \tau)$.
3. $^g$-closed [4], if $A^* \subseteq U$ whenever $A \subseteq U$ and $U$ is $g^*$-open in $(X, \tau)$.

Definition 1.2: A subset $A$ of a topological space $(X, \tau)$ is called
1. $I^g$-closed [3], if $A^* \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $X$.
2. $I^g$-closed [10], if $A^* \subseteq U$ whenever $A \subseteq U$ and $U$ is semi-open in $X$.
3. $W_I^g$-closed [5], if $int(A^*) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi-open in $X$.
4. $W_{I^*g}$-closed [5], if $int(A^*) \subseteq U$ whenever $A \subseteq U$ and $U$ is $g^*$-open in $X$.

Definition 1.3: A function $f:(X, \tau, I) \rightarrow (Y, \sigma)$ is said to be
1. $g$-continuous [11], if for every open set $V \subseteq \sigma$, $f^{-1}(V)$ is $g$-open in $(X, \tau)$.
2. $g^*$-continuous [9], if for every open set $V \subseteq \sigma$, $f^{-1}(V)$ is $g^*$-open in $(X, \tau)$.
Definition 1.4: A function \( f: (X, \tau, I) \rightarrow (Y, \sigma) \) is said to be \( I_g \)-continuous [12], if \( f^{-1}(V) \) is \( I_g \)-closed in \( (X, \tau, I) \) for every closed set \( V \) in \( (Y, \sigma) \).

2. \( wI_g \)-CONTINUOUS AND \( wI_{g*} \)-CONTINUOUS

Definition 2.1: A function \( f: (X, \tau, I) \rightarrow (Y, \sigma) \) is said to be
1. weakly \( I_g \)-continuous (briefly \( wI_g \)-continuous) if \( f^{-1}(V) \) is weakly \( I_g \)-closed set in \( (X, \tau, I) \) for every closed set \( V \) in \( (Y, \sigma) \).
2. weakly \( I_{g*} \)-continuous (briefly \( wI_{g*} \)-continuous) if \( f^{-1}(V) \) is weakly \( I_{g*} \)-closed set in \( (X, \tau, I) \) for every closed set \( V \) in \( (Y, \sigma) \).

Definition 2.2: A function \( f: (X, \tau, I_1) \rightarrow (Y, \sigma, I_2) \) is said to be
(i) \( wI_g \)-irresolute if \( f^{-1}(V) \) is \( wI_g \)-closed in \( (X, \tau, I_1) \) for every \( wI_g \)-closed set \( V \) in \( (Y, \sigma, I_2) \).
(ii) \( wI_{g*} \)-irresolute if \( f^{-1}(V) \) is \( wI_{g*} \)-closed in \( (X, \tau, I_1) \) for every \( wI_{g*} \)-closed set \( V \) in \( (Y, \sigma, I_2) \).

Theorem 2.3: Every continuous function is \( wI_g \)-continuous.

Proof: Let \( f \) be an continuous function and let \( V \) be a closed set in \( (Y, \sigma) \). Then \( f^{-1}(V) \) is closed set in \( (X, \tau, I) \). Since every closed set is \( wI_g \)-closed. Hence \( f^{-1}(V) \) is \( wI_g \)-closed set in \( (X, \tau, I) \). Therefore \( f \) is \( wI_g \)-continuous.

Example 2.4: Let \( X = Y = \{a, b, c\} \), \( \tau = \{\varphi, \{b\}, \{b, c\}\} \), \( \sigma = \{\varphi, \{c\}\} \) and \( I = \{\varphi, \{b\}\} \). Let the function \( f(X, \tau, I) \rightarrow (Y, \sigma) \) be the identity function. Then the function \( f \) is \( wI_g \)-continuous but not continuous.

Theorem 2.5: Every continuous function is \( wI_{g*} \)-continuous.

Proof: Let \( f \) be an continuous function and let \( V \) be a closed set in \( (Y, \sigma) \). Then \( f^{-1}(V) \) is closed set in \( (X, \tau, I) \). Since every closed set is \( wI_{g*} \)-closed. Hence \( f^{-1}(V) \) is \( wI_{g*} \)-closed set in \( (X, \tau, I) \). Therefore \( f \) is \( wI_{g*} \)-continuous.

