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ON P*gα-CLOSEDSETS IN TOPOLOGICAL SPACES

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ABSTRACT

In 1998, T.NOIRI, MAKI, and UMEHARA have defined and studied the concept of generalized preclosedsubsets and generalized preclosed function In topology. In1991, M.K.R.S.VEERA KUMAR have defined and studied the concept of g-preclosed sets and g*-preclosedfunction. The aim of this paper is to introduce the concept of $p^*g\alpha$-closed sets in a topological space and characterization comparing with each other types if generalized closed functions.*

1. INTRODUCTION

In 1970 Levine [1] defined and studied generalized closed sets in topological spaces.

In 1998 T. Noiri [2] M. Maki [2] and J. Umehala [2] defined generalized pre closed sets in topology.

Maki *et. al* [3] considered the concept of generalized pre-closed sets to show that every topological space is pre-T1/2. Dontchev also showed that every topological space is pre-T1/2. In this paper, we introduce the class properly fits between the class of pre-closed sets and the class of generalized pre-closed sets.

Maki *et. al* [4], Bhattacharya and Lahiri [11] dontchev [6] and Gnanambal [5] introduced generalized semi pre-closed and generalized pre-regular closed sets are briefly known as gsp-closed and gpr closed sets.

M.K.R.S Veerakumar[8] introduced the set g^*p-closed set.

Levine [14] Bhattacharya and Lahiri [11] Dontchev [6] and Gnanambal [5]introduced and studied T1/2 spaces , semi-T1/2 spaces, semi-pre T1/2 and pre-regular T1/2 spaces by applying this we introduce and study some new classes of spaces namely. pgTα spaces, $gT\alpha^*$ spaces, $gT\alpha^{**}$ spaces, gpT spaces.

We obtain some inter relationships between these spaces.

We also introduce the notion of $p^*g\alpha$- continuity and study its properties.

We also introduce $p^*g\alpha$ irresolute and also its properties.

2. PRE REQUISITES

Throughout this paper (X, τ) (Y, σ) and (Z, η) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned,

Let us recall the following definitions which are useful in the sequal

Definition 2.01: A subset A of a space (X, τ) is called
1) A semi open set [15] if $A \subseteq cl(int A)$ and a semi closed set if $int(cl(A)) \subseteq A$.
2) A pre-open set [18] if $A \subseteq int(cl(A))$ and a pre-closed set if $cl(int(A)) \subseteq A$.
3) An α-open set [7] if $A \subseteq int(cl(A))$ and an α-closed set if $cl(int(cl(A))) \subseteq A$.
4) A semi pre-open set [6] if $A \subseteq cl(int(cl(A)))$ and a semi pre-closed set if $int\ cl(int(cl(A))) \subseteq A$.

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The semi-closure (resp. pre closure, α-closure, semi pre-closure) of a subset A of space (X, τ) is the intersection of all semi-closed (resp. pre-closed, α-closed, semi-pre closed) sets that contain A and is denoted by scl(A) (resp. pcl(A), αcl(A), spcl(A)).

The following definitions are useful in the sequel.

Definition 2.02: A subset A of a space (X, τ) is called

1. A generalized closed (briefly g-closed) set [14] if cl(A) ⊆ U A ⊆ U and U is open in (X, τ).
2. A generalized closed (briefly αg-closed) set [7] if αcl(A) ⊆ U is open in (X, τ). The complement of an αg-closed set is called an αg-open set.
3. A generalized g^*-pre closed (briefly g^*p-closed set [17] if pcl(A) ⊆ U whenever A ⊆ U and U is a α-open in (X, τ)
4. A generalized preclosed (briefly gp-closed) set [2] if pcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
5. A generalized α-closed (briefly gα-closed) set [4] if αcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
6. A generalized semi-preclosed (briefly gsp-closed) set [6] if pcl(A) ⊆ U and is regular open in (X, τ).
7. A generalized preregular closed (briefly gpr-closed) set [13] if pcl(A) ⊆ U whenever A⊆ U.

Definition 2.03: A function f: (X, τ) → (Y, σ) is said to be

1. Semi-continuous [15] if f^{-1} (V) is semi-open in (X, τ) for every open set V of (Y, σ).
2. Pre-continuous [18] if f^{-1} (V) is preclosed in (X, τ) for every closed set V of (Y, σ).
3. α-continuous [7] if f^{-1} (V) is α-closed in (X, τ) for every closed set V of (Y, σ).
4. g-continuous [1] if f^{-1} (V) is g-closed in (X, τ) for every closed set V of (Y, σ).
5. gα-continuous [4] if f^{-1} (V) is gα-closed in (X, τ) for every closed set V of (Y, σ).
6. αg-continuous [5] if f^{-1} (V) is αg-closed in (X, τ) for every closed set V of (Y, σ).
7. gsp-continuous [6] if f^{-1} (V) is gsp-closed in (X, τ) for every closed set V of (Y, σ).
8. gp-continuous [17] if f^{-1} (V) is gsp-closed in (X, τ) for every closed set V of (Y, σ).
9. gpr-continuous [5] if f^{-1} (V) is gpr-closed in (X, τ) for every closed set V of (Y, σ).
10. gp-irresolute [16] if f^{-1} (V) is gp-closed in (X, τ) for every gsp-closed set V of (Y, σ).
11. gsp-irresolute [6] if f^{-1} (V) is gsp-closed in (X, τ) for every gsp-closed set V of (Y, σ).
12. g^*p-continuous [17] if f^{-1} (V) is g^*p-closed in (X, τ) for every closed set V of (Y, σ).
13. *gα-continuous [] if f^{-1} (V) is *gα-closed in (X, τ) for every closed set V of(Y, σ).

