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FIXED POINT THEOREM FOR FIVE MAPPINGS IN METRIC SPACE

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ABSTRACT

In this paper we prove a common fixed point theorem taking five mappings satisfying a weakly commuting condition. Our result generalize the results of Fisher [2], Pathak [9], Rao and Rao [10], Saluja and Dehariya [12].

***Key Words:** Fixed point, Metric Space, Weakly commuting Mappings.*

***Subject Classification:** 47H10, 54H25.*

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2. INTRODUCTION & PRELIMINARIES:

There are many generalization of Banach Contraction Principle. Some of them are obtained by adding the terms Kannan[8], Fisher[2], Rhoades[11] generalized this principle in many ways. Jaggi [5], Jaggi and Das [6] extended this principle for rational expression. On the same way Bhardwaj et. al., Yadava et.al. also worked for rational expression. Initially Jungck [7] proved a common fixed point theorem for commuting mappings. This result was extended and generalized in various ways by many authors. On the way Sessa [13] gave the concept of weak commutativity.

In the present paper we prove a fixed point theorem for five mappings taking weakly commuting condition for certain mappings. Our result is motivated by Pathak[9], Rao and Rao[10], Saluja and Dehariya [12].

Definition 2.1[3, 4]: Let K be a non empty subset of a metric space (X, d), $T : K \to X \text{ and } F : K \to CB(X)$. The pair (F,T) is said to be weakly commuting if for every $x, y \in K \text{ with } x \in Fy \text{ and } Ty \in K,$ we have

$$d\left(Tx, FTy\right) = d\left(Ty, Fy\right)$$

Definition 2.2[4]: Let K be a non empty subset of a metric space (X, d), $T : K \to X \text{ and } F : K \to CB(X)$. The pair (F,T) is said to be point-wise R-weakly commuting on K if for given $x \in K \text{ and } Tx \in K,$ there exists some R=R(x)>0 such that

$$d\left(Ty, FTx\right) \le R\, d\left(Tx, Fx\right) \qquad \text{for each } y \in K \cap Fx.$$

Definition 2.3: Let R, S, T, U and V are five mappings of a metric space (X, d). Then the pair (R S, T U) is called weakly commuting pair of two mappings with respect to V if

$$(2.3.1) \qquad d\left(SVRSV\left(x\right), TUV\left(x\right)\right) \le d\left(RSVSV\left(x\right), TUV\left(x\right)\right)$$

$$(2.3.2) \qquad d\left(RSV\left(x\right), UVTUV\left(x\right)\right) \le d\left(RSV\left(x\right), TUVUV\left(x\right)\right)$$

$$(2.3.3) \qquad d\left(VRSV\left(x\right), TUV\left(x\right)\right) \le d\left(RSVV\left(x\right), TUVV\left(x\right)\right)$$

$$(2.3.4) \qquad d\left(RSV\left(x\right), VTUV\left(x\right)\right) \le d\left(RSV\left(x\right), TUVV\left(x\right)\right)$$

3. MAIN RESULT:

Theorem 3.1: Let R, S, T, U and V are five self mappings of complete metric space (X, d) satisfying the conditions:
(A) The pair (R, S, T, U) is weakly commuting pair of two mappings with respect to V.

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$$(B)\quad d\left(RSV(x),TUV(y)\right)^2 \le \alpha \frac{d\left(x,TUV(y)\right)d\left(y,RSV(x)\right)}{1+d\left(x,TUV(y)\right)d\left(y,RSV(x)\right)+d\left(x,TUV(y)\right)d(x,y)}$$
$$+\beta\, d\left(y,TUV(y)\right)d\left(x,RSV(x)\right)+\gamma\, d\left(y,TUV(y)\right)d\left(y,RSV(x)\right)$$
$$+\delta\, d\left(RSV(x),TUV(y)\right)d(x,y)+\eta\, d(x,y)^2$$

for all $x,y\in X,\ x\ne y,$ *where* $\alpha,\beta,\gamma,\delta,\eta\ge 0$ *and* $\alpha+\beta+2\gamma+\delta+\eta<1.$

Then R, S, T, U and V have a unique common fixed point in X.

