PRIME CORDIAL AND SIGNED PRODUCT CORDIAL LABELING FOR THE EXTENDED DUPLICATE GRAPH OF KITE GRAPH

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ABSTRACT

In this paper, we prove that the extended duplicate graph of kite graph is Prime cordial and signed product cordial labeling.

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Key words: Graph labeling, prime cordial labeling, signed product cordial labeling, Kite graph.

1. INTRODUCTION

Graph theory is well known subject in mathematics and computer science. Graph theory is now a major tool in mathematical research, marketing and so on. The concept of graph labeling was introduced by Rosa [2] in 1967. Graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. After the introduction of graph labeling, various labeling of graphs such as graceful labeling, cordial labeling, prime cordial labeling, magic labeling, anti magic labeling etc., have been studied in over 2100 papers [1]. The concept of signed product cordial labeling was introduced by J. Baskar Babujee and he proved that many graphs admit signed product cordial labeling [5]. Sundaram, Ponraj and Somasundaram have introduced the notion of prime cordial [4]. The concept of duplicate graph was introduced by E. Sampath kumar and he proved many results [3]. K. Thirusangu, P. P. Ulaganathan and B. Selvam have proved that the duplicate graph of a path graph $P_m$ is cordial [6]. K. Thirusangu, B. Selvam and P. P. Ulaganathan have proved that the extended duplicate graph of twig graphs is cordial and total cordial [7].

2. PRELIMINARIES

In this section, we give the basic definitions relevant to this paper. Let $G = (V, E)$ be a finite, simple and undirected graph with $p$ vertices and $q$ edges.

Definition 2.1 Kite Graph: The kite graph is obtained by attaching a path of length ‘$m$’ with a cycle of length ‘$n$’ and it is denoted as $K_{n,m}$. It has $m+3$ vertices and $m+3$ edges. Kite graphs is also known as the Dragon Graphs or Canoe Paddle Graphs.

Illustration: 1

![KITE GRAPH](image_url)

Fig. 1: $K_{3,5}$
Definition 2.2 Duplicate Graph: Let $G (V, E)$ be a simple graph and the duplicate graph of $G$ is $DG = (V_1, E_1)$, where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ and $f: V \rightarrow V'$ is bijective (for $v \in V$, we write $f(v) = v'$ for convenience) and the edge set $E_1$ of $DG$ is defined as the edge $ab$ is in $E$ if and only if both $ab'$ and $a'b$ are edges in $E_1$.

Definition 2.3 Extended duplicate graph of Kite graph: Let $DG = (V_1, E_1)$ be a duplicate graph of the kite graph $G(V, E)$. Extended duplicate graph of kite graph is obtained by adding the edge $v_2v'_2$ to the duplicate graph. It is denoted by $EDG (K_3,m, m \geq 1)$. Clearly it has $2m+6$ vertices and $2m+7$ edges.

Illustration: 2 EXTENDED DUPLICATE GRAPH OF KITE GRAPH

Fig. 2: $EDG (K_3,5)$

Definition 2.4 Prime Cordial labeling: A function $f: V \rightarrow \{1, 2, ..., |V|\}$ such that each edge $v_iv_j$ is assigned the label ‘1’ if $\gcd(f(v_i), f(v_j)) = 1$ and ‘0’ if $\gcd(f(v_i), f(v_j)) > 1$ is said to be Prime cordial labeling, if the number of edges labeled ‘0’ and the number of edges labeled ‘1’ differ by at most one.

Definition 2.5 Signed product cordial labeling: A vertex labeling of graph $G f: V(G) \rightarrow \{-1, 1\}$ with induced edge labeling $f^*: E(G) \rightarrow \{-1, 1\}$ defined by $f^*(uv) = f(u)f(v)$ is called a signed product cordial labeling if $|u_f(-1) - u_f(1)| \leq 1$ and $|e_f(-1) - e_f(1)| \leq 1$, where $u_f(-1)$ is the number of vertices labeled with $-1$, $v_f(1)$ is the number of vertices labeled with $1$, $e_f(-1)$ is the number of edges labeled with $-1$ and $e_f(1)$ is the number of edges labeled with $1$.

