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ON STABILITY OF CUBIC FUNCTIONAL EQUATION IN NON-ARCHIMEDEAN FUZZY NORMED SPACES

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ABSTRACT

This paper deals with stability of cubic mappings in non-Archimedean Fuzzy normed spaces by alternative proof which gives better estimation than [8]. Finally, some applications of our results in stability of cubic mapping in non – Archimedean Fuzzy normed space are obtained.

Key words and phrases: *Non-Archimedean fuzzy norm, fuzzy norm, cubic functional equation, valuation.*

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INTRODUCTION

In 1940, Ulam [21] posed the first stability problem while in 1941; Hyers [10] obtained some results regarding stability of linear functional equation. Th. M. Rassias [22] generalized the result of Hyers for additive and linear mappings and also obtained analogous stability problems of quadratic mappings. In 1978 P.M.Gruber [19] stated that stability problems are of particular interest in probability theory and in case of functional equation of different types. A.K. Mirmostafafe and M.S. Moslehian [1] introduced the notion of non- Archimedean Fuzzy norm and also obtained some results regarding stability of Cauchy and Jensen Functional equation in the context of non-archimedean Fuzzy norm space using the approach of Hyper- Ulam-Russias. Since then various authors [1, 3-5, 7, 10-11, 13, 16-17] investigated stability problems regarding Jensen, Cauchy, Quadratic, Cubic with more general domains and ranges. Further, the stability problem of quadratic equation has been investigated by a number of authors and references there in. In addition Alsina [6], Mihet and Radu [11] investigated the stability in the settings of fuzzy, probabilistic and random normed spaces. In [4] Mimostafaee and Moslehian introduced the idea of fuzzy stability of functional equations. The functional equation $f(2x+y)+f(2x-y) = 2f(x+y) - 2f(x-y) + 12f(x)$ is called cubic functional equation since the function $f(x) = cx^3$ is its solution. Every solution of the cubic functional equation is said to be a cubic mapping. The stability problem for cubic functional equation was proved by Jun and Kim [16] for mapping $f: X\rightarrow Y$ where X is a real normed space and Y is Banach Space. In the sequel, we will adopt the usual terminology, notations and convention of the theory of non-archimedean Fuzzy normed space. In this paper we consider the Hyers-Ulam-Rassias stability of the cubic functional equation in non-archimedean fuzzy normed spaces. Finally some applications of the above mentioned results are obtained in non-archimedean Fuzzy normed spaces.

Definition 2.1: Let K be a field. A non-Archimedean absolute value on K is a function (valuation) $|\bullet|: K\rightarrow R$ such that for any $a, b \in K$ we have

(i) $|a| \geq 0$ and equality holds if and only if $a = 0$,
(ii) $|ab| = |a|\,|b|$
(iii) $|a + b| \leq \max\{|a|, |b|\}$

The condition (iii) is called the strict triangle inequally. By (ii), we have $|1| = |-1| = 1$. Thus, by induction, it follows from (iii) that $|n| \leq 1$ for each integer n we always assume in addition that $|\cdot|$ is non trivial, i.e. that there is an $a_0 \in K$ such that $|a_0| \notin \{0, 1\}$

An example of a non-Archimedean valuation is the mapping $|.|$ taking everything but 0 into 1 and $|0| = 0$. This valuation is called trivial.

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Definition 2.2: Let X be a vector space over a scalar field K with a non-Archimedean valuation $|.|$. A function $\|.\|: X \to [0, \infty)$ is said to be a non-archimedean norm if it satisfies the following conditions:

(i) $\|x\|=0$ if and only if $x=0$;
(ii) $\|rx\|=|r|\|x\|$ ($r \in K$, $x \in X$);
(iii) the strong triangle inequality $\|x+y\| \leq \max\{\|x\|,\|y\|\}$ ($x, y \in X$)

Then $(X, \|.\|)$ is called a non-Archimedean normed space.

Definition 2.3: Let X be a linear space over a non-Archimedean field K. A function $N: X \times R \to [0, 1]$ is said to be a non-Archimedean fuzzy norm on X if for all $x, y \in X$ and all $t \in R$.

