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ON FUZZY PAIRWISE-T_0 AND FUZZY PAIRWISE-T_1 BICLOSURE SPACES

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ABSTRACT

The purpose of this paper is to introduce the concept of fuzzy biclosure space as a natural generalization of fuzzy closure space defined in **[5]** *and also introduce and study the separation axioms viz. FP T_0 and FP T_1 in it. The notion of subspace of fuzzy biclosure space, sum of family of pairwise disjoint fuzzy biclosure spaces and product of a family of a fuzzy biclosure space are also introduced and studied the concept of T_0-fuzzy biclosure space and T_1-fuzzy biclosure space. We obtain some important results which establish the appropriateness of definition. In particular, we find that T_0 and T_1 satisfy the hereditary, productive and projective properties. Both T_0 and T_1 fuzzy biclosure space are "good extensions" of the corresponding concepts in a biclosure spaces.*

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Key words: *fuzzy biclosure space, Subspace, Sum fuzzy biclosure space, product fuzzy biclosure space, FP T_0 fuzzy biclosure space, FPT_1 fuzzy biclosure space, Good extensions.*

1. INTRODUCTION

The concepts of closure spaces were introduced by Birkhoff and Cech independently. Later on Boonpok [2] introduced the notion of biclosure spaces. Such spaces are equipped with two arbitrary closure operator .Further the concept of fuzzy closure spaces has been introduced by Mahhour and Ghanim [5] and Srivastava *et al* [7]. Mahhour and Ghanim generalizes the concept of Cech closure spaces while Srivastava *et al* generalizes the concept of Birkhoff closure spaces. Later on Tapi and Navalakhe [8] introduced and studied the concept of fuzzy biclosure spaces.

In this paper we have introduced the concept of fuzzy biclosure space as generalization of Srivastava *et al [6, 7]*. We have introduced T_0 and T_1 fuzzy biclosure spaces. We have studied T_0 and T_1 separation axioms in fuzzy biclosure spaces, in detail. Several important results have been obtain e.g. it has been observed that T_0 axioms and T_1 axioms in a fuzzy biclosure space satisfy the hereditary, productive and projective properties. Also both are "good extensions" of the corresponding concepts in closure spaces.

2. PRELIMINARIES

Here I and I_0 will denote the intervals [0, 1] and (0, 1] respectively. For a set X, I^x denotes the set of all functions from X to I. A constant fuzzy set taking value $\alpha \in [0, 1]$ will be denoted by $\underline{\alpha}$. If $A \subseteq X$, $\mathbf{1}_A$ denotes the characteristic function of A, by A itself. Any fuzzy set u in $A \subseteq X$ will be identified with the fuzzy set in X, which takes the same value as u for x ε A and 0 for $x \in X - A$. Now, we recall the definition of closure operations on a set X.

Definition 2.1 [3]: A function C: $2^x \to 2^x$ is called a closure operation on X if it satisfies the following condition:
(C1) $C(\varphi) = \varphi$
(C2) $A \subseteq C(A) \quad \forall A \in 2^x$
(C3) $C(A \cup B) = C(A) \cup C(B) \quad \forall A, B \in 2^x$

The pair (X, C) is called a closure space.

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Definition 2.2[1]: A map C: $2^x \to 2^x$ is said to be a closure operation on X if the following conditions hold for any A, B $\in 2^x$

(c1) C (φ) = φ
(c2) A $\subseteq$ C (A)
(c3) A $\subseteq$ B$\Rightarrow$ C (A) $\subseteq$ C (B)
(c4) C(C (A)) = C (A)

Definition 2.3 [7]: A function c: $I^x \to I^x$ is called a fuzzy closure operation on X if it satisfies the following conditions:

(c1) c ($\underline{\alpha}$) = $\underline{\alpha}$, $\alpha \in [0, 1]$

(c2) A $\subseteq$ c (A) $\forall$ A $\in I^x$

(c3) A $\subseteq$ B $\Rightarrow$ c (A) $\subseteq$ c (B) $\forall$ A, B $\in I^x$

(c4) c(c (A)) = c(A)

The pair (X, c) is called a fuzzy closure space. It is called fuzzy closure operation on X. This definition is obviously an analogue of Birkhoff closure operator.

