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VAGUE FUZZY HYPRID WEIGHTING OPERATOR IN MULTIPLE ATTRIBUTE GROUP DECISION MAKING PROBLEMS

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ABSTRACT

In this paper, the weighted averaging operator from Renyi's, Daroczy's and R-norm entropies, vague fuzzy weighted averaging operator and vague fuzzy hyprid weighted averaging operator for vague sets are proposed. Also a general model for decision making utilising these operators is proposed for vague sets together with a new distance function defined based on the distance functions from the literature.

INTRODUCTION

Some values of the multi attribute decision models are often subjective. The weights of the criteria and the scoring values of the alternatives against the subjective (judgmental) criteria contain always some uncertainties. It is therefore an important question how the final ranking or the ranking values of the alternatives is sensitive to the changes of some input parameters of the decision model. The simplest case is when the value of the weight of a single criterion is allowed to vary. For additive multi attribute models, the ranking values of the alternatives are simple linear functions of this single variable and attractive graphical tools can be applied to present a simple sensitivity analysis to a user.

For a wide class of multi attribute decision models there are different methods to determine the stability intervals or regions for the weights of different criteria. These consist of the values that the weights of one or more criteria can take without altering the results given by the initial set of weights, all other weights being kept constant. There are proposed linear programming models to find the minimum modification of the weights required to make a certain alternative ranked first. Models are presented as an approach of a more complex sensitivity analysis with the change of the scores of the alternatives against the criteria, as well.

SECTION-1: PREVIOUS WORKS

Since the theory of fuzzy sets was proposed in 1965, it has been applied in many uncertain information processing problems successfully, since in the real world there is vague information about different applications. Gau & Buehrer [1994] pointed out the drawback of using the single membership value in fuzzy set theory. In order to tackle this problem, they proposed the notion of vague sets (VSs), which allow using interval-based membership instead of using point-based membership as in FSs.

The interval-based membership generalization in VSs is more expressive in capturing vagueness of data. However, VSs are shown to be equivalent to that of IFSs. For this reason, the interesting features for handling vague data that are unique to VSs are largely ignored. Atanasov [1986, 1989] proposed Intuition fuzzy set theory. Gau & Buehrer [1994] proposed the concept of vague set.

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Bustince & Burillo [1996] proposed that the vague set was intuitionistic fuzzy sets and unified the intuition fuzzy set and the vague set. As the vague set took the membership degree, non-membership degree and hesitancy degree into account, and has more ability to deal with uncertain information than traditional fuzzy set, lots of scholars pay attentions to the research of vague set. Atanassov & Gargov [1989] extended the intuition vague set and proposed the concept of interval intuition vague set, also named interval vague set.

A vague set (VS), as well as an intuitionistic fuzzy set (IFS), is a further generalization of an fuzzy set. Instead of using point-based membership as in FSs, interval-based membership is used in a VS. The interval-based membership in VSs is more expressive in capturing vagueness of data. In the literature, the notions of IFSs and VSs are regarded as equivalent, in the sense that an IFS is isomorphic to a VS.

Furthermore, due to such equivalence and IFSs being earlier known as a tradition, the interesting features for handling vague data that are unique to VSs are largely ignored. Decision-making is the process of finding the best option from all of the feasible alternatives. Sometimes, decision-making problems considering several criteria are called multi-criteria decision-making (MCDM) problems.

The MCDM problems may be divided into two kinds. One is the classical MCDM problems, among which the ratings and the weights of criteria are measured in crisp numbers. Another is the fuzzy multiple criteria decision-making (FMCDM) problems, among which the ratings and the weights of criteria evaluated on imprecision and vagueness are usually expressed by linguistic terms, fuzzy numbers or intuition fuzzy numbers.

A MAGDM problem is to find a desirable solution from a finite number of feasible alternatives assessed on multiple attributes, both quantitative and qualitative. In order to choose a desirable solution, the decision maker often provides his/her preference information which takes the form of numerical values, such as exact values, interval number values and fuzzy numbers. However, under many conditions, numerical values are inadequate or insufficient to model real-life decision problems. Indeed, human judgments including preference information may be stated in vague information. Hence, MAGDM problems under vague environment is an interesting area of study for researchers in the recent days.