(N1) $N(x, t) = 0$ for $t \leq 0$;
(N2) $x = 0$ if and only if $N(x, t) = 1$ for all $t > 0$;
(N3) $N(cx, t) = N\left(x, \frac{t}{|c|}\right)$ if $c \neq 0$; $c \in K$.
(NA4) $N(x + y, \max\{s, t\}) \geq \min\{N(x, s), N(y, t)\}$
(N5) $\lim_{t\to\infty} N(x, t) = 1$.

The pair (X, N) is called a non-Archimedean fuzzy normed space. Clearly, if (NA4) holds then so is

(N4) $N(x + y, s + t) \geq \min\{N(x, s), N(y, t)\}$.

A classical vector space over a complex or real field satisfying (N1) – (N5) is called fuzzy normed space. It is easy to see that (NA4) is equivalent to the following condition

(NA4′) $N(x + y, t) \geq \min\{N(x, t), N(y, t)\}$ ($x, y \in X$; $t \in R$).

Example 2.4: Let $(X, \|\cdot\|)$ be a non-Archimedean normed linear space. Then

$$N(x,t) = \begin{cases} \dfrac{t}{t+\|x\|}, & t \leq \|x\|, x \in X \\ 0, & t > \|x\|, x \in X \end{cases}$$

is a non-Archimedean fuzzy norm on X.

Example 2.5: Let $(X, \|\cdot\|)$ be a non-Archimedean linear space. Then

$$N(x,t) = \begin{cases} 0, & t \leq \|x\| \\ 1, & t > \|x\| \end{cases}$$

is a non-Archimedean fuzzy norm on X.

Definition 2.6: Let (X, N) be a non-Archimedean fuzzy normed linear space. Let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is said to be convergent if there exists $x \in X$ such that

$\lim_{n\to\infty} N(X_n - X, t) = 1$ for all $t > 0$

In that case, x is called the limit of the sequence $\{x_n\}$ and we denote it by N-lim $x_n = x$.

A sequence $\{x_n\}$ in X is called Cauchy if for each $\in > 0$ and each $t > 0$. There exists n_0 such that for all $n \geq n_0$ and all $p > 0$ we have $N(x_{n+p} - x_n, t) > 1-\in$. Due to

$N(x_{n+p} - x_n, t) \geq \min\{N(x_{n+p} - x_{n+p-1}, t), \ldots, N(x_{n+1} - x_n, t)\}$

The sequence $\{x_n\}$ is Cauchy if for each $\in>0$ and each $t > 0$. There exists n_0 such that for all $n \geq n_0$ we have

$N(x_{n+1} - x_n, t) > 1-\in$

It is easy to see that every convergent sequence is a non-Archimedean fuzzy normed space is Cauchy.

If each Cauchy sequence is convergent, then non-Archimedean fuzzy normed space is called a non-Archimdean fuzzy Banach space.

3. In the rest of this paper, unless otherwise explicitly stated,

We will assume that K is a non-archimedean field, X is a vector space over K, (Y, N) is a non-Archimedean fuzzy Banach space over K and (Z, N′) is a (Archimedean or non-Archimedean) fuzzy normed space. The functional equation

$$cf(x, y) = f(2x + y) + f(2x - y) - 2f(x + y) + 2f(x - y) + 12f(x)$$

is said to be the cubic functional equation since cx^3 is its solution. Every solution of the cubic functional equation is said to be cubic mapping.

In this paper, we establish the stability of the cubic functional equations in non-Archimedean fuzzy normed space.

Proposition 3.1[14]: Suppose that a complete generalized metric space (E, d) (i.e one for which d may assume infinite values) and a strictly contractive mapping J: E→E with the Lipschitz Constant $0 < L < 1$ are given. Then, for a given element $x \in E$, exactly one of the following assertions is true: either

(a) $d(J^n x, J^{n+1} x) = \infty$ for all $n \geq 0$,

or

(b) there exists k such that $d(J^n x, J^{n+1} x) < \infty$ for all $n \geq k$.

Actually, if (b) holds, then the sequence $\{J^n x\}$ is convergent to a fixed point x* of J and

(b_1) x* is the unique fixed point of J in $F = \{y \in E, d(J^*x, x) < \infty\}$;

(b_2) $d(y, x^*) \leq d(y, Jy)/1-L$ for all $y \in F$.