Definition 2.4 [3]: A map C: P(X)$\to$ P(X) defined on the power set P(X) of a set X is called a closure operator on X and the pair (X, C) is called a closure space if the following axioms are satisfied:

(C1) C(φ) = φ
(C2) A $\subseteq$ C(A) $\forall$ A $\subseteq$X
(C3) A $\subseteq$ B $\Rightarrow$ C(A) $\subseteq$ C(B) $\forall$ A, B $\subseteq$X

Now we define fuzzy biclosure space which is parrallel line as in **[7]**

Definition 2.5: A function c_i: $I^x \to I^x$, i=1, 2 is called a fuzzy biclosure operation on X if the following axioms are satisfied:

(c1) c_i (α) = α, $\alpha \in [0, 1]$, i= 0, 1

(c2) A $\subseteq c_i$ (A), $\forall$ A $\in I^x$

(c3) A $\subseteq$ B $\Rightarrow$ c_i(A) $\subseteq$ c_i(B), $\forall$ A, B $\in I^x$

(c4) $c_i(c_i(A)) = c_i(A)$, $\forall$ A, B $\in I^x$

Definition 2.6: A subset A of a fuzzy biclosure space (X, c_1, c_2) is said to be fuzzy closed if: c_1 (c_2 (A)) =A

The complement of fuzzy closed set is known as fuzzy open set.

Definition 2.7: We say that the property "FP" in a fuzzy biclosure space is a good extension of the corresponding property 'P' in a biclosure space if (X, C_1, C_2) satisfies 'P' iff (X, ωC_1, ωC_2) satisfies "FP".

Proposition 2.1: Let (X, C_1, C_2) be a fuzzy biclosure space. Then for all$A \subseteq X$. A is C_i- closed iff 1_A is ωC-closed.

Definition 2.8: A biclosure spaces (X, C_1, C_2) is T_0if it satisfied the following condition:

C_i ({x}) = C_i ({y}) $\Rightarrow$ x=y

Proposition 2.2: Let (X, C_1, C_2) be a closure space. Then the following two statements are equivalent.

i. $\forall$ x, y $\in$X, x$\neq$ y,$\exists$ C_i- closed set U s.t x $\in$ U, y$\notin$ U or x $\notin$ U, y $\in$ U.
ii. (X, C_1, C_2) is T_0.

Proof (i) $\Rightarrow$(ii): Let x, y $\in$ X, x$\neq$ y, Then due to (i), $\exists$ C_i- closed set U s.t x $\in$ U, y$\notin$ U or x $\notin$ U, y $\in$ U.

Let us suppose that x ε U, y $\notin$ U. Then {x}$\subseteq$ U any hence C_i ({x})$\subseteq$ C_i (U) =U, but y$\notin$ U so, y$\notin$ C_i ({x}).
Showing that C_i ({x}) $\neq$ C_i ({y}).

(ii) $\Rightarrow$(i): Let x, y $\in$ X, x $\neq$y, from (ii) we have C_i ({x}) $\neq$ C_i ({y}) which implies that either x $\notin C_i$ ({y}) or y $\notin$ C_i ({x}). [Since otherwise {x}$\subseteq$ C_i ({y}) and {y}$\subseteq$ C_i ({x}), which gives C_i ({x}) $\subseteq$ C_i ({y}) and C_i ({y})$\subseteq$ C_i ({x}), thus C_i ({x}) = C_i ({y}), a contradiction].

Let us assume that $x \notin C_i(\{y\})$. Then we have got a C_i- closed set *viz* $C_i(\{y\})$ such that $x \notin C_i(\{y\})$ but $y \in C_i(\{y\})$.

Definition 2.9: A biclosure space (X, C_1, C_2) is called T_1 if $\{x\}$ is Ci- closed $\forall\ x \in X$.