Different types of aggregation operators are found in the literature for aggregating the information. A very common aggregation method is the ordered weighted averaging (OWA) operator. It provides a parameterized family of aggregation operators that includes as special cases the maximum, the minimum and the average criteria. Since its appearance, the OWA operator has been used in a wide range of applications. Induced intuitionistic fuzzy operators are already in the literature.

SECTION-2: DISTANCES BETWEEN VAGUE SETS

In conventional fuzzy set a membership function assigns to each element of the universe of discourse a number from the unit interval to indicate the degree of belongingness to he set under consideration. The degree of non-belongingness is just automatically the complements to 1 of the membership degree. However, a human being who expresses the degree of membership of given element in a fuzzy set very often does not express corresponding degree of non-membership as the complement to 1. This reflects a well-known psychological fact that the linguistic negation not always identifies with logical negation.

Thus Atanassov [1986] introduced the concept of an intuitionistic fuzzy set which is characterized by two functions expressing the degree of belongingness and the degree of non-belongingness, respectively. This idea, which is a natural generalization of usual fuzzy set, seems to be useful in modeling many real life situations, like negotiation processes, etc.

Atanasov [1999] and Szmidt & Kacprzyk [2003] suggested some methods for measuring distances between intuitionistic fuzzy sets that are generalizations of the well-known Hamming distance, Euclidean distance and their normalized counterparts, also in the present paper we propose another generalization of those distances based on the Hausdorff metric is given.

Since Zadeh has introduced fuzzy sets in 1965, many new approaches and theories treating imprecision and uncertainty have been proposed. Some of these theories, like intuitionistic fuzzy sets theory, are extensions of the classical fuzzy set theory. Another, well-known generalization of an ordinary fuzzy set is, the so-called, interval-valued fuzzy set. Therefore, one may easily notice that our definition of the distances between intuitionistic fuzzy sets, based on the Hausdorff metric, could be immediately expressed in terms of interval-valued fuzzy sets.

Definition 2.1: Let $A = (t_A(x_i), f_A(x_i))$, $B = (t_B(x_i), f_B(x_i))$ be two vague values. Then the Euclidean distance between A and B is given as follows:

$$d(A,B) = \sqrt{\frac{1}{2}\sum_{i=1}^{n} \left[(t_A(x_i) - t_B(x_i))^2 + ((1 - f_A(x_i)) - (1 - f_B(x_i)))^2 \right]}$$

SECTION-3: THE VWA OPERATOR FOR GROUP DECISION MAKING

MAGDM with vague theory: Multi Attribute Group Decision Making (MAGDM) problems are wide spread in real life situations. A MAGDM problem is to find a desirable solution from a finite number of feasible alternatives assessed on multiple attributes, both quantitative and qualitative. To choose a desirable solution, the decision maker often provides his/her preferred information in the form of numerical values, such as exact values, interval number values and fuzzy numbers.

However, under many conditions, numerical values are inadequate or insufficient to model real-life decision problems. Indeed, human judgments including preference information may be stated in intuitionistic fuzzy information. Hence, MAGDM problems under intuitionistic fuzzy environment are an interesting area of study for researchers recently.

Processing of MAGDM problems with intuitionistic fuzzy information or vague fuzzy information, sometimes, leads to attribute values taking the form of intuitionistic or vague fuzzy number, respectively. The information about attribute weights may sometimes be known, partially known or be completely unknown. MAGDM problems are assumed to have a predetermined, limited number of decision alternatives.

Solving a MAGDM problem involves sorting and ranking, and can be viewed as an alternative method for combining information in a problem's decision matrix together with additional information from the decision maker to determine a final ranking or selection from the alternatives. Besides the information contained in the decision matrix, all but the simplest MAGDM techniques require additional information from the decision matrix to arrive at a final ranking/ selection.

Szmidt & Kacprzyk [2003] proposed some solution concepts like the intuitionistic fuzzy core and consensus winner in group decision making with intuitionistic fuzzy preference relations. They also developed an approach to aggregate the individual intuitionistic fuzzy preference relations into a social fuzzy preference relation based on fuzzy majority equated with a fuzzy linguistic quantifier.