3. MAIN RESULTS

3.1 Prime Cordial labeling

In this section, we present an algorithm and prove the existence of Prime cordial labeling for the extended duplicate graph of kite graph $K_3,m, m \geq 1$.

Algorithm: 3.1: procedure [Prime Cordial labeling for $EDG (K_3,m, m \geq 1)$]

$V \leftarrow \{v_1, v_2, ..., v_{m+3}, v'_1, v'_2, ..., v'_{m+3}\}$
$E \leftarrow \{e_1, e_2, ..., e_{m+4}, e'_1, e'_2, ..., e'_{m+4}\}$
$v_1 \leftarrow 1; v_3 \leftarrow 6; v_5 \leftarrow 5
v'_3 \leftarrow 3; v'_5 \leftarrow 4; v'_5 \leftarrow 2
$

if $0 < m < 3$

for $i = 1$ to $m$
do
$v_{3+i} \leftarrow 7+i
v'_{3+i} \leftarrow 4+3i$
end for

if $m = 3$

$v_m \leftarrow 11; v'_m \leftarrow 12
$

for $i = 1$ to $2$
do
$v_{3+i} \leftarrow 7+i
v'_{3+i} \leftarrow 4+3i$
end for
Theorem 3.1: The extended duplicate graph of kite graph \( K_{3,m} \), \( m \geq 1 \) is prime cordial.

Proof: Let \( K_{3,m} \), \( m \geq 1 \) be a kite graph. In order to label the vertices, define a function \( f: V \rightarrow \{1, 2 \ldots |V|\} \) as given in algorithm 3.1.

If \( m \geq 1 \), the vertices \( v_1, v_2, v_3, v'_1, v'_2 \) and \( v'_3 \) receive labels 1, 6, 5, 3, 4 and 2 respectively.

Case-(i): If \( 0 < m < 3 \); for \( 1 \leq i \leq m \), the vertices \( v_{3i+1} \) receive labels \( '7+i' \) and \( v'_{3i+1} \) receive labels \( '3i+4' \).

Case-(ii): If \( m = 3 \); for \( 1 \leq i \leq 2 \), the vertices \( v_{3i+1} \) receive labels \( '7+i' \) and the vertices \( v'_{3i+1} \) receive labels \( '3i+4' \); the vertex \( v_m \) receive label 11 and the vertex \( v'_m \) receive label 12.

Case-(iii): If \( m > 3 \); \( \frac{m+1}{3} \leq i \leq \frac{m+2}{3} \), the vertices \( v_{3i+1} \) receive labels \( 2+6i \) and the vertices \( v'_{3i+1} \) receive labels \( 1+6i \);
\( \frac{m+1}{3} \leq i \leq \frac{m+4}{3} \), the vertices \( v_{3i+2} \) receive labels \( 3+6i \) and the vertices \( v'_{3i+2} \) receive labels \( 4+6i \);
\( \frac{m+1}{3} \leq i \leq \frac{m+5}{3} \), the vertices \( v_{3i+3} \) receive labels \( 5+6i \) and the vertices \( v'_{3i+3} \) receive labels \( 6+6i \).

Thus in all the cases, the entire \( 2m+6 \) vertices are labeled by 1, 2 ...\( 2m+6 \).

The induced function \( f^*: E \rightarrow \{0, 1\} \) is defined as
\[
 f^*(v_i, v_j) = \begin{cases} 
 1 & \text{if } g.c.d \left( f(v_i), f(v_j) \right) = 1 \\
 0 & \text{if } g.c.d \left( f(v_i), f(v_j) \right) > 1 
\end{cases}
\]

Case-(i): If \( m = 3n-2 \); \( n \in \mathbb{N} \), the induced function yields the label \( '1' \) for the edge \( e_2 \); label \( '0' \) for the edges \( e'_1 \) and \( e_{m+4} \); for \( 1 \leq i \leq (m+2)/3 \), the edges \( e_{3i} \) and \( e'_{3i+1} \) receive label \( '0' \); for \( 1 \leq i \leq (m+2)/3 \) and \( 1 \leq j \leq 2 \), the edges \( e'_{3i+1} \) receive label \( '1' \); for \( 1 \leq i \leq (m+5)/3 \), the edges \( e_{3i-2} \) receive label \( '1' \); for \( 1 \leq i \leq (m-1)/3 \) and m >1 the edges \( e_{3i+2} \) receive label \( '0' \).