Proof: Let x_0 be an arbitrary point of X. We define a sequence
$\{x_n\}$ *by* $x=x_{2n},\ y=x_{2n-1}$ *and* $RSV\left(x_{2n,}\right)=x_{2n+1},TUV\left(x_{2n-1,}\right)=x_{2n}$

$$d\left(x_{2n},x_{2n+1}\right)^2 = d\left(RSV\left(x_{2n}\right),TUV\left(x_{2n-1}\right)\right)^2$$
$$\le \alpha\frac{d\left(x_{2n},TUV\left(x_{2n-1}\right)\right)d\left(x_{2n-1},RSV\left(x_{2n}\right)\right)}{1+d\left(x_{2n},TUV\left(x_{2n-1}\right)\right)d\left(x_{2n-1},RSV\left(x_{2n}\right)\right)+d\left(x_{2n},TUV\left(x_{2n-1}\right)\right)d\left(x_{2n},x_{2n-1}\right)}$$
$$+\beta d\left(x_{2n},RSV\left(x_{2n}\right)\right)d\left(x_{2n-1},TUV\left(x_{2n-1}\right)\right)+\gamma d\left(x_{2n-1},TUV\left(x_{2n-1}\right)\right)d\left(x_{2n-1},RSV\left(x_{2n}\right)\right)$$
$$+\delta d\left(RSV\left(x_{2n}\right),TUV\left(x_{2n-1}\right)\right)d\left(x_{2n-1},x_{2n}\right)+\eta d\left(x_{2n-1},x_{2n}\right)^2$$
$$\le \alpha\frac{d\left(x_{2n},x_{2n}\right)d\left(x_{2n-1},x_{2n+1}\right)}{1+d\left(x_{2n},x_{2n}\right)d\left(x_{2n-1},x_{2n+1}\right)+d\left(x_{2n},x_{2n}\right)\,d\left(x_{2n},x_{2n-1}\right)}$$
$$+\beta d\left(x_{2n},x_{2n+1}\right)d\left(x_{2n-1},x_{2n}\right)+\gamma d\left(x_{2n-1},x_{2n}\right)d\left(x_{2n-1},x_{2n+1}\right)$$
$$+\delta d\left(x_{2n+1},x_{2n}\right)d\left(x_{2n-1},x_{2n}\right)+\eta d\left(x_{2n-1},x_{2n}\right)^2$$

By using inequalities, we obtain

$$d\left(x_{2n},x_{2n+1}\right)^2 \le \beta d\left(x_{2n},x_{2n+1}\right)d\left(x_{2n-1},x_{2n}\right)+\gamma d\left(x_{2n-1},x_{2n}\right)\left[d\left(x_{2n-1},x_{2n}\right)+d\left(x_{2n},x_{2n+1}\right)\right]$$
$$+\delta d\left(x_{2n+1},x_{2n}\right)d\left(x_{2n-1},x_{2n}\right)+\eta d\left(x_{2n-1},x_{2n}\right)^2$$
$$\le \beta d\left(x_{2n},x_{2n+1}\right)d\left(x_{2n-1},x_{2n}\right)+\gamma d\left(x_{2n-1},x_{2n}\right)^2+\gamma d\left(x_{2n-1},x_{2n}\right)d\left(x_{2n},x_{2n+1}\right)$$
$$+\delta d\left(x_{2n+1},x_{2n}\right)d\left(x_{2n-1},x_{2n}\right)+\eta d\left(x_{2n-1},x_{2n}\right)^2$$

$$\le (\beta+\gamma+\delta)d\left(x_{2n},x_{2n+1}\right)d\left(x_{2n-1},x_{2n}\right)+(\gamma+\eta)d\left(x_{2n-1},x_{2n}\right)^2$$