Szmidt & Kacprzyk [2002] introduced several distance functions and similarity measures for IFSs which were later used in various MAGDM problems. They investigated MADM with intuitionistic fuzzy information and constructed several linear programming models to generate optimal weights for attribute. They also presented a new method for handling multiple attribute fuzzy decision making problems, where the characteristics of the alternatives are represented by intuitionistic fuzzy sets.

Herrera *et al.* [1999] developed an aggregation process for combining numerical, interval valued and linguistic information, and then proposed different extensions of this process to deal with contexts where the information can appear as IFSs or multi-granular linguistic information.

Xu &Chen [2006] developed some geometric aggregation operators for MADM problems. They developed an evaluation function for the decision making problem to measure the degrees to which alternatives satisfy/do not satisfy the decision maker's requirement.

Chen & Tan [1994] presented new techniques for handling multiple attribute fuzzy decision making problems based on vague set theory. Also Hong & Choi [2000] provided some new techniques for handling multiple attribute fuzzy decision making problems based on vague set theory.

Robinson & Amirtharaj [2001, 2012a, 2012b] Xu [2002, 2005] Szmidt & Kacprzyk [2002] and Wei & Wang [2007], and Yager [1988] contributed novel approaches to the field of fuzzy decision making.

In this chapter, a new operator is defined the vague weighted averaging (VWA) operator for vague sets.

Definition 3.1: Let $\tilde{a}_j = (t_j,\ 1 - f_j)$, j = 1, , …, n be a collection of vague values, and let the vague fuzzy weighted averaging operator VWA is defined as: VWA: $Q_n \to Q$ ifVMA$_w(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \sum_{j=1}^{n} w_j\, \tilde{a}_j$ = (1- $\prod_{j=1}^{n}(1 - t_j)^{w_j}$, $\prod_{j=1}^{n}(1 - f_j)^{w_j}$ where the weight vector w = $(w_1, w_2, \ldots, w_m)^T$ of the attributes can be determined in advance. Note that $w_i > 0$ for each i = 1 to n, and $\sum_{j=1}^{n} w_j = 1$.

Definition 3.2: Let $\tilde{a}_j = (t_j, f_j), j = 1, 2, ..., n$ be a collection of vague values, and let the vague fuzzy hybrid weighting average operator VHA is defined as: VHA: $Q_n \rightarrow$ QifVHA$_w(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n)$ = (1- $\prod_{j=1}^{n}(1-t_j)^{w_j}$, 1- $\prod_{j=1}^{n}(1-f_j)^{w_j}$ where the weight vector w = $(w_1, w_2, \ldots, w_m)^T$ of the attributes can be determined in advance. Note that $w_i > 0$ for each i = 1 to n, and $\sum_{j=1}^{n} w_j = 1$.

Definition 3.3 (Weight generating method) (Renyi's, Daroczy's and *R*-norm entropies): For an arbitrary positive integer n, any weighting vector w = $(w_1, w_2, \ldots, w_n)^T$ is generated by the following rules.

$$w_i^* = 0 \quad \text{if} \quad 1 \le i < r,$$

$$w_r^* = \frac{6(n-1)\alpha - 2(n-r-1)}{(n-r+1)(n-r+2)}$$

$$w_n^* = \frac{2(2n-2r+1) - 6(n-1)\alpha}{(n-r+1)(n-r+2)}$$

$$w_i^* = \frac{n-i}{n-r} w_r^* + \frac{i-r}{n-r} w_n^* \text{ if } r < i < n$$

This vector w satisfies $w_i > 0$ for each i = 1 to n, and $\sum_{j=1}^{n} w_j = 1$.

Commutativity 3.4: VMA$_w$ $(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n)$ = VMA$_w$ $(\breve{a}_1, \breve{a}_2, ..., \breve{a}_n)$, where $(\breve{a}_1, \breve{a}_2, ..., \breve{a}_n)$ is any permutation of $(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n)$.

