International Journal of Mathematical Archive-2(8), 2011, Page: 1242-1248
Available online through www.ijma.info ISSN 2229 – 5046

UNSTEADY HYDROMAGNETIC FREE CONVECTION FLOW OF A DISSIPATIVE AND RADIATING FLUID PAST A VERTICAL PLATE THROUGH POROUS MEDIA WITH CONSTANT HEAT FLUX

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(Received on: 21-07-11; Accepted on: 03-08-11)

ABSTRACT

This paper investigates the unsteady free convective flow through porous medium past a vertical plate in the presence of magnetic field with constant heat generation. Boundary layer equations are derived and the resulting approximate non linear ordinary differential equations are solved analytically. The velocity, temperature and skin friction parameters are illustrated graphically.

***Keywords:** Unsteady free convection, Vertical plate, Heat generation, Magnetic field.*

1. INTRODUCTION:

Radiation effects on convective flows are important in the context of process involving high temperatures. In many engineering areas such as nuclear power plants, gas, turbines and various propulsion devices for aircraft, missiles and space vehicles. The major difficulties arise in the study of fluid radiation are discussed by A.Azzam [2].The effect of free convection on accelerated flow of a viscous incompressible fluid past an infinite vertical plate with suction has many important technological applications in the astrophysical, geophysical and engineering problems .The study of the flow of an electrically conducting and radiating fluid over porous media has studied due to its numerous applications such applications include MHD pumps, induction pumps,MHD generators , oilexploration, nuclear power plants ,gas turbines, air crafts and space vehicles among many others Soundagelkar [14] considered the free convection effects on the oscillatory flow past an infinite vertical porous plate with constant suction. England and Emery [3] have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate.Soundalgekar and Takhar [17] have considered thin gray gas past a semi infinite vertical plate.Seigel [12] first studied transient free convection flow past a semi-infinite vertical plate by an integral method.Since then many researchers have been published papers on free convection flow past a semi-infinite verticalplate. Soundalgekar et al.[15] studied free convection flow past a vertical porous plate. Yamamoto et al.[18] investigated the acceleration of convection in a porous permeable medium along an arbitrary but smoothsurface. Raptis [11] studied free convection in a porous medium bounded by an infinite plate.Alagoa [1] studied the transient flow of magnetized plasma due to thermal perturbations. He showed that conduction parameters and viscosity largely govern the plasma flows initiated by temperature perturbation with exponential decay modes. Israel-Cookey and Tay [6] extended the earlier work of Alagoa [1] by the inclusion of radiative heat transfer, which is significant in high operating temperatures, such as in oil recovery and nuclear waste management. Their results showed that at small times, radiation parameters influenced the velocity of the transient flow of the plasma fluid inwhich a sudden temperature distribution is introduced in the presence of constant magnetic field. External free convection boundary-layer flows are usually treated by neglecting the effect of viscous dissipation, Gebhart and Mollendorf [5], and Gebhart [4] have shown that viscous dissipative heat is important when the free convective flow field is of extreme size or the flow is at extremely low temperature or in a high gravity field. Singh and Sacheti [14] reported the results of a finite difference analysis of unsteady hydromagnetic free convection flow with constant heat flux where it was observed that the magnetic parameter had a retarding effect on the flow velocity. Singh and Sacheti [13] followed the study by Soundalgekar and Hiremath [16], which

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looked at the flow past an impulsively started infinite isothermal vertical plate in a dissipative fluid. A few other works of interest in this area include Ogulu and Prakash [9], Kim [7], and Makinde [8], A.Ogulu and O.D. Makinde [10].

The object of this study is to consider unsteady hydromagnetic free convection flow for a dissipative and radiating fluid trough porous medum applying a simple perturbation technique. Most of the studies mentioned above have applied one numerical technique or the other, whereas here we have used a much simpler time-saving technique.