Since $A.M\ge G.M$, *then*

$$d\left(x_{2n},x_{2n+1}\right)^2 \le \frac{(\beta+\gamma+\delta)}{2}\left[d\left(x_{2n},x_{2n+1}\right)^2+d\left(x_{2n-1},x_{2n}\right)^2\right]+(\gamma+\eta)d\left(x_{2n-1},x_{2n}\right)^2$$

$$d\left(x_{2n},x_{2n+1}\right)^2 \le \left\{\frac{\eta+\left(\frac{\beta+3\gamma+\delta}{2}\right)}{1-\left(\frac{\beta+\gamma+\delta}{2}\right)}\right\}d\left(x_{2n-1},x_{2n}\right)^2$$

$$d\left(x_{2n},x_{2n+1}\right)^2 \le k\ d\left(x_{2n-1},x_{2n}\right)^2$$

$$where\quad k=\left\{\frac{\eta+\left(\frac{\beta+3\gamma+\delta}{2}\right)}{1-\left(\frac{\beta+\gamma+\delta}{2}\right)}\right\}<1, \sin ce\ \alpha+\beta+2\gamma+\delta+\eta<1.$$

$$(3.1.1)\qquad d\left(x_{2n},x_{2n+1}\right)\le k^{\frac{1}{2}}\ d\left(x_{2n-1},x_{2n}\right)$$

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Similarly on taking $x = x_{2n+1},\ y = x_{2n}\ and\ RSV\left(x_{2n+1,}\right) = x_{2n+2}, TUV\left(x_{2n,}\right) = x_{2n+1}$

in condition (3.1) (B), we obtain

$$
\begin{aligned}
d\left(x_{2n+1}, x_{2n+2}\right)^2 &= d\left(RSV\left(x_{2n+1}\right), TUV\left(x_{2n}\right)\right)^2 \\
&\le \alpha \frac{d\left(x_{2n+1}, TUV\left(x_{2n}\right)\right) d\left(x_{2n}, RSV\left(x_{2n+1}\right)\right)}{1 + d\left(x_{2n+1}, TUV\left(x_{2n}\right)\right) d\left(x_{2n}, RSV\left(x_{2n+1}\right)\right) + d\left(x_{2n+1}, TUV\left(x_{2n}\right)\right) d\left(x_{2n}, x_{2n+1}\right)} \\
&\quad + \beta d\left(x_{2n+1}, RSV\left(x_{2n+1}\right)\right) d\left(x_{2n}, TUV\left(x_{2n}\right)\right) + \gamma d\left(x_{2n}, TUV\left(x_{2n}\right)\right) d\left(x_{2n}, RSV\left(x_{2n+1}\right)\right) \\
&\quad + \delta d\left(RSV\left(x_{2n+1}\right), TUV\left(x_{2n}\right)\right) d\left(x_{2n+1}, x_{2n}\right) + \eta d\left(x_{2n+1}, x_{2n}\right)^2 \\
&\le \alpha \frac{d\left(x_{2n+1}, x_{2n+1}\right) d\left(x_{2n}, x_{2n+2}\right)}{1 + d\left(x_{2n+1}, x_{2n+1}\right) d\left(x_{2n}, x_{2n+2}\right) + d\left(x_{2n+1}, x_{2n+1}\right) d\left(x_{2n}, x_{2n+1}\right)} \\
&\quad + \beta d\left(x_{2n+1}, x_{2n+2}\right) d\left(x_{2n}, x_{2n+1}\right) + \gamma d\left(x_{2n}, x_{2n+1}\right) d\left(x_{2n}, x_{2n+2}\right) \\
&\quad + \delta d\left(x_{2n+2}, x_{2n+1}\right) d\left(x_{2n}, x_{2n+1}\right) + \eta d\left(x_{2n}, x_{2n+1}\right)^2 \\
&\le \beta d\left(x_{2n+1}, x_{2n+2}\right) d\left(x_{2n}, x_{2n+1}\right) + \gamma d\left(x_{2n}, x_{2n+1}\right)\left[d\left(x_{2n}, x_{2n+1}\right) + d\left(x_{2n+1}, x_{2n+2}\right)\right] \\
&\quad + \delta d\left(x_{2n+2}, x_{2n+1}\right) d\left(x_{2n}, x_{2n+1}\right) + \eta d\left(x_{2n}, x_{2n+1}\right)^2 \\
&\le (\beta + \gamma + \delta) d\left(x_{2n+2}, x_{2n+1}\right) d\left(x_{2n+1}, x_{2n}\right) + (\gamma + \eta) d\left(x_{2n+1}, x_{2n}\right)^2 \\
&\le \left(\frac{\beta + \gamma + \delta}{2}\right)\left[d\left(x_{2n+2}, x_{2n+1}\right)^2 + d\left(x_{2n+1}, x_{2n}\right)^2\right] + (\gamma + \eta) d\left(x_{2n+1}, x_{2n}\right)^2
\end{aligned}
$$

