International Journal of Mathematical Archive-7(9), 2016, 189-194
Available online through www.ijma.info ISSN 2229 – 5046

TRIPLE CONNECTED COMPLEMENTARY ACYCLIC DOMINATION OF A GRAPH

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(Received On: 26-07-16; Revised & Accepted On: 07-09-16)

ABSTRACT

Let G = (V, E) be a non trivial connected graph. A subset S of V(G) is called a triple connected dominating set if S is a dominating set and induced sub graph $\langle S \rangle$ is triple connected. The minimum cardinality taken over all triple connected dominating set is called the triple connected domination number of G and it is denoted by $\gamma_{tc}(G)$. A subset S of V of a non trivial connected graph G is said to be a triple connected complementary acyclic dominating set if S is a triple connected dominating set and induced subgraph $\langle V-S \rangle$ is acyclic .The minimum cardinality taken over all triple connected complementary acyclic dominating sets is called the triple connected complementary acyclic domination number of G and is denoted by $\gamma^{tc}_{c-a}(G)$ .We determine this number for some standard graphs and obtain bounds for general graphs. Its relationship with other graph theoretical parameters are also investigated.

Keywords: *Domination number, Triple connected graph, Triple connected domination number, Triple connected complementary acyclic domination number.*

AMS Subject Classification: *05C69.*

INTRODUCTION

By a graph we mean a finite, simple, connected and undirected graph G(V,E), where V denotes its vertex set and E its edge set. Unless otherwise stated the graph G has n vertices and m edges. Degree of a vertex v is denoted by d(v), the maximum degree of a graph G is denoted by $\Delta(G)$. We denote a cycle on n vertices by C_n, a path on n vertices by P_n, and a complete graph on n vertices by K_n. A graph G is connected if any two vertices of G are connected by a path. A maximal connected sub graph of a graph G is called a component of G. The number of components of G is denoted by $\omega(G)$. The complement of G is the graph with vertex set V in which two vertices are adjacent if and only if they are not adjacent in G. A tree is a connected acyclic graph. A bipartite graph (or bi graph) is a graph whose vertex set can be partitioned into two disjoint non empty sets V_1 and V_2 such that every edge has one end in V_1 and another end in V_2. A complete bipartite graph is a bipartite graph where every vertex of V_1 is adjacent to every vertex in V_2. The complete bipartite graph with partitions of order $|V_1|$= m and $|V_2|$= n, is denoted by $K_{m,,n}$. A star, denoted by $K_{1,n-1}$ is a tree with one root vertex and n-1 pendant vertices. A wheel graph, denoted by W_n is a graph with n vertices, formed by joining a single vertex to all vertices of C_{n-1}. If S is a subset of V, then $\langle S \rangle$ denotes the vertex induced sub graph of G induced by S. The open neighborhood of a set S of vertices of a graph G, denoted by N(S) is the set of all vertices adjacent to some vertex in S. $S \cup N(S)$ is called the closed neighborhood of S, denoted by N[S]. A cut-vertex (cut edge) of a graph G is a vertex (edge) whose removal increases the number of components. A vertex cut, or separating set of a connected graph G is a set of vertices whose removal results in a disconnected graph. The connectivity or vertex connectivity of a graph G, denoted by k(G)(where G is not complete) is the size of a smallest vertex cut. A connected sub graph H of a connected graph G is called a H-cut if $\omega(G-H)=2$. The chromatic number of a graph G, denoted by $\chi(G)$ is the smallest number of colors needed to color all the vertices of a graph G in which adjacent vertices receive distinct colors. Terms not defined here are used in the sense of [2].

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A subset S of V is called a dominating set of G if every vertex in V -S is adjacent to at least one vertex in S. The domination number $\gamma(G)$ of G is the minimum cardinality taken over all dominating sets in G. A dominating set S of a connected graph G is said to be a connected dominating set of G if the induced sub graph $\langle S \rangle$ is connected. The minimum cardinality taken over all connected dominating sets is the connected domination number and is denoted by $\gamma_c(G)$. Many authors have introduced different types of domination parameters by imposing conditions on the dominating set [7]. Recently, the concept of triple connected graphs has been introduced by Paulraj Joseph *et.al* [4] by considering the existence of a path containing any three vertices of G. They have studied the properties of triple connected graphs and established many results on them. A graph G is said to be triple connected if any three vertices of G lie on a path in G. All paths, cycles, complete graphs and wheels are some standard examples of triple connected graphs. A subset S of V of a nontrivial connected graph G is said to be a triple connected dominating set, if S is a dominating set and the induced sub graph $\langle S \rangle$ is triple connected. The minimum cardinality taken over all triple connected dominating sets is called the triple connected domination number of G and is denoted by $\gamma_{tc}(G)$. Any triple connected dominating set with γ_{tc} vertices is called a γ_{tc}-set of G. A dominating set S of G is called a strong complementary acyclic dominating set if S is a strong dominating set and the induced sub graph $\langle V-S \rangle$ is acyclic. The minimum cardinality of a strong complementary acyclic dominating set of G is called the strong complementary acyclic domination number of G and is denoted by $\gamma_{c-a}^{st}(G)$. We use this idea to develop the concept of triple connected complementary acyclic dominating set and triple complementary acyclic domination number of a graph.