Example 2.6: Let \( X = Y = \{a, b, c\} \), \( \tau = \{\varphi, \{b\}, \{b, c\}\} \), \( \sigma = \{\varphi, \{c\}\} \) and \( I = \{\varphi, \{b\}\} \). Let the function \( f(X, \tau, I) \rightarrow (Y, \sigma) \) be the identity function. Then the function \( f \) is \( wI_{g*} \)-continuous but not continuous.

Theorem 2.7: Every \( I_g \)-continuous function is \( wI_{g*} \)-continuous.

Proof: Let \( f \) be an \( I_g \)-continuous function and let \( V \) be a closed set in \( (Y, \sigma) \). Then \( f^{-1}(V) \) is \( I_g \)-closed set in \( (X, \tau, I) \). Since every \( I_g \)-closed set is \( wI_{g*} \)-closed. Hence \( f^{-1}(V) \) is \( wI_{g*} \)-closed set in \( (X, \tau, I) \). Therefore \( f \) is \( wI_{g*} \)-continuous.

Example 2.8: Let \( X = Y = \{a, b, c, d\} \), \( \tau = \{\varphi, \{a, b\}, \{a, b, c\}\} \), \( \sigma = \{\varphi, \{a, b\}, \{a\}\} \) and \( I = \{\varphi, \{a\}\} \). Let the function \( f(X, \tau, I) \rightarrow (Y, \sigma) \) be defined by \( f(a) = b, f(b) = c, f(c) = a, f(d) = d \). Then the function \( f \) is \( wI_{g*} \)-continuous but not \( I_g \)-continuous.

Theorem 2.9: Every \( g* \)-continuous function is \( wI_{g*} \)-continuous.

Proof: Let \( f \) be an \( g* \)-continuous function and let \( V \) be a closed set in \( (Y, \sigma) \). Then \( f^{-1}(V) \) is \( g* \)-closed set in \( (X, \tau, I) \). Since every \( g* \)-closed set is \( wI_{g*} \)-closed set. Hence \( f^{-1}(V) \) is \( wI_{g*} \)-closed set in \( (X, \tau, I) \). Therefore \( f \) is \( wI_{g*} \)-continuous.

Example 2.10: Let \( X = Y = \{a, b, c, d\} \), \( \tau = \{\varphi, \{b\}, \{a, b, c\}\} \), \( \sigma = \{\varphi, \{c\}\} \) and \( I = \{\varphi, \{c\}\} \). Let the function \( f(X, \tau, I) \rightarrow (Y, \sigma) \) be the identity function. Then the function \( f \) is \( wI_{g*} \)-continuous but not \( g* \)-continuous.

Theorem 2.11: Every \( g \)-continuous function is \( wI_{g*} \)-continuous.

Proof: Let \( f \) be an \( g \)-continuous function and let \( V \) be a closed set in \( (Y, \sigma) \). Then \( f^{-1}(V) \) is \( g \)-closed set in \( (X, \tau, I) \). Since every \( g \)-closed set is \( wI_{g*} \)-closed set. Hence \( f^{-1}(V) \) is \( wI_{g*} \)-closed set in \( (X, \tau, I) \). Therefore \( f \) is \( wI_{g*} \)-continuous.

Example 2.12: Let \( X = Y = \{a, b, c, d\} \), \( \tau = \{\varphi, \{b\}, \{c\}\} \), \( \sigma = \{\varphi, \{c\}\} \) and \( I = \{\varphi, \{b\}\} \). Let the function \( f(X, \tau, I) \rightarrow (Y, \sigma) \) be the identity function. Then the function \( f \) is \( wI_{g*} \)-continuous but not \( g \)-continuous.
Theorem 2.13: Every \( I_g \)-continuous function is \( wI_g \)-continuous.

**Proof:** Let \( f \) be an \( wI_g \)-continuous function and let \( V \) be a closed set in \((Y, \sigma)\). Then \( f^{-1}(V) \) is \( wI_g \)-closed set in \((X, \tau, I)\). Since every \( wI_g \)-closed set is \( wI_g \)-closed, hence \( f^{-1}(V) \) is \( wI_g \)-closed set in \((X, \tau, I)\). Therefore \( f \) is \( wI_g \)-continuous.

Example 2.14: Let \( X = Y = \{a, b, c, d\} \), \( \tau = \{\varnothing, \{a,b\}, \{c,d\}, X\} \), \( \sigma = \{\varnothing, \{c,d\}, Y\} \) and \( I = \{\varnothing, \{d\}\} \). Let the function \( f: (X, \tau, I) \rightarrow (Y, \sigma) \) be the identity function. Then the function \( f \) is \( wI_g \)-continuous but not \( I_g \)-continuous.