Definition 2.04: A space (X, τ) is called a

1. $gT\alpha^*$ space [16] if every gp-closed set is p^*gα closed.
2. $gT\alpha^{**}$ space [14] if every g-closed set is p^*gα closed.
3. gpTspace [17] if every g^*p-closed set is p^*gα closed.
4. Every Tg space is an pgTα space

Notation 2.05: For a space (X, τ), C(X, τ) (resp sc(X, τ), αc(X, τ), Gα(X, τ), GC(X, τ), G^*PC(X, τ), GPC(X, τ)αGC(X, τ)) denote the class of all closed (resp. semi closed, α-closed, gα-closed, g-closed, g^*p-closed, gp-closed, αg-closed subsets of (X, τ)).

BASIC PROPERTIES OF p*gα-CLOSED SETS

Definition 3.1: A subset A of (X, τ) is said to be p*g α-closed if p cl(A) ⊆ U whenever A ⊆ U and U is *g α- open in (X, τ).

Theorem 3.2: Every closed set is a p*gα-closed set.

Proof: Let A ⊆ U is p*gα open set in X. Since A is closed set cl(A)=A, then cl(A)⊆ U but p cl(A) ⊆ cl(A) ⊆ U, A is p*gα-closed.

Following example shows that the above implication is not reversible.

Example 3.3: Let X = {a, b, c} andτ = {ϕ, X, {a, b}}, p*gα closed(X, τ) = {ϕ, X, {a},{b},{c},{a, b},{b, c},{a, c}} Here {b, c} is a p*gα closed set of (X, τ) but it is not a closed set of (X, τ).

Theorem 3.4: Every pre-closed set is p*gα-closed set.

Proof: Let A ⊆ U, where U is p*gα open set in XSince A is pre-closed set, p cl(A) = A⊆ U, p cl(A) ⊆ U, A is p*gα closed.

Following examples shows that the above implication is not reversible.

Example 3.5: Let X = {a, b, c} and τ = {ϕ, X, {a}, {a, b}} pre-closed set = {ϕ, X, {b}, {c}, {b, c}, p*gα closed set = {ϕ, X, {b},{c},{b, c},{a, c}}. Here {a, c} is a p*gα-closed set of (X, τ) it is not a pre-closed set of (X, τ).

Theorem 3.6: Every p*gα closed set is gp-closed set.

Proof: Let A⊆ U, where U is open set in X. Since every open set is p*gα-open, Since A is p*gα-closed set, therefore p cl(A) ⊆ U, Hence A is g p closed.

Following example shows that the above implication is reversible.

Example 3.7: Let X = {a, b, c} and τ = {ϕ, X, {a}, {b}, {c}}

Here p*gα-closed = {ϕ, X, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}gp closed = {ϕ, X, {a}, {b},{c}, {a, b}, {b, c}, {a, c}}

Theorem 3.8: Every p*gα-closed set is g*p closed set.

Proof: Let A ⊆ U where U is an g-open set X. Since every g-open set is an p*gα-openset, Since A is p*gα-closed set, p cl(A) ⊆ 0,Hence A is p*gα-closed.

Following examples shows that the above implication is reversible.

Example 3.9: Let X = {a, b, c} and τ = {ϕ, X, {a}} g*p closed set = {ϕ, X, {b}, {c}, {b, c}}, p*g closed set = {ϕ, X, {b},{c}, {b, c}}

Example 3.10: Let X = {a, b, c} and τ = {ϕ, X, {a}, {b}, {a, b}}, g*p closed = {ϕ, X, {b}, {c}, {b, c}, [a, c}} p*g closed = {ϕ, X, {b}, {c}, {b, c}, {a, c}}

Remark 3.11: Every p*g closedness is independent of g-closed set.

Example 3.12: Let X = {a, b, c} and τ = {ϕ, X, {a}}, g-closed – {ϕ, X, {b},{c}, {a, b}, {b, c}, {a, c} p*gα-closed – {ϕ, X, {b}, {c}, {b, c}}

Theorem 3.13: Every p*gα-closed set is gsp closed.

Proof: Let A⊆ U where U is an open set in X. Since every open set is *gα open set, Since A is p*gα-closed set, p cl (A) ⊆ U, But spcl (A) ⊆ p cl (A) ⊆ U,spcl (A) ⊆ U,A is gsp –closed.

Following example show that the above implication is not reversible.

Example 3.14: Let X = {a, b, c} and τ = {ϕ, X, {a}, {b}, {c}}, p*gα-closed: {ϕ, X, {b}, {c}, {a, b}, {b, c}, {a, c}}gsp-closed: {ϕ, X, {a}, {b}, {c}, {a, b}, {b, c}}. Here {a} is gsp closed set of (X, τ) but not in p* closed set of (X, τ).

Theorem 3.15: Every p*gα-closed set is gs-closed set.

Proof: Let A ⊆ U, U is an open set X, Since every open set is *gα open set, Since A is p*gα closed set p cl(A) ⊆ U, But scl (A) ⊆pcl (A) ⊆ U,scl (A) ⊆ U,A is gs-closed set.

Following examples shows that the above implications is not reversible.

Example 3.16: Let X = {a, b, c} and τ = {ϕ, X, {a}} gs-closed = {ϕ, X, {b}, {c}, {a, b}, {b, c}, {a, c}} p*gα-closed = {ϕ, X, {b}, {c}, {b, c}Here {a, b} is gs-closed set of (X, τ) but not in p*gα closed set of (X, τ).

Remark 3.17: Every p*gα-closed sets are independent of semi closed sets and semi pre-closed sets.

Let X= {a, b, c} and τ = {ϕ, X, {a}, {b}}p*gα-closed = {ϕ, X, {b},{c},{b, c}, {a, c}}, semi closed set = {ϕ, X, {b}, {c}, {b, c}}semi pre-closed = {ϕ, X, {b},{c},{b, c}}The set A = {a, c} is an p*gα-closed set but not in semi closed and semi pre closed set of (X, τ).