Thus all the \( 2m+7 \) edges namely the edges \( e_3, e_5, e_6, e_8, … , e_{m+2} \) and \( e_{m+4} \) receive label \( '0' \); the edges \( e'_1, e'_2, e'_3, e'_7, e'_{10}, … , e'_{m+3} \) receive label \( '0' \); the edges \( e_{1}, e_{2}, e_{4}, e_{7}, e_{10}, … , e_{m+3} \) receive label \( '1' \); the edges \( e_{2}, e_{3}, e_{5}, e_{6}, e_{8}, e_{9}, … , e'_{m+3} \) receive label \( '1' \) which differ by atmost one and satisfies the required condition.

Case-(ii): If \( m = 3n-1 \); \( n \in \mathbb{N} \), the induced function yields the label \( '1' \) for the edges \( e_2 \) and \( e'_{m+1} \); label \( '0' \) for the edges \( e_{m+4} \) and \( e'_{1} \); for \( 1 \leq i \leq (m+1)/3 \), the edges \( e_{3i} \), \( e_{3i+2} \) and \( e'_{3i+1} \) receive label \( '0' \); for \( 1 \leq i \leq (m+1)/3 \) and \( 1 \leq j \leq 2 \), the edges \( e'_3, e'_4, e'_7, e'_{10}, … , e'_{m+3} \) receive label \( '0' \); the edges \( e_{1}, e_{2}, e_{4}, e_{7}, e_{10}, … , e_{m+3} \) receive label \( '1' \); the edges \( e_{2}, e_{3}, e_{5}, e_{6}, e_{8}, e_{9}, … , e'_{m+1} \) receive label \( '1' \) which differ by atmost one and satisfies the required condition.
Case-(iii): If \( m = 3n; n \in \mathbb{N} \), the induced function yields the label ‘1’ for the edge \( e_2 \); label ‘0’ for the edges \( e_1 \) and \( e_{m+4} \); for \( 1 \leq i \leq (m/3) \), the edges \( e_{3i+2} \) and \( e'_{3i+1} \) receive label ‘0’; for \( 1 \leq i \leq (m+3)/3 \), the edges \( e_{3i} \) receive label ‘0’; the edges \( e'_{3i-2} \) and \( e'_{3i+j-2} \), \( 1 \leq j \leq 2 \) receive label ‘1’.

Thus all the \( 2m + 7 \) edges namely the edges \( e_3, e_5, e_6, e_8, e_9 \ldots \ldots, e_{m+2} \) receive label ‘0’; the edges \( e'_{1}, e'_{4}, e'_{7}, e'_{10}, \ldots, e'_{m+1} \) receive label ‘0’; the edges \( e_1, e_2, e_4, e_7, e_{10} \ldots \ldots, e_{m+1} \) receive label ‘1’; the edges \( e'_{2}, e'_{3}, e'_{5}, e'_{6}, e'_{8}, e'_{9}, \ldots, e'_{m+2} \) receive label ‘1’ which differ by atmost one and satisfies the required condition.

Hence the extended duplicate graph of kite graph \( K_{3,m} \), \( m \geq 1 \) is prime cordial labeling.

Illustration: 3 Prime Cordial labeling for the graph EDG \((K_{3,5})\)

![Diagram of EDG \((K_{3,5})\)](image)

3.2 Signed product Cordial labeling

In this section, we present an algorithm and prove the existence of signed product cordial labeling for the extended duplicate graph of kite graph \( K_{3,m} \), \( m \geq 1 \).