Theorem 1.1[4]: A tree T is triple connected if and only if $T \cong P_n; n \geq 3$.

Theorem 1.2 [4]: A connected graph G is not triple connected if and only if there exists a H-cut with $\omega(G-H) = 3$ such that $|V(H) \cap N(C_i)| = 1$ for at least three components C_1,C_2 and C_3 of G-H.

2. TRIPLE CONNECTED COMPLEMENTARY ACYCLIC DOMINATION NUMBER

Definition 2.1: A subset S of V of a non trivial connected graph G is said to be a triple connected complementary acyclic dominating set if S is a triple connected dominating set and the induced sub graph $\langle V-S \rangle$ is acyclic. The minimum cardinality taken over all triple connected complementary acyclic dominating sets is called the triple connected complementary acyclic domination number of G and is denoted by $\gamma_{c-a}^{tc}(G)$. Any triple connected complementary acyclic dominating set with γ_{c-a}^{tc} vertices is called a γ_{c-a}^{tc} set of G.

Example 2.2: For the graph G in Figure 2.1, S = {v_2,v_6,v_1} is a triple connected dominating set and $\langle V-S \rangle$ is acyclic. Therefore, S is a triple connected c-a dominating set.

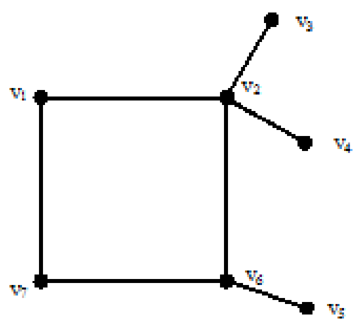


Figure-2.1

Definition 2.3: A triple connected c-a dominating set S of G is minimal if no proper subset of S is a triple connected c-a dominating set of G.

Observation 2.4: Triple connected complementary acyclic dominating set does not exist for all graphs and if exists, then $\gamma_{c-a}^{tc}(G) \geq 3$.

Throughout this paper, we consider only connected graphs for which triple connected complementary acyclic dominating set exists.

Observation 2.5: The complement of the triple connected complementary acyclic dominating set need not be a triple connected complementary acyclic dominating set.

Observation 2.6: Every triple connected complementary acyclic dominating set is a dominating set but not conversely.

Observation 2.7: For any connected graph G, $\gamma_c(G) \le \gamma_{tc}(G) \le \gamma_{c-a}^{tc}(G)$ and for the cycle C_5 the bounds are sharp.

Theorem 2.8: If the induced sub graph of each connected dominating set of G has more than two pendant vertices then G does not contain a triple connected complementary acyclic dominating set.

Proof: The proof follows from theorem 1.1.

EXACT VALUE FOR SOME STANDARD GRAPHS

1. For any cycle of order $n \ge 3, \gamma_{c-a}^{tc}(G) \begin{cases} n \text{ if n=3} \\ \text{n-1 if n=4} \\ \text{n-2, otherwise} \end{cases}$
2. For any complete bipartite graph of order $n \ge 4$, $\gamma_{c-a}^{tc}(K_{p,q}) = \begin{cases} \text{3 if n=4, 5} \\ \text{n-3, otherwise} \end{cases}$

   where $p, q \ge 2$ and p + q = n
3. For any complete graph of order $n \ge 4$, $\gamma_{c-a}^{tc}(K_n) = \begin{cases} 3 \text{ if } n = 4 \\ n-2, \text{ otherwise} \end{cases}$.
4. For any wheel of order $n \ge 4, \gamma_{c-a}^{tc}(W_n) = \begin{cases} n-1 \text{ if n = 4,} \\ \text{n-2, otherwise} \end{cases}$.