Theorem 3.18: Union of two p*gα- closed sets is p*gα closed set again.

Proof: Let A and B are p*gα-closed sets.Let AUB ⊆ U, U is an *gα-open set.Since A and B are p*gα-closed sets. (i.e) pcl(A) ⊆ U and pcl(B) ⊆ U, pcl (AUB) = pcl(A) U pcl(B) ⊆ U, pcl (AUB) ⊆ U,AUB is p*gα-closed set of (X, τ).

Theorem 3.19: Let A be a p*gα-closed set of (X, τ), Then
(i) pcl(A)-A doesnot contain any non-empty *gα-closed set.
(ii) If A ⊆ B ⊆ C pcl(A) then B is also a p*gα closed set of (X, τ).

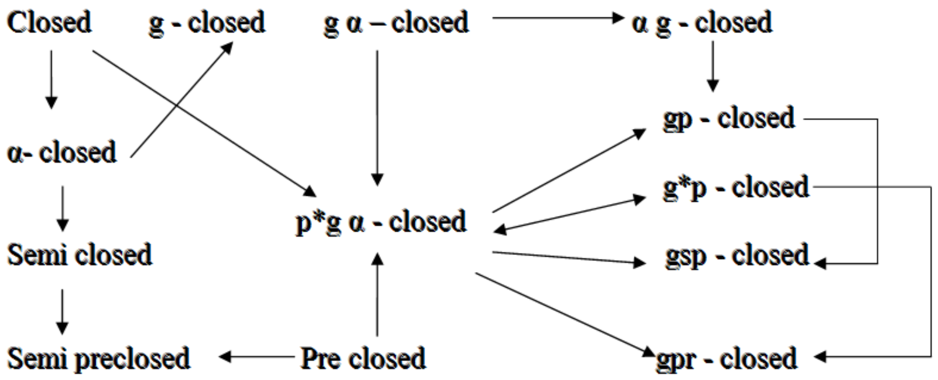
Proof:
(i) Let F be a *gα-closed set contained in (X, τ), pcl(A)-A, pcl(A) ⊆ X-F, Since X-F is *gα-open set with A ⊆ X- F and A is a p*gα-closed, Then F ⊆ (X-pcl(A)) n (pcl(A)-A) ⊆ (X-pcl(A)) n pcl(A) = ϕ, F = ϕ.
(ii) Let U be a *gα-open set of (X, τ), Such that B ⊆ C, then A ⊆ U since A ⊆ U and A is p*gα-closed, pcl(A) ⊆ U, Then pcl(B) ⊆pcl(pcl(A)) = pcl(A), B ⊆pcl(A),Pcl(B) ⊆pcl(A) ⊆ U, Hence B is also a P*gα-closed set of (X, τ).

Theorem 3.20: If a subset A of topological space (X, τ) is regular open then it is p*gα-open set.

Proof: Let A be the regular open. Then A^c is p*gα-closed. $A^c = pcl(A^c) \subseteq U$, A^c is p*gα-closed, Hence A is p*gα-open.

Theorem 3.21: Let A be an open set and B be an *gα-open set, then AUB is *gα-open set.

Proof: Suppose that A is an open set, Suppose that B is an *gα-open set, Since every open set is *gα-open set, So that A is an *gα-open set, Then AUB is *gα-open set,Since union of two *gα-open set is again an *gα-open set of (X, τ).



4. Applications of p*gα-closed sets

Definition 4.1: A space (X, τ) is called $_{pg}T_{\alpha}$ space if very p*gα-closed set is closed.

Theorem 4.2: If (x, τ) is an $_{pg}T_{\alpha}$ space, then every singleton of X is either *gα – closed or open.

Proof: Let X є X and suppose that {X} is not a *g -closed set of (X, τ). Then X/{x} is not a *gα-open. This implies that X is the only *gα-open set containing X/{x}. So X/{x} in a p*gα-closed set of (X, τ). Since (X, τ) is an $_{pg}T_{\alpha}$ space, X/{x} is closed or equivalently {x} is open in (X, τ).

Theorem 4.3: Every semi pre $T_{1/2}$ space is an$_{pg}T_{\alpha}$ -space.

Proof: Let A be a p*gα-closed set of (X, τ), since every p*gα-closed set is gsp-closed, A is gsp-closed, since (X, τ) is a semi pre $T_{1/2}$ space, A is closed, (X, τ) is an $_{pg}T_{\alpha}$ -space.

Example 4.4: Let X = {a, b, c} and τ = {ϕ, X, {a, b}}, gsp-closed = {ϕ, X, {c}, {b, c}, {a, c}}, p*gα-closed = {ϕ, X, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}

Here (X, τ) is an $_{pg}T_{\alpha}$ -space but not semi pre $T_{1/2}$ space. Since {a}, {b} is an p*gα-closed but not a closed.

Application of p*gα-closed sets:

Definition 4.5: A space (X, τ) is called a $_{g}T_{\alpha}^{*}$ if every gp closed set is p*gα-closed.

Theorem 4.6: If (X, τ) is a $_{g}T_{\alpha}^{*}$ space, then every singleton of X is either closed or p*gα-open.

Proof: Let x є X and suppose that {X} is not a closed set of (X,τ). Then X/{x} is not open. This implies X is the only open set containing X/{x}. So X/{x} is a gp-closed set of (X, τ) is a $_{g}T_{\alpha}^{*}$space, X/{x} is a p*gα-closed set or equivalently {x} is p*gα open in (X, τ).