Algorithm: 3.2: procedure (signed product cordial labeling for EDG \((K_{3,m})\))

V \( \leftarrow \{v_1, v_2, \ldots, v_{m+3}, v'_1, v'_2, \ldots, v'_{m+3}\} \)

E \( \leftarrow \{e_1, e_2, \ldots, e_{m+4}, e'_1, e'_2, \ldots, e'_{m+3}\} \)

if \( m = 4n-3 \)

for \( i = 0 \) to \((m-1)/4\) do

for \( j = 0 \) to \(1\) do

\( v_{1+4i+j} \leftarrow -1 \)

\( v'_{3+4i+j} \leftarrow 1 \)

\( v'_{1+4i+j} \leftarrow 1 \)

\( v'_{3+4i+j} \leftarrow -1 \)

end for

end for

else

if \( m = 4n-2 \)

for \( i = 0 \) to \((m+2)/4\) do

\( v_{1+4i} \leftarrow -1 \)

\( v'_{1+4i} \leftarrow 1 \)

end for

for \( i = 0 \) to \((m-2)/4\) do

for \( j = 0 \) to \(1\) do

\( v_{2+4i+j} \leftarrow -1 \)

\( v'_{3+4i+j} \leftarrow 1 \)

\( v'_{2+4i+j} \leftarrow 1 \)

\( v'_{3+4i+j} \leftarrow -1 \)

end for

end for

else

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Theorem 3.2: The extended duplicate graph of kite graph $K_{3,m}$, $m \geq 1$ is Signed product cordial labeling.

Proof: Let $K_{3,m}$, $m \geq 1$ be a kite graph. In order to label the vertices, define a function $f: V \rightarrow \{-1, 1\}$ as given in Algorithm 3.2.

Case-(i): If $m = 4n-3$; $n \in N$, for $0 \leq i \leq (m-1)/4$ and $0 \leq j \leq 1$, the vertices $v_{1+4i+j}$ and $v'_{1+4i+j}$ receive label ‘-1’; the vertices $v_{3+4i+j}$ and $v'_{3+4i+j}$ receive label ‘1’. Hence all the $2m+6$ vertices namely the vertices $v_1, v_2, v_3, v_4, \ldots, v_{m-1}, v_m, v_{m+1}$ receive label ‘-1’; the vertices $v_3, v_7, v_8, \ldots, v_{m+2}, v_{m+3}$ receive label ‘1’; the vertices $v', v_2, v_5, v_6, \ldots, v_{m-2}, v_{m+2}$ receive label ‘-1’. Hence all the $2m+6$ vertices namely the vertices $v_1, v_2, v_3, v_4, \ldots, v_{m-1}, v_m, v_{m+1}$ receive label ‘1’. The extended duplicate graph of kite graph $K_{3,m}$, $m \geq 1$ is Signed product cordial labeling.

Case-(ii): If $m = 4n$, $n \in N$, for $0 \leq i \leq (m-2)/4$ and $0 \leq j \leq 1$, the vertices $v_{1+4i}$ and $v'_{1+4i}$ receive label ‘-1’; the vertices $v_{3+4i}$ and $v'_{3+4i}$ receive label ‘1’. Hence all the $2m+6$ vertices namely the vertices $v_1, v_2, v_3, v_4, \ldots, v_{m-1}, v_m, v_{m+1}$ receive label ‘-1’; the vertices $v_3, v_7, v_8, \ldots, v_{m+2}, v_{m+3}$ receive label ‘1’; the vertices $v', v_2, v_5, v_6, \ldots, v_{m-2}, v_{m+2}$ receive label ‘-1’. Hence all the $2m+6$ vertices namely the vertices $v_1, v_2, v_3, v_4, \ldots, v_{m-1}, v_m, v_{m+1}$ receive label ‘1’. The extended duplicate graph of kite graph $K_{3,m}$, $m \geq 1$ is Signed product cordial labeling.

Case-(iii): If $m = 4n+1$; $n \in N$, for $0 \leq i \leq (m+1)/4$ and $0 \leq j \leq 1$, the vertices $v_{1+4i+j}$ and $v'_{1+4i+j}$ receive label ‘-1’; the vertices $v_{3+4i+j}$ and $v'_{3+4i+j}$ receive label ‘1’. Hence all the $2m+6$ vertices namely the vertices $v_1, v_2, v_3, v_4, \ldots, v_{m-1}, v_m, v_{m+1}$ receive label ‘-1’; the vertices $v_3, v_7, v_8, \ldots, v_{m+2}, v_{m+3}$ receive label ‘1’; the vertices $v', v_2, v_5, v_6, \ldots, v_{m-2}, v_{m+2}$ receive label ‘-1’. Hence all the $2m+6$ vertices namely the vertices $v_1, v_2, v_3, v_4, \ldots, v_{m-1}, v_m, v_{m+1}$ receive label ‘1’. The extended duplicate graph of kite graph $K_{3,m}$, $m \geq 1$ is Signed product cordial labeling.