$$
d\left(x_{2n+2}, x_{2n+1}\right)^2 \le \left\{\frac{\eta + \left(\frac{\beta + 3\gamma + \delta}{2}\right)}{1 - \left(\frac{\beta + \gamma + \delta}{2}\right)}\right\} d\left(x_{2n}, x_{2n+1}\right)^2
$$

$$
d\left(x_{2n+2}, x_{2n+1}\right)^2 \le k\, d\left(x_{2n}, x_{2n+1}\right)^2
$$

$$
(3.1.2) \qquad d\left(x_{2n+2}, x_{2n+1}\right) \le k^{\frac{1}{2}} d\left(x_{2n}, x_{2n+1}\right)
$$

$$
where \qquad k = \left\{\frac{\eta + \left(\frac{\beta + 3\gamma + \delta}{2}\right)}{1 - \left(\frac{\beta + \gamma + \delta}{2}\right)}\right\} < 1, \quad \sin ce\ \ \alpha + \beta + 2\gamma + \delta + \eta < 1.
$$

Hence by (3.1.1) and (3.1.2), we have

$$
d\left(x_{2n+2}, x_{2n+1}\right) \le k^{\frac{1}{2}} d\left(x_{2n}, x_{2n+1}\right) \le \left(k^{\frac{1}{2}}\right)^2 d\left(x_{2n-1}, x_{2n}\right)
$$

$In\ \ general,$

$$
d\left(x_{2n+2}, x_{2n+1}\right) \le \left(k^{\frac{1}{2}}\right)^{2n+1} d\left(x_0, x_1\right) \to 0\ as\ n \to \infty.
$$

Thus $\{x_n\}$ is a D-Cauchy sequence in X. Since X is complete, there exists a point $p \in X,\ such\, that\ \lim\limits_{n\to\infty} x_n = p.$

Now, first we shall prove that p is a common fixed point of RSV and TUV.

On taking $x = p, y = x_{2n+1}\ in\, condition\,(3.1B), we\, obtain$

$$
\begin{aligned}
d\left(RSV(p), x_{2n+2}\right)^2 &= d\left(RSV(p), TUV(x_{2n+1})\right)^2 \\
&\le \alpha \frac{d\left(p, TUV(x_{2n+1})\right) d\left(x_{2n+1}, RSV(p)\right)}{1 + d\left(p, TUV(x_{2n+1})\right) d\left(x_{2n+1}, RSV(p)\right) + d\left(p, TUV(x_{2n+1})\right) d\left(p, x_{2n+1}\right)} \\
&\quad + \beta\, d\left(p, RSV(p)\right) d\left(x_{2n+1}, TUV(x_{2n+1})\right) + \gamma\, d\left(x_{2n+1}, TUV(x_{2n+1})\right) d\left(x_{2n+1}, RSV(p)\right) \\
&\quad + \delta\, d\left(RSV(p), TUV(x_{2n+1})\right) d\left(p, x_{2n+1}\right) + \eta\, d\left(p, x_{2n+1}\right)^2 \\
&\le \alpha \frac{d\left(p, x_{2n+2}\right) d\left(x_{2n+1}, RSV(p)\right)}{1 + d\left(p, x_{2n+2}\right) d\left(x_{2n+1}, RSV(p)\right) + d\left(p, x_{2n+2}\right) d\left(p, x_{2n+1}\right)} \\
&\quad + \beta\, d\left(p, RSV(p)\right) d\left(x_{2n+1}, x_{2n+2}\right) + \gamma\, d\left(x_{2n+1}, x_{2n+2}\right) d\left(x_{2n+1}, RSV(p)\right) \\
&\quad + \delta\, d\left(RSV(p), x_{2n+2}\right) d\left(x_{2n+1}, p\right) + \eta\, d\left(x_{2n+1}, p\right)^2
\end{aligned}
$$