2. MATHEMATICAL FORMULATION

We consider the two-dimensional unsteady flow of an incompressible, electrically conducting viscous Boussinesq fluid in a Cartesian $\left(x^1, y^1\right)$ coordinate system with a transversely applied magnetic field, so that in the spirit of Gebhart [4], Singh and Sacheti [13], and Gebhart and Mollendorf [5] the proposed governing equations are

$$\frac{\partial u^1}{\partial t^1} = v\frac{\partial^2 u^1}{\partial y^{1^2}} - \frac{\sigma B_0{}^2}{\rho}u^1 + g\beta\left(T - T_\infty\right) - \frac{\nu}{\kappa}u^1 \tag{1}$$

$$\frac{\partial T}{\partial t^1} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^{1^2}} + \frac{\nu}{c_p}\left(\frac{\partial u^1}{\partial y^1}\right)^2 - \frac{\partial q_r{}^1}{\partial y^1} \tag{2}$$

$$\frac{\partial^2 q_r^1}{\partial y^{1^2}} - 3\alpha^2 q_r^1 - 16\alpha\sigma T_\infty^3\frac{\partial T}{\partial y^1} = 0 \tag{3}$$

where u^1 is the velocity component, T is temperature, t^1 is time, υ is the kinematics coefficient of viscosity, ρ is fluid density, c_p is the specific heat capacity at constant pressure, k is the thermal conductivity, κ is porousity, $q_r{}^1$ is the radiative flux vector, σ is the electrical conductivity, B_0 is the applied magnetic field, g is the acceleration due to gravity, β is the coefficient of volume expansion due to temperature, and subscript ∞ denotes conditions in the free stream. For an optically thin fluid we have (Israel-Cookey et al., [6])

$$\frac{\partial q_r{}^1}{\partial y^1} = 4\alpha^2\left(T - T_\infty\right) \tag{4}$$

Where

$$\alpha^2 = \int_0^\infty \delta\lambda\frac{\partial B}{\partial T} \tag{5}$$

and α^2 is the absorption coefficient, λ is frequency, B is Planck's function, and δ is the radiation absorption coefficient . The initial and boundary conditions are (Singh and Sacheti, [13]).

$$t^1 \le 0 : u^1 = 0, T = T_\infty \text{ for all } y^1 \qquad t^1 > 0 : \begin{cases} u^1 = U, \dfrac{\partial T}{\partial y^1} = -\dfrac{q}{k}, at, y^1 = 0 \\ u^1 \to 0, T = T_\infty, as, y^1 \to \infty \end{cases} \tag{6}$$

where q is the constant heat flux at the plate surface. We find it convenient to now introduce the following non-dimensional quantities and parameters

$$y = \frac{y^1 U}{v},\ t = \frac{t^1 U^2}{v},\ u = \frac{u^1}{U}\ \ \Pr = \frac{\rho v c_p}{k},\ Gr = \frac{g\beta v^2 q}{kU^4},\ M^2 = \frac{\sigma B_0{}^2 v}{\rho U^2}, \theta = \frac{kU\left(T - T_\infty\right)}{qv}$$

$$\kappa = \frac{kU^2}{v^2},\ Ec = \frac{kU^3}{C_P v q},\ Ra = \frac{4\alpha^2 v}{U^2} \tag{7}$$

Where Pr is the Prandtl number, Gr is the Grashof number, M is the Hartmann number, and Ra is the radiation parameter

$$\frac{\partial u}{\partial t}=\left(\frac{\partial^2}{\partial y^2}-M^2-\kappa\right)u+Gr\theta \tag{8}$$

$$\Pr\frac{\partial \theta}{\partial t}=\left(\frac{\partial^2}{\partial y^2}-Ra\Pr\right)\theta+Ec\Pr\left(\frac{\partial u}{\partial y}\right)^2 \tag{9}$$

The appropriate boundary conditions now become

$t\le 0: u=0,\theta=0$ for all y

$$t>0:\begin{cases} u=1,\dfrac{\partial\theta}{\partial y}=-1, at, y=0 \\ u\to 0,\theta=0, as, y\to\infty \end{cases} \tag{10}$$

We seek the solution of Equations (5) and (6) subject to the conditions in Equation (7) as a power series in ε, where ε << 1. For all dependent variables we write

$$u(y,t)=u_0(y)+\varepsilon e^{\mathrm{int}}u_1(y), \qquad \theta(y,t)=\theta_0(y)+\varepsilon e^{\mathrm{int}}\theta_1(y) \tag{11}$$