1. Relative, sum and product fuzzy closure operations

Here we define relative fuzzy closure operation on a subset A of an fbcs X, sum fuzzy closure operation on $X = \cup X_t$ for a family$\{(X_t, c_{1t}, c_{2t}): t \in j\}$ of pairwise disjoint fuzzy biclosure space and product fuzzy biclosure operation for a family of fuzzy biclosure space.

The definition of relative fuzzy biclosure operation and the sum fuzzy biclosure operation are defined as.

Proposition 3.1: Let (X, c_1, c_2) be an fuzzy biclosure space and $A \subseteq X$. Then the function c_{iA}: $I^A \to I^A$ given by $c_{iA}(B) = A \cap c_i(B)$ is a fuzzy closure operation on A.

Proof: Conditions (1)-(3) can be easily verified

For condition **(c4)** we proceed as follows:

$$\begin{aligned} c_{iA}(c_{iA}(B)) &= c_{iA}(A \cap c(B)) \\ &= A \cap c(A \cap c(B)) \\ &\subseteq A \cap c(A) \cap c(c(B)) \\ &= A \cap c(A) \cap c(B) \\ &= A \cap c(B) \quad (\text{since } A \cap c(A) = A) \\ &= c_{iA}(B) \end{aligned}$$

Definition 3.1: Let (X, c_1, c_2) be a fuzzy biclosure space and $A \subseteq X$ .Then the fuzzy closure operation c_{iA} defined above is called the relative fuzzy closure operation on A and the fuzzy biclosure space (A, c_{1A}, c_{2A}) is called a fuzzy closure subspace of (X, c_1, c_2).

Proposition 3.2: If u is a c_i -closed fuzzy set in an fuzzy biclosure space (X, c_1, c_2) then u/A is c_{iA} -closed in (A, c_{1A}, c_{2A}).

Proof: Let u is a c_i -closed fuzzy set in X. Now consider u/A.

Then $c_{iA}(u/A) = A \cap c_i(u \cap A) \subseteq A \cap c_i(u) = A \cap c_i(u) = A \cap u$.Thus $c_{iA}(u/A) \subseteq u/A$.

Further, using (2), we have $u/A \subseteq c_{iA}(u/A)$ and hence u/A is c_{iA} -closed

Proposition 3.3: Let $\mathfrak{F} = \{(X_t, c_{1t}\ c_{2t})): t \in \mathfrak{F}\}$ be a family of pair wise disjoint fuzzy biclosure spaces and $X = \cup X_t$. Then the function $\oplus c_{it}$: $I^x \to I^x$ defined by $\oplus\ c_{it}(A) = \cup_t c_t(X_t \cap A)$ is a fuzzy closure operation on X.

Proof: Here also, conditions (c1) – (c2) can be easily verified

We check now the condition **(c4)** as follows:

$$\begin{aligned} \oplus c_{it}(\oplus c_{it}(A)) &= \oplus C_{it}(\cup c_{it}(X_t \cap A)) \\ &= \cup c_{it}(X_t \cap (\cup c_{it}(X_t \cap A)) \\ &= \cup c_{it}(c_{it}(X_t \cap A)) \qquad (\text{Since } X_t\text{'s are pair wise disjoint}) \\ &= \cup c_{it}(X_t \cap A)) \\ &= \oplus c_{it}(A) \end{aligned}$$

Definition 3.2: Let $\mathfrak{F} = \{(X_t, c_{1t}, c_{2t}): t \in \mathfrak{F}\}$ be a family of pair wise disjoint fuzzy biclosure spaces. Then the fuzzy closure operation $\oplus c_{it}$ defined above is called the sum fuzzy biclosure operation on $\cup X_t$ and the corresponding pair $(X, \oplus c_{1t}, \oplus c_{2t})$ is called the sum fuzzy biclosure space of the family $\mathfrak{F}$.

Now, we define the product fuzzy closure operation for a family of fuzzy biclosure operation.