The converse of above theorem is not true as can be seen by the following example

Example 4.7: Let X= {a, b, c) with τ= {x, ϕ, {a}}, $p^*g\alpha$ open sets of (X, τ) are {x, ϕ, {a}, {a, b}, {a, c}}, GpC (X, τ) = {x, ϕ, {b}, {c}, {a, b}, {b, c}, {a, c}}, $P^*G\alpha$ (x, τ) = {x, ϕ, {b}, {c}, {b, c}}. Here {a} is $p^*g\alpha$ open set and {b}, {c} are closed set but (X, τ) is not a $gT^*\alpha$ space. Since {a, b}, {a, c} is gp-closed set but not a $p^*g\alpha$-closed set of (X, τ).

Theorem 4.8: Every T_{gp} space is a ${}_gT_\alpha{}^*$ space.

Proof: Let A be a gp-closed set of (X, τ). Since (X, τ) is a T_{gp} space, A is closed. Since every closed set is $p^*g\alpha$ closed, A is $p^*g\alpha$ closed set. Therefore (X, τ) is a T_{gp} space.

Example 4.9: Let X= {a, b, c) with τ= {x, ϕ, {a}, {b, c}}
GpC (X, τ) = {x, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}
$P^*G\alpha$ (x, τ)= {x, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}
Here (X, τ) is a $gT^*\alpha$ space but not a gTα space, since {b}, {c} is gp-closed set but not a closed set.

Theorem 4.10: Every T_g space is an ${}_{pg}T_\alpha$ space.

Proof: Let A be a p*gα-closed set of (X, τ). Since every p*gα-closed set is gp-closed, A is a gp-closed.

Since (X, τ) is a T_g space, A is closed, therefore (X, τ) is an ${}_{pg}T_\alpha$ space.

The space in the following example is an ${}_{pg}T_\alpha$ space but not a Tb space.

Example 4.11: Let X = {a, b, c} with τ={X, ϕ, {c}}
P*GαC (X, τ) = {X, ϕ, {a, b}}
GPC (X, τ) = {X, ϕ, {a}, {b}, {a, b}, {b, c},{a, c}}

Here (X, τ) is an ${}_{pg}T_\alpha$ space but not a T_b space. Since {b, c}is a gp-closed but not a closed set.

Theorem 4.12: Every ${}_gT_\alpha{}^*$ space is a ${}_{\alpha g}T$ space.

Proof: Let A be a gp closed set of (X, τ), Since (X, τ) is a ${}_gT_\alpha{}^*$ space, A is $p^*g\alpha$-closed, Since every $p^*g\alpha$-closed set is αg-closed.A is αg-closed set. Therefore (X, τ) is a ${}_{\alpha g}T$ space.

The space in the following example is ${}_{\alpha g}T$ space but not ${}_gT_\alpha{}^*$ space.

Example 4.13: Let X= {a, b, c) with τ= {x, ϕ, {c}}
αGC (X, τ) = {x, ϕ, {a}, {b}, {a, b}, {b, c}, {a, c}}
$P^*G\alpha$ (x, τ)= {x, ϕ, {a, b}}
GpC (X, τ) = {x, ϕ, {a}, {b}, {a, b}, {b, c}, {a, c}}. Here (X, τ) is ${}_{\alpha g}T$ space but not a ${}_gT_\alpha{}^*$ space. Since {b} is a gp-closed set but not a $p^*g\alpha$-closed set.

Theorem 4.14: Every ${}_gT_\alpha{}^*$ space is an${}_\alpha T_{1/2}{}^*$ space.

Proof: Let A be a *gα-closed set of (X, τ).Since every $^*g\alpha$-closed set is gpcτlosed, A is gp-closed. Since (X, τ) is a ${}_gT_\alpha{}^*$space, A is $p^*g\alpha$-closed. Since every $p^*g\alpha$-closed set is $^*g\alpha$-closed set. Therefore (X, τ) is an${}_\alpha T_{1/2}{}^*$ space.

The space is the following example is an ${}_\alpha T_{1/2}{}^*$ space but not a ${}_gT_\alpha{}^*$ space.

Example 4.15: Let X and τ be as in above example 4.13.

Here (X, τ) is an${}_\alpha T_{1/2}{}^*$ space but not a ${}_gT_\alpha{}^*$ space. Since {a, c} is a gp-closed set but not $p^*g\alpha$-closed set.

Theorem 4.16: The space (X, τ) is a T_b space if and only if it is ${}_gT_\alpha{}^*$ space an ${}_{pg}T_\alpha$ space.

Proof:

(i) Necessity part: By theorem 4.8 - and- theorem 4.10
(ii) (ii)Sufficient part: Let A be gp-closed sets of (X, τ). Since (X, τ) is a ${}_gT_\alpha{}^*$ space, A is $p^*g\alpha$-closed set.

Since (X, τ) is an ${}_{pg}T_\alpha$ space, A is closed. Therefore (X, τ) is an T_α space.

Definition 4.17: A space (X, τ) is called an ${}_gT_\alpha{}^{**}$ space if every g-closed set is $p^*g\alpha$-closed.

Theorem 4.18: Every T_g space is an ${}_gT_\alpha^{**}$ space.

Proof: Let A be g-closed set of (X, τ). Since every g-closed set is gp-closed, A is gp-closed. Since (X, τ) is a T_g space, A is closed. Since every closed set is p^*g-closed, A is p^*gα-closed set. Therefore (X, τ) is a ${}_gT_\alpha^{**}$ space.

The space in the following example is an ${}_gT_\alpha^{**}$ space but not a T_b space.

Example 4.19: Let X and τ be as in example 4.13.

Here (X, τ) is an ${}_gT_\alpha^{**}$ space but not a T_b space. Since {b, c} is a gp-closed set but not closed set.

Theorem 4.20: Every ${}_\alpha T_g$ space is an ${}_gT_\alpha^{**}$ space.

Proof: Let A be a g-closed set of (X, τ). Since (X, τ) is a ${}_\alpha T_g$ space, A is closed. Since every closed set is p^*gα-closed, A is p^*gα-closed set. Therefore (X, τ) is an ${}_{pg}T_\alpha$ space ${}_gT_\alpha^{**}$ space.