Lemma 3.2: Let (Z, N’) be a non-Archimedean Fuzzy normed linear space and $\psi : X \to Z$ be a function. Let $E=\{g: X \to Y;\ g(0)=0\}$ and define $d(g, h)=\inf \{a>0 \mid N(g(x)-h(x), a_1^q t) \geq N'(\psi(x),t)$ for all $x \in X$ and $t>0\}$ ($g, h \in E$). Then d is a generalized complete metric on E.

Proof: Let $g, h, k \in E$, $d(g, h) < a_1$ and $d(h, k) < a_2$, then

$$N(g(x)-h(x), a_1^q t) \geq N'(\psi(x), t) \text{ and}$$

$$N(g(x)-k(x), a_2^q t) \geq N'(\psi(x),t) \text{ for each } x \in X \text{ and } t > 0. \text{ Therefore,}$$

$$N(g(x)-k(x), \max(a_1^q t, a_2^q t)) \geq \min\{N(g(x)-h(x), a_1^q t), N(g(x)-k(x), a_2^q t)\}$$
$$\geq N'(\psi(x),t)$$

for each $x \in X$ and $t > 0$. Using $\max\left\{a_1^q, a_2^q\right\} \leq a_1^q + a_2^q \leq \left(a_1 + a_2\right)^q$

($a_1, a_2>0$), and by definition of $d(h, k) \leq a_1+a_2$. This proves the triangle inequality for d. Other properties are direct by using definition of non-Archimedean Fuzzy normed linear space.

Theorem 3.3: Let X be linear space, (Z, N′) be a non -Archimedean fuzzy normed space and $\varphi: X \times X \to Z$ be a function let $f: X \to Y$ be mapping such that

(3.1) $N(cf(x, y), t) \geq N'(\varphi(x, y), t)$ $(x, y \in X, t > 0)$

If for some $\alpha < 8$

(3.2) $N'(\varphi(2x, 0), t) \geq N'(\alpha\, \varphi(x, 0), t)$ $(x \in X, t > 0)$

$f(0) = 0$ and $\lim_{n\to\infty} N'(2^{-3n} \phi(2^n x, 2^n y), t)=1$

for all $x, y \in X$ and $t > 0$, then there exists a unique cubic mapping $T : X \to Y$ such that

(3.3) $N(T(x)-f(x), t) \geq N'(\phi(x, 0), 2(8^p-\alpha^p)^q t)$

Proof: Letting y = 0 in (3.1)

(3.4) $N(2f(2x)-2^4 f(x), t) \geq N'(\phi(x, 0), t)$

Let

$E = \{g : X\to Y;\ g(0) = 0\}$

Define

H: E → E by $H(g)(x) = 2^{-3} g(2x)$ for each $g \in E$ and $x \in X$. By Lemma 3.2

$$d(g, h) = \inf \{a > 0;\ N(g(x)-h(x), a^q t)$$
$$\geq N'(\phi(x, 0), t) \text{ for all } x \in X \text{ and } t > 0\} \text{ for each } g \in E \text{ and } x \in X. \text{ By Lemma 3.2}$$

Defines a metric on E. Let $d(g, h) < a$, by the definition

$$N(g(x)-h(x),\ a^q\ t) \geq N'(\phi\ (x, 0), t)\ (x \in X, t > 0)$$

By (3.2), for each $x \in X$, $t > 0$

$$\begin{aligned} N(H(g)(x) - H(h)(x),\ 2^{-3}\ a^q\ t) &= N(2^{-3} g(2x) - 2^{-3} h(2x),\ 2^{-3}\ a^q\ t) \\ &\geq N(\phi\ (2x, 0), t) \\ &\geq N'(\alpha\ \phi\ (x, 0), t) \end{aligned}$$

Hence, by definition,

$$d(H(g), H(h)) \leq \left(\frac{\alpha}{8}\right)^p a. \text{ Therefore}$$

$$d(H(g), H(h)) \leq \left(\frac{\alpha}{8}\right)^p d(g, h)\ (g, h \in E)$$

This means that H is a contractive mapping with Lipschitz constant

$$L = \left(\frac{\alpha}{8}\right)^p < 1$$

Then by (3.4) $d(f, H(f)) \leq \left(\frac{1}{2^4}\right)^p$.