Let $\{(X_j, c_{1j}, c_{2j}): j \in J\}$ be a family of fuzzy biclosure spaces and let $X = \prod_{j \in J} Xj$ and p_j: $X \to X_j$ be the mapping. Let c_i: $I^x \to I^x$ be the mapping given by

$c(u) = \inf\{v \in I^x: v \geq u$, and $v = \wedge_j\ p_j^{-1}(u_j)$ where each u_j is c_j –closed$\}$.

Then c is a fuzzy closure operation on X.

Definition 3.3: Let $\{(X_j, c_{1j}, c_{2j}): j \in J\}$ be a family of fuzzy biclosure spaces then the fuzzy biclosure operation defined above is called the product fuzzy biclosure operation on $X=\prod x_j$ and the fuzzy biclosure space (X, c_1, c_2) is called the product fuzzy biclosure space of the family $\{(X_j, c_{1j}, c_{2j}): j\in J\}$.

Definition 3.4 [1]: Let c_1 and c_2 be two fuzzy closure operations on X.The fuzzy biclosure space (X, c_1, c_2) is said to be coarser than (X, c_1^*, c_2^*) (or that (X, c_1^*, c_2^*) is finer than (X,c_1,c_2)) if $c_i(A) \subseteq c_i^*(A)$ for all $A\in I^x$.

This definition is a natural generalization of **[1]**

2. T_0 and T_1 -FUZZY BICLOSURE SPACES

Definition 4.1: An fuzzy biclosure space (X, c_1, c_2) is said to be fuzzy pairwise T_0 (in short FPT_0) if for all $x, y\in X$, $x\neq y$, $\exists$ a c_i- closed fuzzy set u such that $u(x) \neq u(y)$.

Definition 4.2: A fuzzy biclosure space (X, c_1, c_2) is said to be fuzzy pairwise T_1 (in short FPT_1) $\{x\}$ is c_i- closed $\forall\, x \in X$.

Theorem 4.1: T_0-ness in a fuzzy biclosure space, satisfies the hereditary property.

Proof: Let (X, c_1, c_2) be T_0 space and let (Y, c_1^*, c_2^*) be a subspace of (X, c_1, c_2).

To show that- (Y, c_1^*, c_2^*) is T_0.

Let y_1, y_2 be two distinct points of Y. Since $Y\subseteq X$. y_1, y_2 are also two distinct points of X. Since (X, c_1, c_2) is a T_0 space, $\exists$ a c_i- closed fuzzy set u such that $u (y_1) \neq u (y_2)$. Then

$u_y (y_1) = u\cap Y(y_1) = \inf (u (y_1), Y (y_1)) = u (y_1)$

$u_y (y_2) = u\cap Y(y_2) = \inf (u (y_2), Y (y_2)) = u (y_2)$

$\because \quad u (y_1) \neq u (y_2)$

$\therefore \quad u_y (y_1) \neq u_y (y_2)$

(Y, c_1^*, c_2^*) is also T_0.

Theorem 4.2: T_1-ness in a fuzzy biclosure space, satisfies the hereditary property.

Proof: Let (X, c_1, c_2) be T_1 space and let (Y, c_1^*, c_2^*) be a subspace of (X, c_1, c_2).

To show that- (Y, c_1^*, c_2^*) is T_1.

Since (X, c_1, c_2) be T_1 space. Therefore $\{x\}$ is c_i- closed $\forall \quad x \in X$,

$\Rightarrow$ X-{x} is c_i-open

$\Rightarrow$ Y$\cap$ (X-{x}) is c_i^*-open

$\Rightarrow$Min {Y, (X-{x})} is c_i^*-open

$\Rightarrow$X-{x} is c_i^*-open

$\Rightarrow$$\{x\}$ is c_i^*-closed

$\Rightarrow$$(Y, c_1^*, c_2^*)$ is also T_1.