Idempotency 3.5: If VMA$_w$ $(\breve{a}_1, \breve{a}_2, ..., \breve{a}_n)$, where $\tilde{a}_j = (t_j, f_j), \tilde{a} = (t, f)$ for all j,then VMA$_w$ $(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \tilde{a}$

Monotonicity 3.6: If $\tilde{a}_i \le \tilde{a}_i$ for all i, then VMA$_w$ $(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) \le$ VMA$_w$ $(\breve{a}_1, \breve{a}_2, ..., \breve{a}_n)$,

SECTION-4: PROPOSED MODEL OF MAGDM

Assumptions 4.1: Let $A = \{A_1, A_2, ..., A_n\}$ be a set of alternatives, $G = \{G_1, G_2, ..., G_n\}$ be the set of alternatives, $\omega = (\omega_1, \omega_2, ..., \omega_n)$ is the weighting vector of the attribute G_j, j=1,2,…,n, where $\omega_j \in [0,1], \sum_{j=1}^{n} \omega_j = 1$. Let $D = \{D_1, D_2, ..., D_t\}$ be the set of decision makers, $V = (V_1, V_2, ..., V_n)$ be the weighting vector of the decision makers, with $V_k \in [0,1], \sum_{k=1}^{t} V_k = 1$.

Let $\tilde{R}_k = \left(\tilde{r}_{ij}^{(k)}\right)_{m \times n} = \left(t_{ij}^{(k)}, f_{ij}^{(k)}\right)_{m \times n}$ be the vague decision matrix, where $t_{ij}^{(k)}$ is the degree of the truth membership value that the alternative A_i satisfies the attribute G_j given by the decision maker D_k and $f_{ij}^{(k)}$ is the degree of false membership value that the alternative for the alternative A_i, where $t_{ij}^{(k)}, f_{ij}^{(k)} \subset [0,1]$ and, $t_{ij}^{(k)} + f_{ij}^{(k)} \le 1$, i =1,2,…,m, j=1,2,…,n, and k=1,2,…,t.

4.2: An algorithm for a developed model of MAGDM:

Here the steps mentioned below are studied for a model of MAGDM.

Algorithm: The following steps are now given:
Step-1: Utilize the vague decision matrix $R^k = (\tilde{r}_{ij})^{(k)}{}_{m \times n} = ((t_{ij})^k, 1 - (f_{ij})^k)_{m \times n}$, and the FWA operator which has the associated weighting vector w = $(w_1, w_2, \ldots, w_m)^T$ generated from the definition (3.3). Let $(\tilde{r}_{ij})^k = (t_{ij}{}^k, 1 - f_{ij}{}^k)$, i = 1, , , m; j = 1,2,.., n be a matrix of vague values for each k =1 to t. Let R^k = ($(\tilde{r}_{ij})^{(k)}$) be the collection of *t* number of m × n matrices of each the form $R^k = ((\tilde{r}_{ij})^{(k)})$ where k = 1,2,…,t. Then the operator FWA: $[(M_{m \times n})^k \rightarrow M_{m \times n}]$ $R^1, R^2, \ldots, R^k) \rightarrow R(r_{ij})$ is defined by VWA $((\tilde{r}_{ij})^{(1)}), (\tilde{r}_{ij})^{(2)}), \ldots, (\tilde{r}_{ij})^{(k)}))$ which is found due to the definition (3.1). Here $V = (V_1, V_2, ..., V_t)$ be the weighting vector of the decision maker or generaled from the definition (3.3).

Step-2: Utilizing the information from the collective decision matrix R = $(C_{ij}, D_{ij})_{m\times n}$ **found in the step 1.** Then VFHWA operator R = $\tilde{r}_i = (t_i, 1- f_i)$ is defined by $(1-(\prod_{j=1}^{n}(1-c_{ij})^{w_j}), 1-(\prod_{j=1}^{n}(1-c_{ij})^{w_j})$, i =1,2,…,m derive the collective overall preference values of the alternative A_i, which have weight w_i in such a way that the weighting vector as w = $(w_1, w_2, \ldots, w_m)^T$ generated from the definition (3.3).