Theorem 2.15: Every \( g \)-continuous function is \( wI_g \)-continuous.

**Proof:** Let \( f \) be a \( g \)-continuous function and let \( V \) be a closed set in \((Y, \sigma)\), then \( f^{-1}(V) \) is \( g \)-closed set in \((X, \tau, I)\). Since every \( g \)-closed set is \( wI_g \)-closed set. Hence \( f^{-1}(V) \) is \( wI_g \)-closed set in \((X, \tau, I)\). Therefore \( f \) is \( wI_g \)-continuous.

Example 2.16: Let \( X = Y = \{a, b, c, d\} \), \( \tau = \{\varnothing, \{a,b\}, \{a,b,c\}, X\} \), \( \sigma = \{\varnothing, \{d\}, \{c,d\}, Y\} \) and \( I = \{\varnothing, \{a\}\} \). Let the function \( f: (X, \tau, I) \rightarrow (Y, \sigma) \) be the identity function. Then the function \( f \) is \( wI_g \)-continuous but not \( g \)-continuous.

Theorem 2.17: Every \( I_g \)-continuous function is \( wI_g \)-continuous.

**Proof:** Let \( f \) be an \( I_g \)-continuous function and let \( V \) be a closed set in \((Y, \sigma)\), then \( f^{-1}(V) \) is \( I_g \)-closed set in \((X, \tau, I)\). Since every \( I_g \)-closed set is \( wI_g \)-closed set. Hence \( f^{-1}(V) \) is \( wI_g \)-closed set in \((X, \tau, I)\). Therefore \( f \) is \( wI_g \)-continuous.

Example 2.18: In example 2.17, let the function \( f: (X, \tau, I) \rightarrow (Y, \sigma) \) be the identity function. Then the function \( f \) is \( wI_g \)-continuous but not \( I_g \)-continuous.

Theorem 2.19: Every \( I_g \)-continuous function is \( wI_g \)-continuous.

**Proof:** Let \( f \) be an \( I_g \)-continuous function and let \( V \) be a closed set in \((Y, \sigma)\). Then \( f^{-1}(V) \) is \( I_g \)-closed set in \((X, \tau, I)\). Since every \( I_g \)-closed set is \( wI_g \)-closed set. Hence \( f^{-1}(V) \) is \( wI_g \)-closed set in \((X, \tau, I)\). Therefore \( f \) is \( wI_g \)-continuous.

Example 2.20: Let \( X = Y = \{a, b, c, d\} \), \( \tau = \{\varnothing, \{b\}, \{a,b,c\}, X\} \), \( \sigma = \{\varnothing, \{a\}, \{a,c,d\}, Y\} \) and \( I = \{\varnothing, \{d\}\} \). Let the function \( f: (X, \tau, I) \rightarrow (Y, \sigma) \) be the identity function. Then the function \( f \) is \( wI_g \)-continuous but not \( I_g \)-continuous.

Theorem 2.21: Every \( wI_g \)-continuous function is \( wI_g \)-continuous.

**Proof:** Let \( f \) be a \( wI_g \)-continuous function and let \( V \) be a closed set in \((Y, \sigma)\). Then \( f^{-1}(V) \) is \( wI_g \)-closed set in \((X, \tau, I)\). Since every \( wI_g \)-closed set is \( wI_g \)-closed. Hence \( f^{-1}(V) \) is \( wI_g \)-closed set in \((X, \tau, I)\). Therefore \( f \) is \( wI_g \)-continuous.

Example 2.22: Let \( X = Y = \{a, b, c, d\} \), \( \tau = \{\varnothing, \{d\}, \{a, b, c\}, X\} \), \( \sigma = \{\varnothing, \{a\}, \{a\}, Y\} \) and \( I = \{\varnothing, \{b\}\} \). Let the function \( f: (X, \tau, I) \rightarrow (Y, \sigma) \) be the identity function. Then the function \( f \) is \( wI_g \)-continuous but not \( wI_g \)-continuous.

Theorem 2.23: A map \( f: (X, \tau, I) \rightarrow (Y, \sigma) \) is \( wI_g \)-continuous iff the inverse image of every closed set in \((Y, \sigma)\) is \( wI_g \)-closed in \((X, \tau, I)\).