Example 4.21: Let X and τ be as in example 4.13.

Here (X, τ) is an ${}_gT_\alpha^{**}$ space. Since {a, c} is g-closed set but not closed set.

Theorem 4.22: Every ${}_gT_\alpha^{**}$ space is an ${}_{gsp}T_c$ space.

Proof: Let A be a g-closed set of (X, τ). Since (X, τ) is a ${}_gT_\alpha^{**}$ space, A is p^*gα-closed. Since every p^*gα-closed set is gsp-closed, A is gsp-closed set. Therefore (X, τ) is an ${}_{pg}T_\alpha$-space.

The space in the following example is an ${}_{pg}T_\alpha$ space but not an ${}_gT_\alpha^{**}$ space.

Example 4.23: Let X and τ be as in example 4.7.

Here (X, τ) is an ${}_{gsp}T_c$ space but not an ${}_gT_\alpha^{**}$ space. Since {a} is closed set but not p^*gα-closed set.

Theorem 4.24: Every ${}_gT_\alpha^{*}$ space is an ${}_gT_\alpha^{**}$ space.

Proof: Let A be a g-closed set of (X, τ). Since every g-closed set is gp-closed. Since (X, τ) is a ${}_gT_\alpha^{*}$ space, A is p^*gα-closed. Therefore (X, τ) is an ${}_gT_\alpha^{**}$ space.

The space in the following example is an ${}_gT_\alpha^{**}$ space but not ${}_gT_\alpha^{*}$ space.

Example 4.25: Let X and τ be as in example 4.7.

Here (X, τ) is an ${}_gT_\alpha^{**}$ space but not a ${}_gT_\alpha^{*}$ space. Since {a, b} is an gp-closed set but not p^*gα-closed set.

Remark 4.26: ${}_gT_\alpha^{*}$ space and ${}_gT_\alpha^{**}$ space independent of each other.

Example 4.27: Let X = {a, b, c} with τ = {X, ϕ, {a}}.
GC(X, τ) = {ϕ, X, {b}, {c}, {a, b}, {b, c}, {a, c}}.
P^*GαC(X, τ) = {ϕ, X, {b}, {c}, {b, c}}

Since {a, b} is g-closed set but not p^*gα-closed set.

Example 4.28: Let X and τ be as in example 4.13.

Here (X, τ) is an ${}_gT_\alpha^{**}$ space. But not ${}_{pg}T_\alpha$ space.

Since {a, b} is p^*gα-closed set but not closed set.

Definition 4.29: A space (X, τ) is called a ${}_{gp}T$ space if every g^*p closed set is p^*gα-closed set.

Theorem 4.30: Every T_{gp} space is a ${}_{gp}T$ space.

Proof: Let A be a g^*p-closed set of (X, τ). Since (X, τ) is a T_{gp} space, A is closed. Since every closed set is p^*gα-closed, A is p^*gα-closed. Therefore (X, τ) is an ${}_{gp}T$ space.

The space in the following example is a ${}_{gp}T$ space but not a T_{gp} space.

Example 4.31: Let X = {a, b, c} with τ = {X, ϕ, {a}, {b, c}}
$G^{*}PC(X, \tau)$ = {X, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}

Here (X, τ) is a ${}_{gp}T$ space but not T_{gp} space.

Since {c} is a g^{*}p-closed set but not closed set.

Theorem 4.32: Every T_{gp} space is a ${}_{gp}T$ space.

Proof: Let A be a g^{*}p-closed set of (X, τ). Since every g^{*}p-closed set is g-closed, A is g closed set. Since (X, τ) is an T_g space A is closed. Since every closed set is p^{*}gα-closed. A is p^{*}gα-closed. Therefore (X, τ) is an ${}_{gp}$ T space.

The space in the following example is a ${}_{gp}T$ space but not T_g space.

Example 4.33: Let X and τ be as in above example. Here (X, τ) is a ${}_{gp}T$ space but not a T_g space. Since {c} is g-closed set but not closed set.

Theorem 4.34: Every ${}_{g}T_{\alpha}^{*}$ space is ${}_{gp}T$ space.

Proof: Let A be a g^{*}p-closed set of (X, τ). Since every g^{*}p-closed set is gp-closed, A is gp-closed set.

Since (X, τ) is a ${}_{g}T_{\alpha}^{*}$ space, A is p^{*}gα-closed. Therefore (X, τ) is an ${}_{gp}T$ space.

The space in the following example is a ${}_{gp}T$ space but not a ${}_{g}T_{\alpha}^{*}$ space.

Example 4.35: Let x = {a, b, c} with τ = {X, ϕ, {a}}

Here (X, τ) is an ${}_{gp}T$ space but not a ${}_{g}T_{\alpha}^{*}$ space. Since {a, b} is a gp-closed set but not a p^{*}gα-closed set.

Theorem 4.36: The space (X, τ) is a T_{gp} space if and only if it is a ${}_{gp}T$ space and an ${}_{pg}T_{\alpha}$ space.

Proof:
(i) Necessity part: By theorem 4.30 and theorem 4.10
(ii) Sufficient part: Let A be a g^{*}p-closed set of (X, τ). Since (X, τ) is a ${}_{gp}T$ space, A is p^{*}gα-closed. Since (X, τ) is an${}_{pg}T_{\alpha}$ space, A is closed. Therefore (X, τ) is anT_{gp} space.

Remark 4.37:${}_{gp}T$ space and ${}_{g}T_{\alpha}^{**}$ space are independent of each other.

Example 4.38: Let X and τ be as in example 4.13. Here (X, τ) is an ${}_{pg}T_{\alpha}$ space but not an ${}_{gp}T$ space. Since {a} is g^{*}p-closed set but not p^{*}gα-closed set.