Step-3: Calculate the distance between the collective overall preference values and the positive ideal vague value $\tilde{r}^+$, or the negative ideal vague value $\tilde{r}^-$, where $\tilde{r}^+$ = (1,0) and $\tilde{r}^-$ = (0,1). Using the Euclidean distance function we can find the distances between the collective overall preference values $\tilde{r}_i$ and the positive ideal vague value $\tilde{r}^+$ as follows:

$$d(\tilde{r}_i, \tilde{r}^+) = \sqrt{\frac{1}{2}\sum_{i=1}^{n} \left[(t_{\tilde{r}_i}(x_i) - t_{\tilde{r}^+}(x_i))^2 + ((1-f_{\tilde{r}_i}(x_i)) - (1-f_{\tilde{r}^+}(x_i)))^2 \right]}$$

Step-4: Rank all the alternatives A_i, where i = 1, 2,…,m and select the best one in accordance with the distance obtained in step 3.

4.3 - Numerical illustration:

Suppose an investment company, wanting to invest a sum of money in the best option, and there is a panel with five possible alternatives to invest the money;
A_1 is an IT company; A_2 is a multinational company; A_3 is a tools company,
A_4 is an airlines company and A_5 is an automobile company.

The investment company must take a decision according to the four following attributes; G_1 is the risk analysis, G_2 is the growth analysis, G_3 is the socio-political impact analysis and G_4 is the environmental impact analysis.

The five possible alternatives A_i, where i = 1, 2,…,m, are to be evaluated by three decision makers whose weighting vector is V = $(0.12, 0.16, 0.20, 0.24, 0.28)^T$ under the method in definition (3.3) with r = 1, α = 0.4, & n = 5, and above said four attributes whose weighting vector is w = $(0.16, 0.22, 0.28, 0.34)^T$, which is generated from the method in (3.3) with r = 1, α = 0.4, & n = 4:

$$R_1 = \begin{bmatrix} (0.4873,0.7256) & (0.5221,0.7222) & (0.6286,0.8312) & (0.4427,0.9986) \\ (0.3271,0.9001) & (0.6676,0.5413) & (0.4261,0.8126) & (0.7710,0.9442) \\ (0.5238,0.8011) & (0.4278,0.5261) & (0.5527,0.6216) & (0.5687,0.7981) \\ (0.7218,0.6283) & (0.7213,0.8912) & (0.8311,0.9219) & (0.6626,0.8215) \\ (0.6257,0.7983) & (0.8321,0.9426) & (0.6256,0.7119) & (0.4136,0.6295) \end{bmatrix}$$

$$R_2 = \begin{bmatrix} (0.4351,0.7846) & (0.5121,0.7221) & (0.1009,0.6221) & (0.2217,0.7184) \\ (0.6321,0.8221) & (0.6226,0.8108) & (0.3009,0.5129) & (0.6225,0.9105) \\ (0.5387,0.9105) & (0.4124,0.7216) & (0.5010,0.7101) & (0.4491,0.5426) \\ (0.7317,0.8119) & (0.5221,0.8001) & (0.2091,0.4104) & (0.2101,0.4110) \\ (0.5273,0.6217) & (0.3125,0.7278) & (0.4728,0.7182) & (0.6210,0.8109) \end{bmatrix}$$

$$R_3 = \begin{bmatrix} (0.3198,0.8279) & (0.4419,0.9816) & (0.2211,0.5221) & (0.6661,0.7027) \\ (0.7726,0.8901) & (0.6245,0.7815) & (0.6216,0.8225) & (0.7101,0.9005) \\ (0.5201,0.7287) & (0.5821,0.6286) & (0.7117,0.9211) & (0.6105,0.9117) \\ (0.3247,0.4821) & (0.7139,0.8148) & (0.4212,0.5334) & (0.5529,0.7217) \\ (0.7351,0.9113) & (0.8001,0.9112) & (0.2221,0.6121) & (0.4214,0.5005) \end{bmatrix}$$

$$R_4 = \begin{bmatrix} (0.3269,0.9111) & (0.4575,0.8222) & (0.5527,0.8686) & (0.8421,0.9526) \\ (0.6321,0.8108) & (0.5010,0.8126) & (0.7317,0.9191) & (0.7351,0.9005) \\ (0.3247,0.4821) & (0.7227,0.9444) & (0.5529,0.8287) & (0.1553,0.9891) \\ (0.8112,0.9238) & (0.2091,0.9545) & (0.4494,0.8898) & (0.6522,0.7377) \\ (0.1983,0.9916) & (0.3125,0.8278) & (0.7428,0.8482) & (0.6217,0.9197) \end{bmatrix}$$