Hence by proposition 3.1, H has a unique fixed point in the set $\{g \in E;\ d(f, g) < \infty\}$, T: X → Y defined by (3.5)

$$\begin{aligned} T(x) &= N\text{-}\lim_{n\to\infty} H^n(f)(x) \\ &= \lim_{n\to\infty} 2^{-3n} f(2^n x) \end{aligned}$$

and

$$d(f, T) = \frac{d(f, H(f))}{1-L} = \frac{16^{-p}}{1-8^{-p}\alpha^p} = \frac{1}{2^p(8^p-\alpha^p)}$$

It means (3.3) holds.

By (3.5) $cT(x, y) = \lim_{n\to\infty} 2^{-3n} c(f(2^n x, 2^n y))$

Replacing x, y by $2^n x$, $2^n y$ in (3.2)

$$N(2^{-3n} c(f(2^n x, 2^n y), t)) \geq N'(2^{-3n}\ \phi(2^n x, 2^n y), t)$$

By our assumption

$$\lim_{n\to\infty} N'(2^{-3n}\phi\ (2^n x, 2^n y), t) = 1$$

it follows that

$$N(T(x, y), t) = 1 \text{ for all } x, y \in X, t > 0$$

Hence by (N2) T satisfies cubic i.e. T is cubic function. To prove the uniqueness assertion, let us assume that there exists a cubic function T': X→Y which satisfies (3.3). Thus T' is a fixed point of H.

However, by Proposition 3.1, H has only one fixed point.

Hence T ≅ T'.

Theorem 3.4: Let (Z, N′) be non-Archimedean fuzzy normed space and φ: X × X → Z be a function. Let f: X → Y be mapping such that

$$N(c(f(x, y)), t) \geq N'(\varphi(x, y), t)\ (x, y \in X, t > 0)$$

If for some $\alpha > 8$

$N'(\varphi(x/2), 0), t) \geq N'(\varphi(x, 0), \alpha t)$ $(x, y \in X, t > 0)$

$f(0) = 0$ and $\lim_{n\to\infty} N'(2^{3n}\phi\ (2^{-n} x, 2^{-n}y), t)=1$

for all x, y in X and t > 0, then there exists a unique cubic mapping $T : X \rightarrow Y$ such that

$N(T(x)-f(x), t) \geq N'(\phi\ (x, 0), 2(\alpha^p\ 2^p\ )^q\ t)$ $(x, y \in X, t > 0)$.

Proof: After a simple modification in the above proof, we obtain the required result.

Theorem 3.5: Let for each $x \in X$, the functions $s \rightarrow f(sx)$ and $s \rightarrow \varphi(sx, 0)$ be continuous, then for each $x \in X$, then the function $s \rightarrow T(sx)$ is continuous and $T(sx) = s^3\ T(x)$ for each $x \in X$ and $s \in R$.

Proof: fix $x \in X$, $s_0 \in R$, $t > 0$ and $0 < \beta < 1$ take n large enough so that

$$\text{(3.6)}\quad \left(N'(\phi s_0 x,0),(8^p-\alpha^p)^q\left(\frac{8}{\alpha}\right)^n\left(\frac{t}{3^q}\right)\right)>\beta$$

Since, $\lim_{n\to\infty}(8^p-\alpha^p)^q(8/\alpha)^n(t/3^q)=\infty$. Then we have

$$N\left(2^{-3n}f(2^n s_0x)-T(s_0x),\frac{t}{3^q}\right)=N\left(f(2^n s_0x)-T(2^n s_0x),\frac{2^{3n}t}{3^q}\right)$$

$$\geq N'\left(\phi(s_0x,0),2(8^p-\alpha^p)^q\left(\frac{8}{\alpha}\right)^n\left(\frac{t}{3^q}\right)\right)>\beta$$