Substituting Equation (8) into Equations (5)–(6), we obtain the following sequence of approximations:

$$\frac{d^2u_0}{dy^2}-(M^2+\kappa)u_0=-Gr\theta_0 \tag{12}$$

$$\frac{d^2\theta_0}{dy^2}-Ra\Pr\theta_0=-Ec\Pr\left(\frac{du_0}{dy}\right)^2 \tag{13}$$

$$u_0=1,\frac{d\theta_0}{dy}=-1 \quad \text{on}\quad y=0 \qquad u_0\to 0,\theta_0\to 0 \quad \text{as}\quad y\to\infty \tag{14}$$

for O (1) equations

$$\frac{d^2u_1}{dy^2}-\left(in+M^2+\kappa\right)u_1=-Gr\theta_1 \tag{15}$$

$$\frac{d^2\theta_1}{dy^2}-\Pr\left(in+Ra\right)\theta_1=-2Ec\Pr\left(\frac{du_0}{dy}\frac{du_1}{dy}\right) \tag{16}$$

$$u_1=0,\theta_1=0 \qquad \text{on}\quad y=0$$

$$u_1\to 0,\theta_1\to 0 \quad \text{as}\quad y\to\infty \tag{17}$$

for $O(\varepsilon)$ equations.

As observed in Soundalgekar, the Eckert number Ec (viscous dissipation parameter) for incompressible flows is a small so we can further expand our flow variables as

$$u_0(y)=u_{01}(y)+Ecu_{02}(y)$$

$$u_1(y)=u_{11}(y)+Ecu_{12}(y)$$

$$\theta_0(y)=\theta_{01}(y)+Ec\theta_{02}(y)$$

$$\theta_1(y)=\theta_{11}(y)+Ec\theta_{12}(y) \tag{18}$$

Substituting Equation (18) into (12) to (17), we can show that

$$
\begin{aligned}
u(y,t) &= A_2 \exp(-\alpha_2 y) - A_1 \exp(-\alpha_1 y) \\
&+ Ec \left\{ \begin{array}{l} B \exp(-\alpha_2 y) + A_6 \exp(-\alpha_1 y) \\ - A_7 \exp(-2\alpha_2 y) - A_8 \exp(-2\alpha_1 y) \\ + A_9 \exp(-\alpha y) \end{array} \right\} \\
&+ \varepsilon e^{\mathrm{int}} \left\{ \begin{array}{l} ((1 + A_{10})(\exp(-\alpha_4 y)) - A_{10} \exp(-\alpha_3 y)) \\ + Ec \left(\begin{array}{l} D \exp(-\alpha_4 y) - A_{20} \exp(-\alpha_3 y) \\ + A_{21} \exp(-\beta y) + A_{22} \exp(-\Omega y) \\ + A_{23} \exp(-\eta y) + A_{24} \exp(-\xi y) \end{array} \right) \end{array} \right\}
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
\theta(y,t) &= \frac{1}{\alpha_1} \exp(-\alpha_1 y) + Ec \left\{ \begin{array}{l} A \exp(-\alpha_1 y) - A_3 \exp(-2\alpha_2 y) \\ - A_4 \exp(-2\alpha_1 y) + A_5 \exp(-\alpha y) \end{array} \right\} \\
&+ \varepsilon e^{\mathrm{int}} \left\{ \frac{1}{\alpha_3} \exp(-\alpha_3 y) + Ec \left(\begin{array}{l} C \exp(-\alpha_3 y) - A_{16} \exp(-\beta y) \\ - A_{17} \exp(-\Omega y) - A_{18} \exp(-\eta y) \\ - A_{19} \exp(-\xi y) \end{array} \right) \right\}
\end{aligned}
\tag{20}
$$

3. DISCUSSION

In order to study the behavior of velocity (u) and temperature (Θ) fields, a comprehensive numerical computation is carried out for various values of the parameters that describe the flow characteristics, and the results are reported in terms of graphs as shown in Figures .To be more realistic, the value of the Prandtl number is chosen to be Pr = 0.71, which corresponds to air.