EXACT VALUE FOR SOME SPECIAL GRAPHS

1. The Moser Spindle (also called the Moser's Spindle or Moser graph) is an undirected graph, with seven vertices and eleven edges given in Figure 2.2.

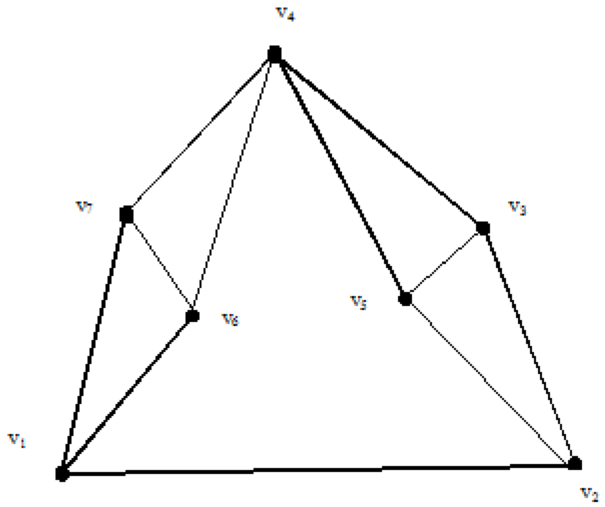


Figure-2.2: S={v_4,v_5,v_6} is a triple connected complementary acyclic dominating set.

For the Moser Spindle graph G, $\gamma_{c-a}^{tc}(G) = 3$.

2. The Wagner graph is a 3-regular graph with 8 vertices and 12 edges given in Figure 2.3.

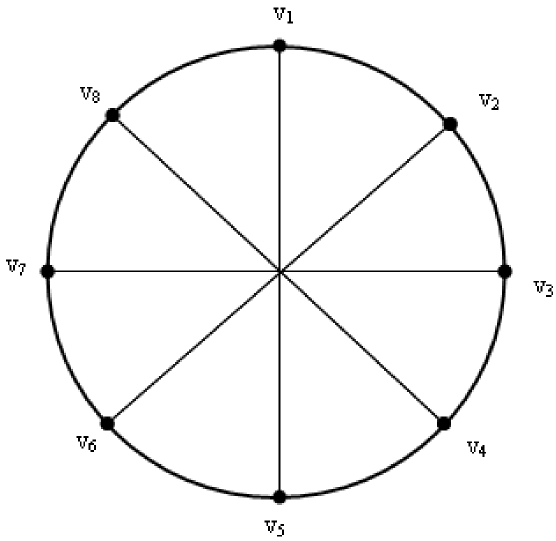


Figure-2.3

For the Wagner graph G, $\gamma_{c-a}^{tc}(G) = 4$.

3. The Bidiakis cube is a 3-regular graph with 12 vertices and 18 edges given in Figure 2.4.

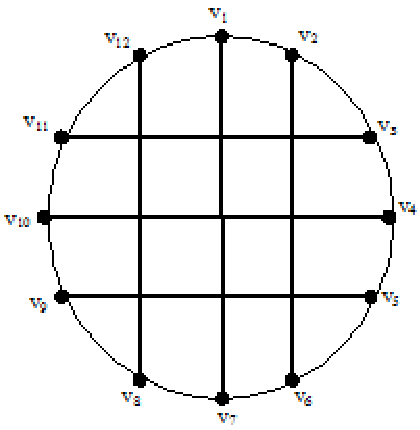
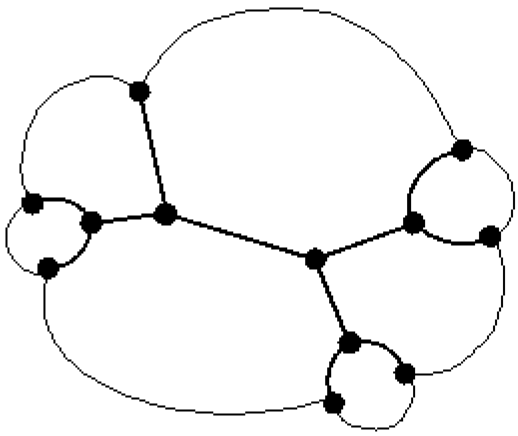


Figure-2.4

For the Bidiakis cube graph G, $\gamma_{c-a}^{tc}(G) = 8$

4. The Frucht graph is a 3-regular graph with 12 vertices, 18 edges and no non trivial symmetries given in Figure 2.5.



For the Frucht graph G, $\gamma_{c-a}^{tc}(G) = 6$.