Thus $s \rightarrow f(2^n\ sx)$ and $s \rightarrow \phi\ (sx, 0)$ are continuous at s_0, we can find some $\delta > 0$ such that, $0<|s-s_0|<\delta$

$$\Rightarrow\quad \begin{cases} N(f(2^n sx)-f(2^n s_0x),\dfrac{2^{3n}t}{3^q})>\beta \\ N'\left(\phi(sx,0)-\phi(s_0x,0),\left(\dfrac{8}{\alpha}\right)^n\left(\dfrac{t}{3^q}\right)\right)>\beta \end{cases}$$

Let $|s-s_0|<\delta$ then (3.7)

$$N(T(sx)-T(s_0x),t)\geq\min\begin{Bmatrix} N\left(T(sx)-2^{-3n}f(2^n s_0x),\dfrac{t}{3^q}\right), \\ N\left(2^{-3n}f(2^n sx)-2^{-3n}f(2^n s_0x),\dfrac{t}{3^q}\right), \\ N\left(2^{-3n}f(2^n s_0x)-T(s_0x),\dfrac{t}{3^q}\right) \end{Bmatrix}$$

$$\Rightarrow\quad N\left(T(sx)-2^{-3n}f(2^n sx),\frac{t}{3^q}\right)\geq N'\left(\phi(sx,0),2(8^p-\alpha^p)^q\left(\frac{8}{\alpha}\right)^n\frac{t}{3^q}\right),$$

$$\geq\min\begin{Bmatrix} N'\left(\phi(sx,0)-\phi(s_0x,0),(8^p-\alpha^p)^q\left(\dfrac{8}{\alpha}\right)^n\dfrac{t}{3^q}\right), \\ N'\left(\phi(s_0x,0),(8^p-\alpha^p)^q\left(\dfrac{8}{\alpha}\right)^n\dfrac{t}{3^q}\right), \end{Bmatrix}$$

$$>\beta$$

The first term of (3.7) is bigger than β. Therefore, for every choice $s_0 \in R$, $x \in X$ and $t > 0$, we can find some $\delta > 0$ such that $N(T(sx, s_0 x)) > \beta$ for every $s \in R$ with $|s - s_0| < \delta$. Thus $s \to T(sx)$ is continuous.

By induction on m, one can easily prove that $T(mx) = m^3\ T(x)$ for every natural number m. It follows that

$$T\left(\frac{m}{k}x\right) = m^3\, T\left(\frac{x}{k}\right) = \left(\frac{m}{k}\right)^3 T(x)\ (m, k \in N)$$

Hence for every rational number r, $T(rx) = r^3\, T(x)$. Let s be a real number, then there exists a sequence $\{r_m\}$ of rational numbers such that $r_m \to s$. Thus by continuity of T(sx) for every $x \in X$.

$$T(sx) = \lim_{m\to\infty} T(r_m x) = \lim_{m\to\infty} r_m^3\, T(x) = s^3\ T(x)$$

4. Applications of stability of cubic mappings in a non-Archimedean fuzzy normed space.

Theorem 4.1: Let $\varphi: X \times Y \to [0, \infty)$ be a mapping such that either
(i) for some $\alpha \neq 8$, $\varphi(2x, 0) \leq \alpha\, \varphi(x, 0)$ for all $x \in X$ and for each $x, y \in X$.
$\lim_{n\to\infty} 2^{-3n}\, \phi\, (2^n x, 2^n y) = 0$ or
(ii) for some $\alpha > 8$, $\alpha\, \phi\, (x, 0) \leq \alpha\, \phi\, (2x, 0)$ for all $x \in X$ and for each $x, y \in X$, $\lim_{n\to\infty} 2^{3n}\, \phi\, (2^{-n} x, 2^{-n} y) = 0$.

Let $f: X\to Y$ satisfy the inequality $\|cf(x, y)\| \leq \varphi(x, y)$ for each $x, y \in X$ and $f(0) = 0$, then there exists a unique cubic mapping $T: X \to Y$ such that

$$\|f(x)- T(x)\| \leq \frac{\phi(x,0)}{2\left|\alpha^p - 8^p\right|^q} \tag{4.1}$$

And, if mappings $s \to f(st)$ and $s - \varphi(sx, 0)$ are continuous, then $s \to T(sx)$ is continuous and $T(sx) = s^3\, T(x)$ for each $s \in R$ and $x \in X$.