**Proof:** Necessary: Let \( v \) be an open set in \((Y, \sigma)\). Since \( f \) is \( wI_g \)-continuous, \( f^{-1}(v^c) \) is \( wI_g \)-closed in \((X, \tau, I)\). But \( f^{-1}(v^c) = X - f^{-1}(v) \). Hence \( f^{-1}(v) \) is \( wI_g \)-closed in \((X, \tau, I)\).

**Sufficiency:** Assume that the inverse image of every closed set in \((Y, \sigma)\) is \( wI_g \)-closed in \((X, \tau, I)\). Let \( v \) be a closed set in \((Y, \sigma)\). By our assumption \( f^{-1}(v^c) = X - f^{-1}(v) \) is \( wI_g \)-closed in \((X, \tau, I)\), which implies that \( f^{-1}(v) \) is \( wI_g \)-closed in \((X, \tau, I)\). Hence \( f \) is \( wI_g \)-continuous.

Remark 2.24:
(i) The union of any two \( wI_g \)-continuous function is \( wI_g \)-continuous.
(ii) The intersection of any two \( wI_g \)-continuous function is need not be \( wI_g \)-continuous.
Theorem 2.25: Let \( f: (X, \tau, I_1) \rightarrow (Y, \sigma, I_2) \) and \( g: (Y, \sigma, I_2) \rightarrow (Z, \eta, I_3) \) be any two functions. Then the following hold.

(i) \( g \circ f \) is \( \textit{w} \)-continuous if \( f \) is \( \textit{w} \)-continuous and \( g \) is continuous.

(ii) \( g \circ f \) is \( \textit{w} \)-irresolute if \( f \) is \( \textit{w} \)-irresolute and \( g \) is \( \textit{w} \)-continuous.

(iii) \( g \circ f \) is \( \textit{w} \)-irresolute if \( f \) is \( \textit{w} \)-irresolute and \( g \) is irresolute.

Proof:

(i) Let \( v \) be a closed set in \( Z \). Since \( g \) is continuous, \( g^{-1}(v) \) is closed in \( Y \). \( \textit{w} \)-continuous of \( f \) implies, \( f^{-1}(g^{-1}(v)) \) is \( \textit{w} \)-closed in \( X \) and hence \( g \circ f \) is \( \textit{w} \)-continuous.

(ii) Let \( v \) be a closed set in \( Z \). Since \( g \) is \( \textit{w} \)-continuous, \( g^{-1}(v) \) is \( \textit{w} \)-closed in \( Y \). Since \( f \) is \( \textit{w} \)-irresolute, \( f^{-1}(g^{-1}(v)) \) is \( \textit{w} \)-closed in \( X \). Hence \( g \circ f \) is \( \textit{w} \)-continuous.

(iii) Let \( v \) be a \( \textit{w} \)-closed set in \( Z \). Since \( g \) is \( \textit{w} \)-irresolute, \( g^{-1}(v) \) is \( \textit{w} \)-closed in \( Y \). Since \( f \) is \( \textit{w} \)-irresolute, \( f^{-1}(g^{-1}(v)) \) is \( \textit{w} \)-closed in \( X \). Hence \( g \circ f \) is \( \textit{w} \)-irresolute.

Theorem 2.26: Let \( X=A \cup B \) be a topological space with topology \( \tau \) and \( Y \) be a topological space with topology \( \sigma \). Let \( f: (A, \tau, I_1) \rightarrow (Y, \sigma, I_2) \) and \( g: (B, \tau/B) \rightarrow (Y, \sigma, I_2) \) be \( \textit{w} \)-continuous maps such that \( f(x)=g(x) \) for every \( x \in A \cap B \). Suppose that \( A \) and \( B \) are \( \textit{w} \)-closed sets in \( X \). Then the combination \( \alpha: (X, \tau, I_1) \rightarrow (Y, \sigma) \) is \( \textit{w} \)-continuous.

Proof: Let \( F \) be any closed set in \( Y \). Clearly \( \alpha^{-1}(F)=f^{-1}(F) \cup g^{-1}(F) = \text{CUD} \) where \( C = f^{-1}(F) \) and \( D = g^{-1}(F) \). But \( C \) is \( \textit{w} \)-closed in \( A \) and \( B \) is \( \textit{w} \)-closed in \( X \). Hence \( \alpha \) is \( \textit{w} \)-continuous.

Theorem 2.27: A map \( f: (X, \tau, I_1) \rightarrow (Y, \sigma) \) is \( \textit{w} \)-continuous iff the inverse image of every closed set in \( (Y, \sigma) \) is \( \textit{w} \)-closed in \( (X, \tau, I_1) \).