Theorem 4.3: Let $\{(X_t, c_{1t}, c_{2t}): t\in \mathcal{J}\}$ be a family of pair wise disjoint FPT_0 fuzzy biclosure space. Then their sum fuzzy biclosure space $(X, \oplus c_{1t} \oplus c_{2t})$ is also FPT_0.

Proof: Let $x, y \in X, x \neq y$. There are two possibilities:

Case (i): Both $x, y \in X_t$ for some $t \in \mathcal{J}$ and

Case (ii): $x \in X_{t1}, y \in X_{t2}$ for $t_1, t_2 \in \mathcal{J}, t_1 \neq t_2$.

We consider these two cases separately.

Case (i): Since (X_t, c_{1t}, c_{2t}) is FPT_0, $\exists$ c_{it}-closed fuzzy set say u_t such that $u_t(x) \neq u_t(y)$. Treating u_t as a fuzzy set in X, it can be checked that u_t is $\oplus c_{it}$–closed and obviously satisfies our requirement.

Case (ii): Here Treating X_{t1} as a fuzzy set in X, it can be checked that X_{t1} is $\oplus c_{it}$ –closed and is such that $X_{t1}(x) \neq X_{t1}(y)$.

Hence the sum fuzzy biclosure space $(X, \oplus c_{1t}, \oplus c_{2t})$ is also FPT_0.

Theorem 4.4: Let $\{(X_t, c_{1t}, c_{2t}): t \in \mathcal{J}\}$ be a family of pair wise disjoint FPT_1 fuzzy biclosure space. Then their sum fuzzy biclosure space $(X, \oplus c_{1t}, \oplus c_{2t})$ is also FPT_1.

Proof: Let $x \in X = \cup X_t$ then $x \in X_t$ for some $t_1 \in \mathcal{J}$. Now, since (X_t, c_{1t}, c_{2t}) is c_{t1} –closed and then treating {x} is as a fuzzy set in X, we get {x} is $\oplus c_{it}$ –closed. Hence $(X, \oplus c_{1t}, \oplus c_{2t})$ is also FPT_1.

Theorem 4.5: Let $\{(X_j, c_{1j}, c_{2j}): j \in J\}$ be a family of $FP\ T_0$ -fuzzy biclosure spaces then there product (X, c_1, c_2) is a FPT_0 space iff each co-ordinate fuzzy biclosure spaces (X_j, c_{1j}, c_{2j}) is T_0.

Proof: First let us suppose that $\{(X_j, c_{1j}, c_{2j})$ is T_0 $\forall j \in J$

To show that- (X, c_1, c_2) is a FPT_0.

Now take x, y $\in$ X, x$\neq$y, Let $x=\Pi x_j$ and $y=\Pi y_j$ then since $x \neq y$, $\exists$ $j_1 \in J$ s.t $x_{j1} \neq y_{j1}$, now since (X_j, c_{j1}, c_{j2}) is T_0 $\exists c_{ji}$-closed fuzzy set u_{ji} s.t $u_{j1}(x_{j1}) \neq u_{j1}(y_{j1})$. Consider, $P_{j1}^{-1}(u_{j1})$ then $P_{j1}^{-1}(u_{j1})$ is c-closed and s.t $P_{j1}^{-1}(u_{j1})(x) \neq P_{j1}^{-1}(u_{j1})(y)$, showing that (X, c_1, c_2) is a T_0.

Conversely- Let (X, c_1, c_2) be T_0.