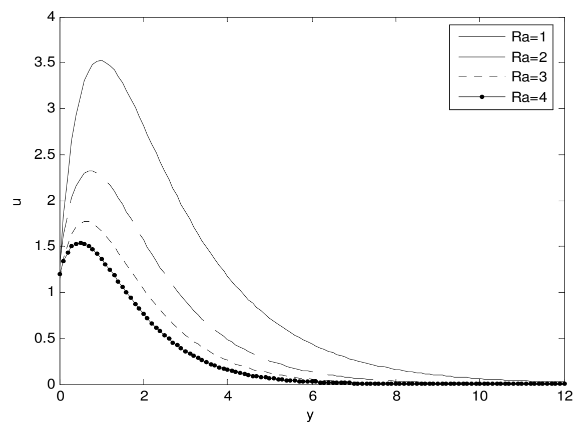

Fig1: Velocity distribution, M=1,Ec=0.01, Gr=5, t=n=1 with κ=1

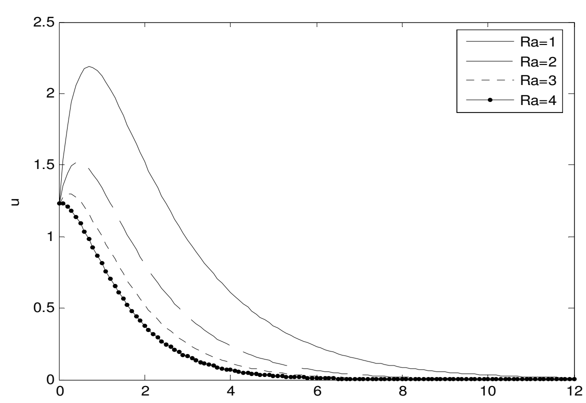

Fig2: Velocity distribution, M=1,Ec=0.01, Gr=5, t=n=1 with κ=3

From figures. 1 and 2 the velocity rises rapidly from unity at the plate, attains a maximum near the plate and decreases to free stream value away from plate.

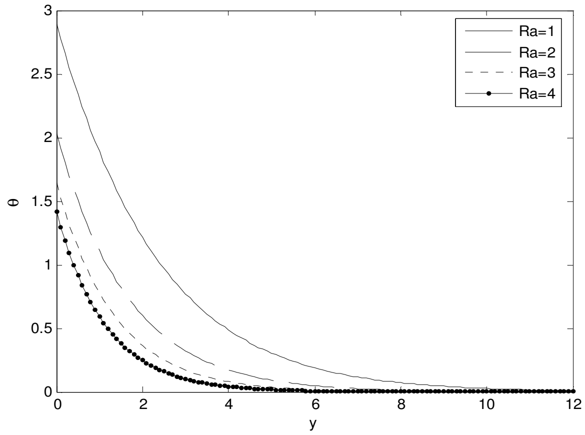

Fig3: Temperature distribution M=1, Ec=0.01,Gr=5,t=n=1with κ=1

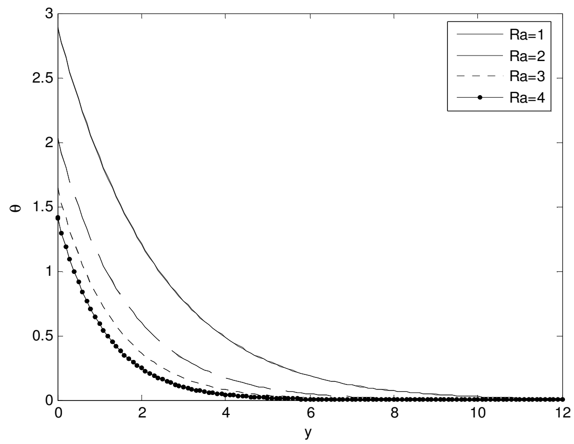

Fig4: Temperature distribution M=1, Ec=0.01,Gr=5,t=n=1with κ=3.

Figures 3 and 4 shows an exponential decrease in the fluid temperature from the plate surface to the free stream value away from the plate. However, it is interesting to note that as the radiation parameter (Ra) increases, both velocity and thermal boundary layer thickness decrease when the plate is cooled by free convection current (Gr > 0).