$making\ \lim it\ n \to \infty,\ we\, have$

$$
\begin{aligned}
d\left(RSV(p), p\right)^2 &\le \alpha \frac{d(p,p)\, d\left(p, RSV(p)\right)}{1 + d(p,p)\, d\left(p, RSV(p)\right) + d(p,p)\, d(p,p)} \\
&\quad + \beta\, d\left(p, RSV(p)\right) d(p,p) + \gamma\, d(p,p)\, d\left(p, RSV(p)\right) \\
&\quad + \delta\, d\left(RSV(p), p\right) d(p,p) + \eta\, d(p,p)^2
\end{aligned}
$$

$d\left(RSV(p), p\right)^2 \le 0$

$\Rightarrow d\left(RSV(p), p\right) \le 0$

$which\, is\ possible\, only\ when\quad RSV(p) = p.$

$Similarly\ \ TUV(p) = p.$

Now, we shall prove that p is the common fixed point of RSV and TUV. Let q be another common fixed Point of RSV and TUV, then by condition (3.1)(B), we have

$$
\begin{aligned}
d(p,q)^2 &= d\left(RSV(p), TUV(q)\right)^2 \\
&\le \alpha \frac{d\left(p, TUV(q)\right) d\left(q, RSV(p)\right)}{1 + d\left(p, TUV(q)\right) d\left(q, RSV(p)\right) + d\left(p, TUV(q)\right) d(p,q)} \\
&\quad + \beta\, d\left(p, RSV(p)\right) d\left(q, TUV(q)\right) + \gamma\, d\left(q, TUV(q)\right) d\left(q, RSV(p)\right) \\
&\quad + \delta\, d\left(RSV(p), TUV(q)\right) d(p,q) + \eta\, d(p,q)^2
\end{aligned}
$$

$$
\begin{aligned}
&\le \alpha \frac{d(p,q)\, d(q,p)}{1 + d(p,q)\, d(q,p) + d(p,q)\, d(q,p)} \\
&\quad + \beta\, d(p,p)\, d(q,q) + \gamma\, d(q,q)\, d(q,p) \\
&\quad + \delta\, d(p,q)\, d(p,q) + \eta\, d(p,q)^2
\end{aligned}
$$

$d(p,q)^2 \le (\alpha + \delta + \eta)\, d(p,q)^2$

$which\, is\ possible\, only\ when\ p = q,\ \sin ce\ \alpha + \beta + 2\gamma + \delta + \eta < 1.$

Now, we shall show that SV and UV have a common fixed point.