$$R_5 = \begin{bmatrix} (0.2686,0.8812) & (0.4427,0.8986) & (0.6676,0.8126) & (0.7717,0.9552) \\ (0.5218,0.7918) & (0.4278,0.7176) & (0.8311,0.9519) & (0.4163,0.7295) \\ (0.5122,0.9100) & (0.2091,0.5515) & (0.2101,0.8126) & (0.3125,0.8287) \\ (0.3198,0.9728) & (0.4491,0.9861) & (0.6261,0.8522) & (0.7101,0.9552) \\ (0.5210,0.6268) & (0.7711,0.9211) & (0.6195,0.7119) & (0.7513,0.9311) \end{bmatrix}$$

4.5 - Explanation: The stpes for the given algorithm are as follows:

Step-1: Utilizing the decision information given in the matrix $\widetilde{R}_k = \left(\tilde{r}_{ij}^{(k)}\right)_{5\times4}, k = 1,2,3$, 4,5 and the *VWA* operator which has the associated weighting vector w = (0.28, 0.24, 0.2, 0.16, 0.12)^T a collective decision matrix $\widetilde{R}_k = \left(\tilde{r}_{ij}^{(k)}\right)_{5\times4}$ is obtained as follows:

$$R = \begin{bmatrix} (0.3948,0.8059) & (0.4850,0,8048) & (0.4587,0.7095) & (0.5999,0.8490) \\ (0.5917,0.8510) & (0.6001,0.7085) & (0.5777,0.7582) & (0.6901,0.8921) \\ (0.4980,0.7588) & (0.4994,0.6887) & (0.5497,0.7507) & (0.4723,0.7767) \\ (0.6554,0.7103) & (0.5392,0.8729) & (0.5841,0.6702) & (0.5680,0.6785) \\ (0.5702,0.7763) & (0.6829,0.8594) & (0.5562,0.7118) & (0.5570,0.7115) \end{bmatrix}$$

Step-2: Utilizing the VFHWA operator, the collective overall preference values of the alternatives A_i j = 1, 2,…, 5 are found mentioned below:

Using the weighting vector w = (0.34, 0.28, 0.22, 0.16),

$\tilde{r}_1$= (0.4718, 0.7960);
$\tilde{r}_2$= (0.6087, 0.8101);
$\tilde{r}_3$= (0.5064, 0.7423);
$\tilde{r}_4$= (0.5961, 0.7594);
$\tilde{r}_5$= (0.6006, 0.7837);

Step-3: Calculating the distances between the collective overall preference values $\tilde{r}_i$ and the positive ideal vague value $\tilde{r}$ = (1, 0).The distances calculated from the following distance function:

$$d(\tilde{r}_i, \tilde{r}) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}\left[(t_{\tilde{r}_i} - t_{\tilde{r}})^2 + ((1 - f_{\tilde{r}_i}) - (1 - f_{\tilde{r}}))^2\right]}$$

Thus $d(\tilde{r}_1, \tilde{r})$ = 0.6755;
$d(\tilde{r}_2, \tilde{r})$ = 0.6361;
$d(\tilde{r}_3, \tilde{r})$ = 0.3937 ;
$d(\tilde{r}_4, \tilde{r})$ = 0.3324;
$d(\tilde{r}_5, \tilde{r})$ = 0.6219;

Step-4: Rank the alternatives based on the shortest distance: $A_1 > A_2 > A_5 > A_3 > A_4$ Thus A_1 is the best alternative.
Let us consider the replacing of Step-3 with the correlation coefficient proposed Robinson & Amirtharaj [2011]. Then the ranking order of the alternatives is obtained as follows: Thus $A_1 > A_2 > A_5 > A_3 > A_4$ Thus A_1 is the best alternative.

From the comparison, it can be observed that there is a change in the ranking of the best alternatives. In the proposed method with a distance function, A_1 is the best alternative, and with the replacement of step-3 in the algorithm with methods as proposed by Robinson & Amirtharaj [2011b], it can be seen that A_1 is the best alternative.

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