Where the quantities involved in equations (19) and (20) are given in the appendix. Having obtained expressions for the velocity field and the temperature field we can define the nondimensional skin-friction at the plate τ as

$$\tau = -\frac{du}{dy} \text{ at } y=0$$

Effect of material parameters on skin-friction

Gr	Ra	Ec	τ
1	2	0.001	-0.6133
1	3	0.001	-0.8487
1	3	0.005	-0.8596
3	2	0.001	1.4238
3	3	0.001	0.7603
3	3	0.005	1.0488
-3	2	0.001	-4.8081
-3	3	0.001	-3.9771
-3	3	0.005	-4.2281

4. CONCLUSIONS

We can therefore conclude that

(1) For cooling of the plate by free convection current (Gr>0)

a. Increase in Ra results a decrease in the velocity boundary layer thickness.
b. Increase in Ra leads to a decrease in the thermal boundary layer thickness.
c. Increase in the viscous dissipation Ec results in a decrease in the skin friction

(2) For heating of the plate by free convection current (Gr<0)

Increase in the viscous dissipation Ec results in an decrease in skin friction.

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6. APPENDIX:

$$\alpha=\alpha_1+\alpha_2,\quad \alpha_1=\frac{\sqrt{4\Pr Ra}}{2},\quad \alpha_2=\frac{\sqrt{4(\kappa+M^2)}}{2},\quad \alpha_3=\frac{\sqrt{4\left(in\Pr+\Pr Ra\right)}}{2},\quad \alpha_1=\frac{\sqrt{4\left(in+M^2+\kappa\right)}}{2},$$

$$A_1=\frac{Gr}{\alpha_1\left(\alpha_1^2-M^2-\kappa\right)},\quad A_2=1+A_1,$$

$$A_3=\frac{A_2^2\Pr}{4\alpha_2^2-\Pr Ra},\ A_4=\frac{A_1^2\Pr}{4\alpha_1^2-\Pr Ra},$$

$$A_5=\frac{2A_1A_2\Pr}{\alpha^2-\Pr Ra},\qquad A=\frac{1}{\alpha_1}\left(1+2\alpha_2A_3+2\alpha_1A_4-\alpha A_5\right),$$

$$A_6=\frac{A}{\alpha_1^2-M^2-\kappa},\ A_7=\frac{A_3}{4\alpha_2^2-M^2-\kappa},$$

$$A_8=\frac{A_4}{4\alpha_1^2-M^2-\kappa},\ A_9=\frac{A_5}{\alpha^2-M^2-\kappa},$$

$$A_{10}=\frac{Gr}{\alpha_3\left(\alpha_3^2-in-M^2-\kappa\right)},\qquad A_{11}=\alpha_4\left(1+A_{10}\right),\quad A_{12}=A_2A_{10}\alpha_3,$$

$$A_{13}=A_2A_3,\qquad A_{14}=A_1A_{10}\alpha_3,\qquad A_{15}=A_1A_{11},\qquad \beta=\alpha_2+\alpha_3,\qquad \Omega=\alpha_2+\alpha_4,$$

$$\eta=\alpha_1+\alpha_3,\qquad \xi=\alpha_1+\alpha_4,$$

$$A_{16}=\frac{2A_{12}\Pr}{\beta^2-in\Pr-\Pr Ra},\qquad A_{17}=\frac{2A_{13}\Pr}{\Omega^2-in\Pr-\Pr Ra},$$

$$A_{18}=\frac{2A_{14}\Pr}{\eta^2-in\Pr-\Pr Ra},\qquad A_{19}=\frac{2A_{15}\Pr}{\xi^2-in\Pr-\Pr Ra},$$

$$A_{20}=\frac{CGr}{\alpha_3^2-in-M^2-\kappa},\qquad A_{21}=\frac{A_{16}Gr}{\beta^2-in-M^2-\kappa},$$

$$A_{22}=\frac{A_{17}Gr}{\Omega^2-in-M^2-\kappa},\qquad A_{20}=\frac{A_{18}Gr}{\eta^2-in-M^2-\kappa},$$

$$A_{24}=\frac{A_{19}Gr}{\xi^2-in-M^2-\kappa},\qquad C=\frac{1}{\alpha_3}\left(1+A_{16}\beta+A_{17}\Omega+A_{18}\eta+A_{19}\xi\right),$$

$$D=1+A_{20}-A_{21}-A_{22}-A_{23}-A_{24}.$$

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