Proof: Consider the non-archimedean fuzzy norm define in example 2.4. If (i) holds, then hypotheses of theorem 3.3 are satisfied. In either case, by above theorem 3.5,we can find a cubic mapping $T: X \to Y$ such that (4.1) holds. If the mapping $s \to f(st)$ and $s - \varphi(sx, 0)$ are continuous, then by Theorem(3.3) and (3.5), we get the result.

Corollary 4.2: Let for some $\in>0$, $f: X\to Y$ satisfy the inequality $\|cf(x, y)\| \leq \in$; $x, y \in X$. Then there is a unique continuous cubic mapping $T: X\to Y$ s. t.

$$\|f(x) - f(0) - T(x)\| \leq \frac{2^q \varepsilon}{2\left|1-2^{3p}\right|^q} \text{ and}$$

$T(sx) = s^3\ T(x)$ for each $s \in R$ and $x \in X$.

Proof: For each $x, y \in X$, we have

$$\|c(f- f(0))\,(x, y)\|^p \leq \|cf(x, y)\|^p + \|c\, f(0, 0)\|^p$$
$$\leq \in^p + \in^p = 2 \in^p .$$

Hence $\|c(f- f(0))\,(x, y)\| < 2^q\ (x, y \in X)$.

By Theorem 3.3 for $\varphi(x, y) = 2^q \in$ for each $x, y \in X$ and $\alpha = 1$, we get deserved conclusion.

The following correspondence between a family of non-archimedean norms on a space and a non-archimedean fuzzy normed on the space, under some additional properties is presented.

Theorem 4.3: Let (X, N) be non-Archimedean fuzzy normed linear space such that
(1) $N(x, t) > 0$ for each $t > 0 \Rightarrow x = 0$
(2) $N(x, \cdot)$ is continuous function of R and strictly increasing on $\{t ; 0 < N(x, t) < 1\}$ for each non zero $x \in X$.

Let $\|x\|\alpha = \inf \{t; N(x, t) \geq \alpha\}$, $\alpha\in (0, 1)$ and $N_1: X\times R \to [0, 1]$ is defined by

$$N_1(x, t) = \begin{cases} \sup\{\alpha \in (0,1); \|x\|_\alpha < t\}, & (x,t) \neq (0,0), \\ 0 & ,(x,t) = (0,0) \end{cases}$$

Then
(i) $\{\|\cdot\|_\alpha\}\ \alpha\in(0, 1)$ is an increasing family of non-Archimedean norms on the linear space X.
(ii) $N = N_1$

Moreover, (X, N) is complete if and only if $(X, \|\cdot\|_\alpha)$ is complete for each $\alpha \in (0, 1)$.

Theorem 4.5: Let condition of theorem 3.3 for p = 1 hold let N and N' satisfy conditions (i) and (ii) of above theorem. If $\{\|\bullet\|_\alpha\}\ \alpha \in (0, 1)$ and $\{\|\bullet\|'_\alpha\}\ \alpha \in (0, 1)$ are the increasing non-Archimedean norms corresponding to N and N', respectively. Then there is unique cubic mapping T: $X \to Y$ such that for each $\alpha \in (0, 1)$,

$$\|f(x)-T(x)\|_\alpha \le \frac{\|\phi(x,0)\|'_\alpha}{2(8-\alpha)}$$

Proof: By theorem 3.3, there is unique cubic mapping T: $X \to Y$ such that

$$N(f(x) - T(x), t) \ge N'\left(\frac{\phi(x,0)}{2(8-\alpha)}, t\right)$$

By definition of $\|\bullet\|_\alpha$ and $\|\bullet\|'_\alpha$

$$\|f(x)-T(x)\|_\alpha \le \frac{\|\phi(x,0)\|'_\alpha}{2(8-\alpha)}$$

If T': $X \to Y$ satisfies

$$\|f(x)-T(x)\|_\alpha \le \frac{\|\phi(x,0)\|'_\alpha}{2(8-\alpha)}$$

for each $\alpha\in(0, 1)$, then by the definition and properly (ii) of above theorem.

$$N(f(x)-T'(x)\ \alpha \le), t) \ge N'\left(\frac{\phi(x,0)}{2(8-\alpha)}, t\right).$$

Hence T = T′.

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