Proof: Necessary: Let \( v \) be an open set in \( (Y, \sigma) \). Since \( f \) is \( \textit{w} \)-continuous, \( f^{-1}(v) \) is \( \textit{w} \)-closed in \( (X, \tau, I_1) \). But \( f^{-1}(v) = X - f^{-1}(c) \). Hence \( f^{-1}(v) \) is \( \textit{w} \)-closed in \( (X, \tau, I_1) \).

Sufficiency: Assume that the inverse image of every closed set in \( (Y, \sigma) \) is \( \textit{w} \)-closed in \( (X, \tau, I_1) \). Let \( v \) be a closed set in \( (Y, \sigma) \). By our assumption \( f^{-1}(v) \) is \( \textit{w} \)-closed in \( (X, \tau, I_1) \), which implies that \( f^{-1}(v) \) is \( \textit{w} \)-closed in \( (X, \tau, I_1) \). Hence \( f \) is \( \textit{w} \)-continuous.

Remark 2.28:

(i) The union of any two \( \textit{w} \)-continuous functions is \( \textit{w} \)-continuous.

(ii) The intersection of any two \( \textit{w} \)-continuous functions need not be \( \textit{w} \)-continuous.

Theorem 2.29: Let \( f: (X, \tau, I_1) \rightarrow (Y, \sigma, I_2) \) and \( g: (Y, \sigma, I_2) \rightarrow (Z, \eta, I_3) \) be any two functions. Then the following hold.

(i) \( g \circ f \) is \( \textit{w} \)-continuous if \( f \) is \( \textit{w} \)-continuous and \( g \) is continuous.

(ii) \( g \circ f \) is \( \textit{w} \)-continuous if \( f \) is \( \textit{w} \)-continuous and \( g \) is \( \textit{w} \)-continuous.

(iii) \( g \circ f \) is \( \textit{w} \)-irresolute if \( f \) is \( \textit{w} \)-irresolute and \( g \) is irresolute.

Proof:

(i) Let \( v \) be a closed set in \( Z \). Since \( g \) is continuous, \( g^{-1}(v) \) is closed in \( Y \). \( \textit{w} \)-continuous of \( f \) implies, \( f^{-1}(g^{-1}(v)) \) is \( \textit{w} \)-closed in \( X \) and hence \( g \circ f \) is \( \textit{w} \)-continuous.

(ii) Let \( v \) be a closed set in \( Z \). Since \( g \) is \( \textit{w} \)-continuous, \( g^{-1}(v) \) is \( \textit{w} \)-closed in \( Y \). Since \( f \) is \( \textit{w} \)-irresolute, \( f^{-1}(g^{-1}(v)) \) is \( \textit{w} \)-closed in \( X \). Hence \( g \circ f \) is \( \textit{w} \)-continuous.

(iii) Let \( v \) be a \( \textit{w} \)-closed set in \( Z \). Since \( g \) is \( \textit{w} \)-irresolute, \( g^{-1}(v) \) is \( \textit{w} \)-closed in \( Y \). Since \( f \) is \( \textit{w} \)-irresolute, \( f^{-1}(g^{-1}(v)) \) is \( \textit{w} \)-closed in \( X \). Hence \( g \circ f \) is \( \textit{w} \)-irresolute.

Theorem 2.30: Let \( X=A \cup B \) be a topological space with topology \( \tau \) and \( Y \) be a topological space with topology \( \sigma \). Let \( f: (A, \tau, I_1) \rightarrow (Y, \sigma, I_2) \) and \( g: (B, \tau/B) \rightarrow (Y, \sigma, I_2) \) be \( \textit{w} \)-continuous maps such that \( f(x)=g(x) \) for every \( x \in A \cap B \). Suppose that \( A \) and \( B \) are \( \textit{w} \)-closed sets in \( X \). Then the combination \( \alpha: (X, \tau, I_1) \rightarrow (Y, \sigma, I_2) \) is \( \textit{w} \)-continuous.

Proof: Let \( F \) be any closed set in \( Y \). Clearly \( \alpha^{-1}(F)=f^{-1}(F) \cup g^{-1}(F) = \text{CUD} \) where \( C = f^{-1}(F) \) and \( D = g^{-1}(F) \). But \( C \) is \( \textit{w} \)-closed in \( A \) and \( B \) is \( \textit{w} \)-closed in \( X \). Hence \( \alpha \) is \( \textit{w} \)-continuous.

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