Observation 2.9: For any connected graph G with n vertices, $\gamma_{tc,coi}(G) = 3$ if and only if $G \cong P_3$ or C_3.

Theorem 2.10: For any connected graph G with n = 3, we have $\gamma_{c-a}^{tc}(G) = n$.

Observation 2.12: For any connected graph G with 3 vertices, $\gamma_{c-a}^{tc}(G) = n$ if and only if $G \cong P_3$ or C_3.

Observation 2.13: For any connected graph G with 4 vertices, $\gamma_{c-a}^{tc}(G) = n-1$.

Theorem 2.14: For any connected graph G with $n \geq 5$,we have $3 \leq \gamma_{c-a}^{tc}(G) \leq n-2$ and the bounds are sharp.

Proof: The lower and upper bounds follows from Definition 2.1. For C_5, the lower bound is attained and for K_5 the upper bound is attained.

The Nordhaus - Gaddum type result is given below.

Theorem 2.15: Let G be a graph such that G and $\overline{G}$ have no isolates of order $n \geq 5$ .Then

i) $\gamma_{c-a}^{tc}(G) + \gamma_{c-a}^{tc}(\overline{G}) \leq 2(n-2)$

ii) $\gamma_{c-a}^{tc}(G) \cdot \gamma_{c-a}^{tc}(\overline{G}) \leq (n-2)^2$.

Proof: The bound directly follows from Theorem 2.14. For the cycle C_5, $\gamma_{c-a}^{tc}(G) + \gamma_{c-a}^{tc}(\overline{G}) = 2(n-2)$ and $\gamma_{c-a}^{tc}(G) \cdot \gamma_{c-a}^{tc}(\overline{G}) = (n-2)^2$.

3. RELATION WITH OTHER GRAPH THEORETICAL PARAMETERS

Theorem 3.1: For any connected graph G with $n \geq 5$ vertices, $\gamma_{c-a}^{tc}(G) + k(G) \leq 2n-3$ and the bound is sharp if and only if $G \cong K_n$.

Proof: Let G be a connected graph with $n \geq 5$ vertices. We know that $k(G) \leq n-1$ and by Theorem 2.14, $\gamma_{c-a}^{tc}(G) \leq n-2$. Hence $\gamma_{c-a}^{tc}(G) + k(G) \leq 2n-3$. Suppose G is isomorphic to K_n. Then clearly $\gamma_{c-a}^{tc}(G) + k(G) = 2n-3$. Conversely, let $\gamma_{c-a}^{tc}(G) + k(G) = 2n-3$. This is possible only if $\gamma_{c-a}^{tc}(G) = n-2$ and $k(G) = n-1$. But and so $G \cong K_n$.

Theorem 3.2: For any connected graph G with $n \geq 5$ vertices, $\gamma_{c-a}^{tc}(G) + \chi(G) \leq 2n-2$ and the bound is sharp if and only if $G \cong K_n$.

Proof: Let G be a connected graph with n vertices. We know that $\chi(G) \leq n$ and by Theorem 2.14 $\gamma_{c-a}^{tc}(G) \leq n-2$. Hence $\gamma_{c-a}^{tc}(G) + \chi(G) \leq 2n-2$. Suppose G is isomorphic to K_n. Then clearly $\gamma_{c-a}^{tc}(G) + \chi(G) = 2n-2$. Conversely, let $\gamma_{c-a}^{tc}(G) + \chi(G) = 2n-2$. This is possible only if $\gamma_{c-a}^{tc}(G) = n-2$ and $\chi(G) = n$. Since $\chi(G) = n$, G is isomorphic to K_n for which $\gamma_{c-a}^{tc}(G) \leq n-2$. (G) = n − 2. Hence $G \cong K_n$.

Theorem 3.3: For any connected graph G with $n \geq 5$ vertices, $\gamma_{c-a}^{tc}(G) + \Delta(G) \leq 2n-3$ and the bound is sharp.

Proof: Let G be a connected graph with n vertices. We know that $\Delta(G) \leq n-1$ and by

Theorem 2.14: $\gamma_{c-a}^{tc}(G) \leq n-2$. Hence $\gamma_{c-a}^{tc}(G) + \Delta(G) \leq 2n-3$ .For K_5 the bound is sharp.

CONCLUSION

We found triple connected complementary acyclic domination number for some standard graphs and obtained some bounds for general graphs. Its relationship with other graph theoretical parameters also investigated.

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Source of support: Nil, Conflict of interest: None Declared