On taking x=SV (p), y= p in condition (3.1)(B) and applying definition (2.3.1), we have

$$d\left(SV(p),p\right)^2 = d\left(SVRSV(p),TUV(p)\right)^2 \le d\left(RSVSV(p),TUV(p)\right)^2$$
$$\le \alpha \frac{d\left(SV(p),TUV(p)\right)d\left(p,RSVSV(p)\right)}{1+d\left(SV(p),TUV(p)\right)d\left(p,RSVSV(p)\right)+d\left(SV(p),TUV(p)\right)d\left(SV(p),p\right)}$$
$$+\beta\, d\left(SV(p),RSVSV(p)\right)d\left(p,TUV(p)\right)+\gamma\, d\left(p,TUV(p)\right)d\left(p,RSVSV(p)\right)$$
$$+\delta\, d\left(RSVSV(p),TUV(p)\right)d\left(SV(p),p\right)+\eta\, d\left(SV(p),p\right)^2$$

$$\le \alpha \frac{d\left(SV(p),p\right)d\left(p,SV(p)\right)}{1+d\left(SV(p),p\right)d\left(p,SV(p)\right)+d\left(SV(p),p\right)d\left(SV(p),p\right)}$$
$$+\beta\, d\left(SV(p),SV(p)\right)d\left(p,p\right)+\gamma\, d\left(p,p\right)d\left(p,SV(p)\right)$$
$$+\delta\, d\left(SV(p),p\right)d\left(SV(p),p\right)+\eta\, d\left(SV(p),p\right)^2$$

$$\le \alpha \frac{d\left(SV(p),p\right)^2}{1+2d\left(SV(p),p\right)^2}+\delta\, d\left(SV(p),p\right)^2+\eta\, d\left(SV(p),p\right)^2$$

$$\le (\alpha+\delta+\eta)\, d\left(SV(p),p\right)^2$$

which is possible only when $p = SV(p)$, *since* $\alpha+\beta+2\gamma+\delta+\eta<1.$

Similarly, on taking x=p, y=UV (p), and applying definition (2.3.2), we have

$$d\left(p,UV(p)\right)^2 = d\left(RSV(p),UVTUV(p)\right)^2 \le d\left(RSV(p),TUVUV(p)\right)^2$$
$$\le \alpha \frac{d\left(p,TUVUV(p)\right)d\left(UV(p),RSV(p)\right)}{1+d\left(p,TUVUV(p)\right)d\left(UV(p),RSV(p)\right)+d\left(p,TUVUV(p)\right)d\left(p,UV(p)\right)}$$
$$+\beta\, d\left(p,RSV(p)\right)d\left(UV(p),TUVUV(p)\right)$$
$$+\gamma\, d\left(UV(p),TUVUV(p)\right)d\left(UV(p),RSV(p)\right)$$
$$+\delta\, d\left(RSV(p),TUVUV(p)\right)d\left(p,UV(p)\right)+\eta\, d\left(p,UV(p)\right)^2$$

$$\le \alpha \frac{d\left(UV(p),p\right)d\left(p,UV(p)\right)}{1+d\left(UV(p),p\right)d\left(p,UV(p)\right)+d\left(UV(p),p\right)d\left(UV(p),p\right)}$$
$$+\beta\, d\left(p,p\right)d\left(UV(p),UV(p)\right)+\gamma d\left(UV(p),UV(p)\right)d\left(UV(p),p\right)$$
$$+\delta\, d\left(UV(p),p\right)d\left(UV(p),p\right)+\eta\, d\left(UV(p),p\right)^2$$

$$\le \alpha \frac{d\left(UV(p),p\right)^2}{1+2d\left(UV(p),p\right)^2}+\delta\, d\left(UV(p),p\right)^2+\eta\, d\left(UV(p),p\right)^2$$
$$\le (\alpha+\delta+\eta)\, d\left(UV(p),p\right)^2$$

which is possible only when $p = UV(p)$, *since* $\alpha+\beta+2\gamma+\delta+\eta<1.$

Thus

(3.1.3) SV (p) =UV (p) =p.

Finally, we shall prove that p is a common fixed point of R, S, T, U and V.