To show that- $\{(X_j, c_{1j}, c_{2j})$ is T_0

Let $x_j, y_j \in X_j$ s.t $x_j \neq y_j$, now consider two points $x=\Pi x_i$ and $y=\Pi y_i$ in X such that $x_i = y_i$ for $i \neq j$ and $x_j = x_j$, $y_j = y_j$ then $x \neq y$ and hence since (X, c_1, c_2) is a T_0 $\exists$ c_i- closed fuzzy set u such that $u(x) \neq u(y)$ now, since u is c_i-closed. $c_i(u)=u=\inf\{v \in I^x / v \geq u$ and $v = \wedge P_i^{-1}(u_i)$ where u_i is c_i- closed $\}$ and $u(x) \neq u(y)$ $\exists$ $v \varepsilon I^x$ such that $v \geq u$, $v = \wedge P_i^{-1}(u_i)$ where u_i is c_i- closed such that $v(x) \neq v(y)$ i.e $\wedge P_i^{-1}(u_i)(x) \neq \wedge P_i^{-1}(u_i)(y)$ or $\inf u_i(x_i) \neq \inf u_i(y_i)$, which implies that $u_j(x_j) \neq u_j(y_j)$ since $u_i(x_i) = u_i(y_i)$ for $i \neq j$ {as $x_i = y_i$ for $i \neq j$ }. Hence $\{(X_j, c_{1j}, c_{2j})$ is T_0.

Theorem 4.6- Let $\{(X_j, c_{1j}, c_{2j}): j\ \varepsilon\ J\}$ be a family of $FP\ T_1$ -fuzzy biclosure spaces then there product (X, c_1, c_2) is a FPT_1 space iff each co-ordinate fuzzy biclosure spaces (X_j, c_{1j}, c_{2j}) is T_1.

Proof: First let us suppose that (X_j, c_{1j}, c_{2j}) is T_1 $\forall j \in J$

To show that- (X, c_1, c_2) is a T_1.

Now take $x \in X$. Let $x = \prod x_j$, then we can write $\{x\} = \wedge_j p_j^{-1}(x_j)$. Now since (X_j, c_{1j}, c_{2j}) is T_1 $\forall j \in J$, $\{x_j\}$ is c_{ij}- closed and hence $p_j^{-1}(x_j)$ is c_i- closed $\forall j\ \varepsilon\ J$

And now using Proposition 2.1, $\wedge_j p_j^{-1}(x_j)$ is c_i- closed. Thus {x} is c_i- closed, showing that (X, c_1, c_2) is T_1.

Conversely- let (X, c_1, c_2) is T_1.

To show that- (X_j, c_{1j}, c_{2j}) is T_1, we have to show that $\{x_j\}$ is c_{ij} – closed $\forall$ $x_j \in X_j$. Now take any $x_j \in X_j$ and consider any $x \in X$ with j^{th} co-ordinate equal to x_j. Let $x = \prod_{i=j} x_j$. Since (X, c_1, c_2) is T_1, {x} is c_i- closed. Hence

$$\{x\} = \inf\{v \in I^x: v \geq u, \text{ and } v = \wedge_i p_i^{-1}(u_i) \text{ where each } u_i \text{ is } c_i\text{–closed}\} \quad (1)$$

Let us denote the family {v∈I^x: v≥u, and v=$\wedge_i\, p_i^{-1}(u_i)$ where each u_i is c_i –closed} by **F.** Take any y_j∈ x_j . Such that $x_j \neq y_j$ and consider $y = \prod y_i$ such that $x_i = y_i$. for i≠j and y_j=y_j. Then x≠y and hence inview of (1), inf v(y)=0.Now, choose any ε>0 then ∃ v^{ε}in **F** such that v^{ε}(y)ε. But $v^{\varepsilon}$is of the form $\wedge_i\, p_i^{-1}(u_i^{\varepsilon})$ where u_i^{ε} is c_i –closed.

So we have

$$\wedge_i\, p_i^{-1}(u_i^{\varepsilon})(y) < \varepsilon \qquad \text{or} \qquad \inf u_i^{\varepsilon}(y'_i) < \varepsilon \tag{2}$$

Here $u_i^{\varepsilon}(y_i)$ =1 for i≠j since v^{ε}≥{x}, i.e. $\wedge_i\, p_i^{-1}(u_i^{\varepsilon})$(x)=1 or that inf($u_i^{\varepsilon}$)($x_i$)=1 which implies that ($u_i^{\varepsilon}$)($x_i$)=1 ∀i ε J and hence $u_i^{\varepsilon}(y_i)$= (u_i^{ε})(x_i)=1 for i≠j. Therefore (2) gives: (u_j^{ε})(y_j) < ε , and now consider $\wedge_i\, u_j^{\varepsilon} = u_j$ (say). Then u_j is C_i –closed and such that $u_j(x_j)$=1 since (u_j^{ε})(x_j)=1 ∀ ε0 and further $u_j(y_j)$=0 since for any ε >0, we can find a c_i –closed fuzzy set such that{ x_j }⊆u_j .