Case-(iv): If $m = 4n+2$; $n \in N$, for $0 \leq i \leq (m+2)/4$ and $0 \leq j \leq 1$, the vertices $v_{1+4i+j}$ and $v'_{1+4i+j}$ receive label ‘-1’; the vertices $v_{3+4i+j}$ and $v'_{3+4i+j}$ receive label ‘1’. Hence all the $2m+6$ vertices namely the vertices $v_1, v_2, v_3, v_4, \ldots, v_{m-1}, v_m, v_{m+1}$ receive label ‘-1’; the vertices $v_3, v_7, v_8, \ldots, v_{m+2}, v_{m+3}$ receive label ‘1’; the vertices $v', v_2, v_5, v_6, \ldots, v_{m-2}, v_{m+2}$ receive label ‘-1’. Hence all the $2m+6$ vertices namely the vertices $v_1, v_2, v_3, v_4, \ldots, v_{m-1}, v_m, v_{m+1}$ receive label ‘1’. The extended duplicate graph of kite graph $K_{3,m}$, $m \geq 1$ is Signed product cordial labeling.

Thus in all the cases, the entire $2m+6$ vertices are labeled in such a way that the number of vertices labeled -1 and 1 differ by at most one, which satisfies the required condition.

The induced function $f^* : E \rightarrow \{-1, 1\}$ is defined as

$$f^* (v_1, v_2) = f(v_1) x f(v_2) ; \quad v_1, v_2 \in V$$
The induced function yields the label ‘-1’ for the edges \( e_1, e'_1 \) and \( e_{m+4} \); label ‘1’ for the edges \( e_2 \) and \( e'_2 \); label ‘1’ for the edges \( e_{2i+1} \) and \( e'_{2i+1} \) if \( 1 \leq i \leq (m+1)/2 \), ‘m’ is odd; label ‘-1’ for the edges \( e_{2i+2} \) and \( e'_{2i+2} \) if \( 1 \leq i \leq (m+1)/2 \), ‘m’ is odd and if \( 1 \leq i \leq m/2 \), ‘m’ is even.

Thus the entire \( 2m+7 \) edges are labeled namely when ‘m’ is odd, \( m+3 \) edges \( e_2, e_3, e_5, e_7, e_9, …, e_{m+2}, e'_2, e'_3, e'_5, e'_7, e'_9, …, e'_{m+2} \) receive label ‘1’ and \( m+4 \) edges \( e_1, e_4, e_6, e_8, …, e_{m+3}, e'_1, e'_4, e'_6, e'_8, …, e'_{m+3} \) receive label ‘-1’ and when ‘m’ is even, \( m+4 \) edges \( e_2, e_3, e_5, e_7, e_9, …, e_{m+2}, e'_2, e'_3, e'_5, e'_7, e'_9, …, e'_{m+2} \) receive label ‘1’ and \( m+3 \) edges \( e_1, e_4, e_6, e_8, …, e_{m+2}, e'_1, e'_4, e'_6, e'_8, …, e'_{m+3} \) receive label ‘-1’ which differ by atmost one and satisfies the required condition.

Hence the extended duplicate graph of kite graph \( K_{3,m} \), \( m \geq 1 \) is signed product cordial labeling.

**Illustration:** 4 Signed product cordial labeling for the graphs \( EDG(K_{3,5}) \) and \( EDG(K_{3,6}) \)

4. **CONCLUSION**

In this paper, we presented algorithms and prove that the extended duplicate graph of kite graph \( K_{3,m} \), \( m \geq 1 \) is prime cordial and signed product cordial labeling.

5. **REFERENCES**


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