On taking x=V (p), y=p and applying definition (2.3.3), we have

$$
\begin{aligned}
d\left(V(p),p\right)^2 &= d\left(VRSV(p),TUV(p)\right)^2 \le d\left(RSVV(p),TUVV(p)\right)^2 \\
&\le \alpha \frac{d\left(V(p),TUVV(p)\right)d\left(p,RSVV(p)\right)}{1+d\left(V(p),TUVV(p)\right)d\left(p,RSVV(p)\right)+d\left(V(p),TUVV(p)\right)d\left(V(p),p\right)} \\
&\quad + \beta\, d\left(V(p),RSVV(p)\right)d\left(p,TUVV(p)\right)+\gamma\, d\left(p,TUVV(p)\right)d\left(p,RSVV(p)\right) \\
&\quad + \delta\, d\left(RSVV(p),TUVV(p)\right)d\left(V(p),p\right)+\eta\, d\left(V(p),p\right)^2 \\
&\le \alpha \frac{d\left(V(p),p\right)d\left(p,V(p)\right)}{1+d\left(V(p),p\right)d\left(p,V(p)\right)+d\left(V(p),p\right)d\left(V(p),p\right)} \\
&\quad +\beta\, d\left(p,p\right)d\left(V(p),V(p)\right)+\gamma d\left(p,p\right)d\left(V(p),p\right) \\
&\quad +\delta\, d\left(V(p),p\right)d\left(V(p),p\right)+\eta\, d\left(V(p),p\right)^2 \\
&\le \alpha \frac{d\left(V(p),p\right)^2}{1+2d\left(V(p),p\right)^2}+\delta\, d\left(V(p),p\right)^2+\eta\, d\left(V(p),p\right)^2 \\
&\le \left(\alpha+\delta+\eta\right)d\left(V(p),p\right)^2
\end{aligned}
$$

which is possible only when $p = V(p)$, *since* $\alpha+\beta+2\gamma+\delta+\eta<1$.

Similarly, on taking x=p, y=U(p) and applying definition (2.3.4), we have p = U(p).

Thus
(3.1.4) U (p) = V (p) = p.

Hence by (3.1.3) and (3.1.4), we have

SV (p) = UV (p) =U (p) =V (p) = S (p) =p.

Also by the uniqueness of p,

RSV (p) =TUV (p) =R(p) =T(p) =p.

Therefore, R(p) =S(p) =T(p) =U(p) =V(p) =p.

Hence p is a common fixed point of R, S, T, Uand V.

Uniqueness: Let w be another common fixed point of R, S, T, U and V. Then by condition (3.1) (B), we have

$$
\begin{aligned}
d\left(p,w\right)^2 &= d\left(RSV(p),TUV(w)\right)^2 \\
&\le \alpha \frac{d\left(p,TUV(w)\right)d\left(w,RSV(p)\right)}{1+d\left(p,TUV(w)\right)d\left(w,RSV(p)\right)+d\left(p,TUV(w)\right)d\left(p,w\right)} \\
&\quad + \beta\, d\left(p,RSV(p)\right)d\left(w,TUV(w)\right)+\gamma\, d\left(w,TUV(w)\right)d\left(w,RSV(p)\right) \\
&\quad + \delta\, d\left(RSV(p),TUV(w)\right)d\left(p,w\right)+\eta\, d\left(p,w\right)^2 \\
&\le \alpha \frac{d\left(p,w\right)d\left(w,p\right)}{1+d\left(p,w\right)d\left(w,p\right)+d\left(p,w\right)d\left(w,p\right)} \\
&\quad + \beta\, d\left(p,p\right)d\left(w,w\right)+\gamma d\left(w,w\right)d\left(w,p\right) \\
&\quad + \delta\, d\left(p,w\right)d\left(p,w\right)+\eta\, d\left(p,w\right)^2
\end{aligned}
$$

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$$\leq \alpha \frac{d(p,w)^2}{1+2d(p,w)^2} + \delta\, d(p,w)^2 + \eta\, d(p,w)^2$$

$$\leq (\alpha+\delta+\eta)\, d(p,w)^2$$

which is possible only when $p = w$, *since* $\alpha+\beta+2\gamma+\delta+\eta<1.$

This completes the proof of the theorem.

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