Example 4.39: Let X = {a, b, c} with τ = {X, ϕ, {a}, {b, c}}

Here (X, τ) is an ${}_{gp}T$ space but not ${}_{pg}T_{\alpha}$ space. Since {a} is p^{*}gα-closed set but not closed set.

Definition 4.40: A space (X, τ) is called an ${}_{p}{*}_{g}T\alpha$ space if every p*gα closed set is pre-closed.

Theorem 4.41: Every ${}_{p}{*}_{g}T_{\alpha}$ space is an ${}_{\alpha}T_{p}{*}$ space.

Proof: Let A be a pre-closed set of (X, τ). Since (X, τ) is a ${}_{p}{*}_{g}T_{\alpha}$ space, A is p*gα-closed. Since every p*gα closed set is pre-closed set.A is pre-closed set.Therefore (X, τ) is an ${}_{\alpha}T_{p}{*}$ space.

Example 4.42: Let X = {a, b, c}, τ = {ϕ, X, {a}, {b}, {a, b}}, pre-closed set = {ϕ, X, {b}, {c}, {b, c}}
p*gα-closed set = {ϕ, X, {b}, {c}, {b, c}, {a, c}}

Here (X, τ) is an ${}_{\alpha}T_{p}{*}$ space but not an ${}_{p}{*}_{g}T_{\alpha}$ space. Since {a, b} is p*gα-closed set.

Theorem 4.43:
(i) P O (τ) c p*gα O (τ)
(ii) A space (X, τ) is ${}_{p}{*}_{g}T_{\alpha}$ if and only if P O (τ) = p*gα O (τ).

Proof:

(i) Let A be pre-open.Then X-A is pre-closed and so p*gα-closed.This implies A is p*gα-open. Hence P O (τ) c p*gα O (τ).

(ii) Necessity:Let (X, τ) be p*gTα. Let A ϵ p*gα O (τ).Then X-A is p*gα-closed.By hypothesis X-A is pre-closed and thus A ϵ P O (τ). Hence p*gα O (τ) = P O (τ).

(iii) Sufficiently:Let P O (τ) = p*gα O (τ).Let A be p*gα-closed.Then X-A is p*gα-open.Hence X-A ϵ P O (τ). Thus A is pre-closed there by implying (X, τ) is p*gTα.

Definition 4.44: A space X is called Pα-space if the intersection of a preclosure with a closed set is closed.

Example 4.45: Let X = {a, b, c}τ = {ϕ, X, {a}, {b}, {a, b}} pcl(A) = {ϕ, X, {c}, {b, c}, {a, c}}. Then (X, τ) is p_α-space.

Remark 4.46: The following diagram shows them relationship among the separation axioms considered in this paper and reference A→B represents A implies B but B need not imply A always (A and B are independent each other.

$p^*g\alpha$-continuity and $p^*g\alpha$-irresoluteness:

We introduce the following definitions

Definition 5.1: A function f : (X, τ) → (Y, σ) is called $p^*g\alpha$-continuous if f^{-1} (V) is a $p^*g\alpha$-closed set of (X, τ) for every closed set V of (Y, σ).

Theorem 5.2: Every continuous map is $p^*g\alpha$-continuous

Proof: Let V be a closed set of (Y, σ), since f is continuous f^{-1} (V) is closed in (X, τ). But every closed set is $p^*g\alpha$ closed set. Hence f^{-1} (V) is $p^*g\alpha$ closed set in (X, τ). Thus f is $p^*g\alpha$ continuous.

Example 5.3: Let X = {a, b, c} = Yτ = {ϕ, X, {a, b}}, σ = {ϕ, Y, {a}}

Define f: (X, τ) → (Y, σ) by f(a) = c, f(b) = a, f(c) = b, $p^*g\alpha$ closed = { ϕ, X, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}

Here f^{-1} (a, b) = {b, c} is not a closed set in (X, τ). Therefore f is not continuous. However f is $p^*g\alpha$ continuous.

Theorem 5.4: Every $p^*g\alpha$ continuous map is gsp continuous.

Proof: Let V be a closed set of (Y, σ). Since f is $p^*g\alpha$ continuous, f^{-1} (V) is $p^*g\alpha$ closed set in (X, τ). But every $p^*g\alpha$ closed set is gsp closed set. Hence f^{-1} (V) is gsp-closed set in (X, τ). Thus f is gsp-continuous.

Example 5.5: Let X = {a, b, c} = Y**,** τ = {ϕ, X, {a}}**,** σ = {ϕ, Y, {b, c}}

Define f: (X, τ) → (Y, σ) by f(a) = a , f(b) = b, f(c) = c, $p^*g\alpha$ closed = { ϕ, X,{b}, {c}, {b, c}}
gsp-closed = { ϕ, X, {b}, {c}, {a, b}, {b, c}, {a, c}}

Here f^{-1}(a, b) = {a, b} is not in $p^*g\alpha$ closed set in (X, τ). Therefore f is not $p^*g\alpha$ continuous. However f is gsp continuous.

Theorem 5.6: Every $p^*g\alpha$ continuous map is gp continuous.

Proof: Let V be a closed set of (Y, σ). Since f is $p^*g\alpha$ continuous f^{-1} (V) is $p^*g\alpha$ closed set in (X, τ). But every $p^*g\alpha$ closed set in gp closed set in (X, τ). Hence f^{-1} (V) is gp-closed set in (X, τ). Thus f is gp-continuous.

The converse of the above theorem need not be one by the following example

Example 5.7: Let X = {a, b, c} = Y, τ = {ϕ, X, {a}}, σ = {ϕ, Y, {b, c}}

Define f: (X, τ) → (Y, σ) by f(a) = a , f(b) = b, f(c) = c, $p^*g\alpha$ closed = { ϕ, X,{b}, {c}, {b, c}}
gp-closed = { ϕ, X, {b}, {c}, {a, b}, {b, c}, {a, c}}

Here f^{-1}(a, b) = {a, b} is not in $p^*g\alpha$ closed set in (X, τ). Therefore f is not $p^*g\alpha$ continuous. However f is gp continuous.