Therefore, c_j({x_j}) ⊆ (u_j) but $u_j(y_j)$=0 hence c_j({x_j})(y_j)=0. Thus, c_j({x_j}) = (x_j) and hence (X_j,c_{1j},c_{2j}) is T_1.

Theorem 4.7: Let (X, C_1, C_2) be a biclosure space then (X, C_1, C_2) is PT_0 iff the fuzzy biclosure space (X, ωC_1, ωC_2) isFPT_0.

Proof: Let (X, C_1, C_2) be T_0. Then (using proposition 2.2) ∃C_i- closed set U s.t, x∈ U, y∉U or x∉U, y∉U. Let as assume that x∈ U, y∉U. now consider the characteristics function I_U Then in view of (proposition 2.1), I_U is ωC_i- closed and is s.t I_U (x) ≠ I_U (y). Hence (X, ωC_1, ωC_2) is T_0.

Conversely- Let (X, ωC_1, ωC_2)$be\ T_0$ then for x, y∈X, x≠ y, ∃ωC_i- closed fuzzy set U, such that

u(x) ≠ u(y) or ωC_i u(x) ≠ ωC_i u(y), where, i=1,2

where

$$\omega C_i\,(u) = \inf\{v\in I^x : v\geq u \text{ and } v^{-1}[1\text{-}\alpha,1] \text{ is } C_i\text{- closed for each } \alpha\in I_0\} \tag{1}$$

Let us denote the family {v∈I^x: v≥u and v^{-1} [1-α, 1] is C_i- closed for each α∈ I_0} by 𝕱**.** Then ωC_i u(x) ≠ ωC_i u(y) means

$$\bigwedge_{v\in \mathbf{F}} v(x) \neq \bigwedge_{v\in \mathbf{F}} v(y)$$

So ∃ v ∈ **F** such that v(x) ≠ v(y). Let us assume that v(x) ∉ v(y). Then choose's' such that v(x) ∉ s∉ v(y). Then 0∉ s∉1 and hence ∃ s ΄∈ I_0such that s=1- s ΄. Now , consider v^{-1}[1- s ΄,1] then in view of (1) is C_i- closed and since v(x) < (1- s ΄) < v(y), we have x∉ v^{-1}[1- s ΄,1] and y∈ v^{-1}[1- s ΄,1], proving that (X, C_1, C_2) is T_0.(using proposition 2.2)

Theorem 4.8: Let (X, C_1, C_2) be a biclosure space then (X, C_1, C_2) is PT_1 iff the fuzzy biclosure space (X, ωC_1, ωC_2) isFPT_1.

Proof: Let us suppose that (X, C_1, C_2) isPT_1, then {x} is C_i- closed. (Using proposition 2.1) {x} is ωC_i –closed i.e. (X, ωC_1, ωC_2) isFPT_1.

Conversely: Let us suppose that (X, ωC_1, ωC_2) isFPT_1, then {x} is ωC_i –closed, (Again using proposition 2.1) {x} is C_i –closed i.e. (X, C_1, C_2) is PT_1.

CONCLUSION

Here we define fuzzy biclosure space as a simple formulation on the parallel lines to its counterparts as given in **[7]**. We introduced and studied relative fuzzy biclosure space, sum fuzzy biclosure space and product fuzzy biclosure space. We also see that fuzzy closure space satisfy good extension property also. We also introduced the notion of T_0 and T_1, fuzzy biclosure space .We see that T_0 and T_1 fuzzy biclosure space satisfy hereditary, productive and projective properties.

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