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APPLICATION OF BERNOULLI EQUATION

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ABSTRACT

It is valid in regions of steady, incompressible flow where net frictional forces are negligible. In this paper we discuss the first order differential equations such as linear and Bernoulli equation. From this we get some idea how differential equations are closely associated with physical applications to study real world problems which are described by first order differential equations. We introduce the equation of continuity and conservation of fluid flow, from which we derive Bernoulli equation.

Keywords: *compressive, conservation of energy, steady flow, streamline, turbulent.*

1. INTRODUCTION

Bernoulli equation is applied to fluid contained in a control volume fixed in space, the control volume has impenetrable boundaries, with the exception of one or more inlets and one or more outlets through which fluid enters and leaves the control volume. During passage of fluid through the control volume, mechanical work is irreversibly transferred by fluid friction into heat, leading to losses. Also the fluid may operate a turbine, performing work on the blades of the machine, or work may be performed on the fluid by a pump. These lead to shaft work, assumed by convention to be positive when performed by the fluid, and negative when performed on the fluid. Both losses and shaft work are included in the energy form of Bernoulli equation on the basis of unit mass of fluid flowing through.

In some applications, there is no machinery such as pump or turbine in the control volume. In such cases, the shaft work term in Bernoulli equation is zero. It is convenient to neglect losses in short section of piping. Where there are losses they are accommodated separately using so called loss coefficient. To handle these types of applications, use a simpler version of Bernoulli equation that contains neither shaft work nor a loss term. It is simply called Bernoulli equation.

2. BASIC ASSUMPTIONS

The Bernoulli equation is an approximate relation between pressure p, velocity v and elevation h. The steady flow of liquid through an enclosed tube or pipe, if we put mass m_1 into the pipe, then the same mass $m_2 = m_1$ must flow out, then the energy form of Bernoulli equation is as follows:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Assume the flow is steady and the overall flow does not change very quickly in time. Derive the rules governing fluid motion by considering the pressure force acting on the individual fluid particle and applying Newton's second law, the force causes acceleration. A force acting in the flow direction causes fluid particles change their speed where as a force acting normal to the flow direction causes streamline curvature.

3. BERNOULLI EQUATION: PRESSURE GRADIENT ALONG STREAMLINES

A fluid particle travelling along a straight line. Let x direction be in the direction of motion. If the particle is in a region of varying pressure and has a finite size l. The pressure drops in the x direction, ($\frac{dp}{dx} < 0$), the pressure at the rear is higher than at the front and the particle experiences a positive net force. By Newton's second law, the force causes acceleration and the particle's velocity increases as it moves along the streamline. Conversely, if the pressure increases in the direction of the flow, the particle decelerates.

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This means that if the pressure drops along a streamline, the velocity increases and vice versa. Mathematically, the Bernoulli equation describes as follows:

The cubic fluid particle experiences a pressure p behind and slightly different pressure p+dp in front. This causes acceleration according to Newton's second law is

$$F = ma = m\frac{dv}{dt} \tag{3.1}$$

where m is the mass and a is the acceleration. The resultant pressure force is F = - Adp, which is negative because it points in the –x direction. The mass of the fluid particle can be determined from its volume and fluid density ρ is

$$m = Al\rho \tag{3.2}$$

The magnitude of the pressure change between front and back can be determined from the pressure gradient in the stream wise direction dp/dx and the size of the particle is

$$dp = l\ \frac{dp}{dx} \tag{3.3}$$

Combining the above equations which gives $-Al\frac{dp}{dx} = Al\rho\frac{dv}{dt}$

That is $d\ = -\rho\frac{dv}{dt}dx$. Since $v = \frac{dx}{dt}$, this can be rewritten as $dp = -\rho v\ dv$

Integrate between any two points, taken as 1 and 2 along the streamline to relate the pressure difference between these points to the velocity difference is

$$\int_1^2 dp\ = -\int_1^2 \rho v\ dv$$

$$p_2 - p_1 = -\left(\frac{\rho}{2}v_2^2 - \frac{\rho}{2}v_1^2\right)$$

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 \tag{3.4}$$

Points 1 and 2 are arbitrary locations along the streamline. Equation (3.4) is used to connect any two locations along a streamline. This is essence in Bernoulli equation.

Note: The streamline need not be straight.

4. BERNOULLI EQUATION: PRESSURE GRADIENT ACROSS CURVED STREAMLINES

Consider a Particle moving along a curved streamline. Assume that the particle's speed is constant.

Define pressure inside and outside is $p_i = p$ and $p_o = p+dp$ and the centripetal force is $F = \frac{m}{R}v^2$.

Similar to before, we have $m = \rho Ah$ and $dp = h\frac{dp}{dn}$

where n is the coordinate in the direction normal to the streamline. Combining the above equations which yields
$F = A\ dp = Ah\frac{dp}{dn} = \rho Ahv^2\frac{1}{R}$ which simplifies to $\frac{dp}{dn} = \rho v^2\frac{1}{R}$

This equation represents pressure gradient across streamlines in terms of the local radius of curvature R and the flow velocity v.

If the streamline is straight, $R\rightarrow\infty$ and $\frac{dp}{dn} = 0$.

Thus, there is no pressure gradient across the straight streamlines.

CONCLUSION

Bernoulli equation applies only along a streamline. There is no explicit relationship between pressure and velocity of neighboring streamlines. All streamlines in a flow originate from a region where there uniform velocity and pressure. In such cases it is possible to apply Bernoulli equation.

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