Theorem 5.8: Every $p^*g\alpha$ continuous map is g^*p continuous.

Proof: Let V be a closed set of (Y, σ). Since f is $p^*g\alpha$ continuous f^{-1} (V) is $p^*g\alpha$ closed set in (X, τ). But every $p^*g\alpha$ closed set in g^*p closed set in (X, τ). Hence f^{-1} (V) is g^*p closed set in (X, τ). Thus f is g^*p continuous.

Example 5.9: Let X = {a, b, c} = Y, τ = {X, ϕ, {c}}**,** σ = {Y, ϕ, {a, b}}

Define f: (X, τ) → (Y, σ) by f(a) = a , f(b) = b, f(c) = c, $p^*g\alpha$ c (X, τ) = { X, ϕ, {a, b}}
g^*p c (X, τ) = { X, ϕ, {a}, {b}, {a, b}}

Here f^{-1} {b} = {a} is not a $p^*g\alpha$ closed set in (X, τ). Therefore f is not $p^*g\alpha$ continuous. However f is g^*p continuous.

Remark 5.10: $p^*g\alpha$ continuity is independent of semi-continuity and semi pre continuity.

Example 5.11: From the above example f^{-1} {b} is not a $p^*g\alpha$-closed set in (X, τ). Therefore f is not $p^*g\alpha$ continuous. However f is semi continuous and semi pre-continuous.

Example 5.12: Let X = {a, b, c} = Y**,** τ = {X, ϕ, {a}, {a, b}}**,** σ = {Y, ϕ, {c}, {b, c}}

Define f:(X, τ) → (Y, σ) by f(a) = a , f(b) = b, f(c) = c, $p^*g\alpha$ c (X, τ) = {X, ϕ, {b}, {c}, {b, c}, {a, c}}
sc(X, τ) = {X, ϕ, {b}, {c}, {b, c}} = spc (X, τ).

Here f^{-1} {{a, c}} = {a, c} is not a semi closed set and semi pre closed set in (X, τ). Therefore f is not semi continuous and semi pre continuous. However f is $p^*g\alpha$ continuous.

5. Application of p*gα-closed

Definition 5.13: A function f: (X, τ) → (Y, σ) is called p*gα-closed irresolute if f^{-1} (v) is a p*gα closed set of (X, τ) for every p*gα closed set of (Y, σ)

Theorem 5.14: Every p*gα-irresolute function is p*gα continues.

Proof: Let V be a closed set of (Y, σ)**,** since every closed set is p*gα-closed set, Therefore V is p*gα-closed set of (Y, σ), Since f is p*gα irresolute f^{-1}(V) is p*gα-closed in (X, τ), f is p*gα continues.

Example 5.15: Let X={a, b, c}=Y**,**τ ={Y, σ, {b}}, σ={X, ϕ, {b}, {b, c}
p*gα-closed (Y, σ) = {{a}, {b}, {c}, {a, b}, {b, c}, {a, c}, ϕ, X}
p*gα c{X, τ}={{a}, {c}, {a, b}, {b, c}, {a, c}, ϕ, Y}
f(a)=a f(b)=b f(c)=c

Here f is p*gα continues but f is not irresolute.

Since {b} is p*gα-closed set in (Y, σ) but f^{-1}(b) = {b} is not in p*gα-closed set in (X, τ).

Theorem 5.16: Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) be any two functions. Then,
(i) gof: (X, τ) → (Z, η) is p*gα continues if g is continues and f is p*gα continues.
(ii) gof: (X, τ) → (Z, η) is p*gα irresolute if both g and f are p*gα irresolute.
(iii) gof: (X, τ) → (Z, η) is p*gα continues if g is p*gα continues and f is p*gα irresolute.

Proof:
1. Let V be closed in (Z, η). Then g^{-1}(V) is closed in (Y, σ).Since g is continues p*gα continuity of f implies $f^{-1}(g^{-1}(V))$ is p*gα-closed in (X, τ). That is $(gof)^{-1}$(v) is p*gα-closed in (X, τ). Hence gof is p*gα continues.
2. Let V be p*gα-closed in (Z, η). Since g is p*gα-irresolute, g^{-1}(V) is p*gα-closed in (Y, σ). As f is p*gα irresolute $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is p*gα-closed in (X, τ). Therefore gof is p*gα irresolute.
3. Let V be closed in (Z, η). Since g is p*gα continues, g^{-1}(V) is p*gα-closed in (Y, σ). As f is p*gα irresolute $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is p*gα-closed in (X, τ). Therefore gof is p*gα continues.

Theorem 5.17: Let f: (X, τ) → (Y, σ) be *gα irresolute and pre closed then for every p*gα- closed set V of (X, τ), f (V) is p*gα-closed in (Y, σ)

Theorem 5.18: Let f: (X, τ) → (Y, σ) be p*gα continuous map (resp.gp continuous, g continuous, g^*p continuous). If (X, τ) is an ${}_{pg}T_\alpha$(resp. ${}_gT_\alpha^*$,${}_gT_\alpha^{**}$, ${}_{gp}T$) space, then f is ($p^*g\alpha$ continuous, $p^*g\alpha$ continuous, $p^*g\alpha$ continuous) continuous.

Proof: Let V be a closed set of (Y, σ). Since f is p*gα continues, $f^{-1}(V)$ is p*gα closed in (X, τ). Since (X, τ) is an pgTα space, f^{-1} (V) is closed in (X, τ). Therefore f is continues.

Theorem 5.19: Let f: (X, τ) → (Y, σ) be a surjective, *gα-irresolute and a closed map. Then f(A) is p*gα-closed set of (Y, σ) for every is p*gα closed set of (X, τ).

Proof: Let A be a is p*gα-closed set of (X, τ), let V be an *gα-open set of (Y, σ). Such that f(A)≤ U. Since f is surjective and *gα-irresolute, $f^{-1}(U)$ is a *gα-open set of (X, τ). Since A≤ $f^{-1}(U)$ and A is p*gα-closed set of (X, τ).pcl (A) ≤ $f^{-1}(U)$, Then f(pcl(A)) ≤ f($f^{-1}(U)$) = U, Since f is closed, f (pcl (A)) = pcl (f (pcl(A))).This implies pcl (f (A)) = pcl (f (pcl (A))) = f (pcl (A)) ≤ U, Therefore f(A) is p*gα-closed set of (Y, σ).

Theorem 5.20: Let f: (X, τ) → (Y, σ) be a surjective, p*gα-irresolute and a closed map. If (X, τ) is an pgTα space, then (Y, σ) is also an pgTα space.

Proof: Let A be a p*gα-closed set of (Y, σ). Since f is p*gα-irresolute, $f^{-1}(A)$ is a p*gα-closed set of (X, τ). Since (X, τ) is an pgTα space, $f^{-1}(A)$ is closed set of (X, τ). Then f($f^{-1}(A)$) = A is closed in (Y, σ). Thus A is closed set of (Y, σ).Therefore (Y, σ) is a pgTα space.

Definition 5.21: A function f: (X, τ) → (Y, σ) is said to be always pre star generalized alpha open (briefly always p*gα open) if for each p*gα open set V of X, f(V) is p*gα-open in Y.

Definition 5.22: A function f: (X, τ)→ (Y, σ) is said to be always pre star generalised closed if for each p*gα -closed set F of X, f(F) is p*gα closed in Y.

Remark 5.23: A bijective functions is always p*gα-open if it is always p*gα-closed.

Theorem 5.24: A surjective function f: (X, τ)→ (Y, σ) is always p*gα -open (respectively p*gα-closed) if and only if for each subset B of Y and each p*gα-closed (respectively p*gα-open) set H of Y such that B < and $f^{-1}(H)$ < f.

Proof: Suppose of is always p*gα open (respectively always p*gα closed). Let B be any subset of Y and f is p*gα-closed (respectively p*gα-open) set of X containing $f^{-1}(B)$. put H = Y-f(X-f). Then H is p*gα-closed (respectively p*gα-open) in Y B < H and $f^{-1}(H)$ < f.

Sufficient: Let U be any p*gα-open (respectively p*gα-closed). Set in X. Put B = Y-f(U), then we have $f^{-1}(B)$ < X-U and X-U is p*gα-closed (respectively p*gα-open). Set in X. There exists p*gα-closed (respectively p*gα-open). Set H of Y such that B = Y-f (U) < H and $f^{-1}(H)$ < X-U. Therefore we obtain f(U) = Y-H and hence f(U) is p*gα open (respectively p*gα-closed)in Y. This shows that f is always p*gα open (respectively always p*gα closed).

6. Pre*generalized αc homeomorphism and their group structure

Definition 6.1: A function f: (X, τ) → (Y, σ) is said to be p*gα-open if the image f(U) is p*gα-open in (Y, σ) for every open set U of (X, τ).

Definition 6.2: A function f: (X, τ)→ (Y, σ) is said to be p*gα open if the image f(U) in p*gα closed in (Y, σ) for every closed set U of (X, τ)

Definition 6.3: A function f: (X, τ) → (Y, σ) is said to be p*gαc homeomorphism (respectively p*gα-homeomorphism) if f is bijective and f and f^{-1} are p*gα-irrespective (respective p*gα continues).

Theorem 6.4: Suppose that f is bijection. Then the following condition are equivalent:

1. f is p*gα homeomorphism.
2. f is p*gα open and p*gα continues.
3. f is p*gα closed and p*gα continues.
4. If f is homeomorphism, then f and f^{-1} are p*gα irresolute.
5. Every p*gα c –homeomorphism is a p*gα homeomorphism.

Proof:

1. First we prove that f^{-1} is p*gα irresolute. Let A be p*gα closed set of (X, τ). To show $(f^{-1})^{-1}(A)$ = f(A) is p*gα-closed in (Y, σ). Let Y be a *gα-open set such that f(A) ≤ U. Then A = (f^{-1}(f(A))) ≤ $f^{-1}(U)$ is *gα open. Since A is p*gα-closed, pcl(A) ≤ $f^{-1}(U)$. we have pcl(f(A))≤ f(pcl(A)) ≤ f($f^{-1}(U)$) = U and so f(A) is p*gα closed. Thus f^{-1} is p*gα irresolute. Since f^{-1} is also a homeomorphism $(f^{-1})^{-1}$ = f p*gα irresolute.
2. Let f is bijective. Since f is p*gαc-homeomorphism f and f^{-1} are p*gα continues. Therefore of is p*gα homeomorphism.

Definition 6.5: For a topological space (X, τ). We define the following three collections of functions:

(i) p*gαch (X, τ) = {f/f: (X, τ) → (X, τ) is a p*gαc-homeomorphism}
(ii) p*gαh (X, τ) = {f/f: (X, τ) → (X, τ) is a p*gα-homeomorphism}
(iii) h(X, τ) = {f/f: (X, τ) → (X, τ) is a homeomorphism}

Corollary 6.6: For a space (X, τ) the following properties hold.

(i) h(X, τ) ≤ p*gαch (X, τ).
(ii) The set p*gαch (X, τ) forms a group under composition of functions.
(iii) The group h(X, τ) is a subgroup of p*gαch (X, τ).
(iv) If f: (X, τ)→ (Y, σ) is a p*gαc -homeomorphism then it induces as isomorphism. f: p*gαch (X, τ)→ p*gαch (Y, σ).

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