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# PRIME CORDIAL AND SIGNED PRODUCT CORDIAL LABELING FOR THE EXTENDED DUPLICATE GRAPH OF KITE GRAPH

**B. SELVAM*[1], K. ANITHA[2] AND K.THIRUSANGU[3]**

[1,3]Department of Mathematics,
S. I. V. E. T. College, Gowrivakkam, Chennai – 600 073, India.

[2]Department of Mathematics,
Sri Sairam Engineering College, Chennai – 600 044, India.

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***ABSTRACT***

*In this paper, we prove that the extended duplicate graph of kite graph is Prime cordial and signed product cordial labeling.*

***AMS Subject Classification:*** *05C78.*

***Key words:*** *Graph labeling, prime cordial labeling, signed product cordial labeling, Kite graph.*

## 1. INTRODUCTION

Graph theory is well known subject in mathematics and computer science. Graph theory is now a major tool in mathematical research, marketing and so on. The concept of graph labeling was introduced by Rosa [2] in 1967. Graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. After the introduction of graph labeling, various labeling of graphs such as graceful labeling, cordial labeling, prime cordial labeling. magic labeling, anti magic labeling etc., have been studied in over 2100 papers [1]. The concept of signed product cordial labeling was introduced by J.BaskarBabujee and he proved that many graphs admit signed product cordial labeling [5]. Sundaram, Ponraj and Somasundaram have introduced the notion of prime cordial [4]. The concept of duplicate graph was introduced by E.Sampath kumar and he proved many results [3]. K.Thirusangu, P.P. Ulaganathan and B. Selvam have proved that the duplicate graph of a path graph $P_m$ is cordial [6]. K.Thirusangu, B. Selvam and P.P. Ulaganathan have proved that the extended duplicate graph of twig graphs is cordial and total cordial [7].

## 2. PRELIMINARIES

In this section, we give the basic definitions relevant to this paper. Let G=G(V, E) be a finite, simple and undirected graph with p vertices and q edges.

**Definition 2.1 Kite Graph:** The kite graph is obtained by attaching a path of length 'm' with a cycle of length 'n' and it is denoted as $K_{n,m}$. It has m+3 vertices and m+3 edges. Kite graphs is also known as the Dragon Graphs or Canoe Paddle Graphs.

**Illustration: 1**

**KITE GRAPH**

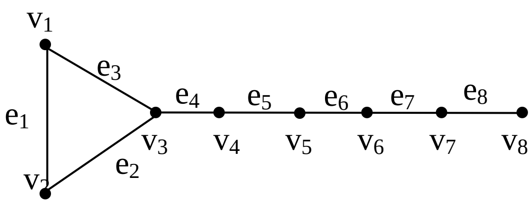


**Fig. 1: $K_{3,5}$**

*Corresponding Author: B. Selvam*[1]*
*[1]Department of Mathematics, S. I. V. E. T. College, Gowrivakkam, Chennai – 600 073, India.*

**Definition 2.2 Duplicate Graph:** Let G (V, E) be a simple graph and the duplicate graph of G is DG = ($V_1$, $E_1$), where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ and $f$: $V \to V'$ is bijective (for $v \in V$, we write $f(v) = v'$ for convenience) and the edge set $E_1$ of DG is defined as the edge $ab$ is in E if and only if both $ab'$ and $a'b$ are edges in $E_1$.

**Definition 2.3 Extended duplicate graph of Kite graph:** Let DG = ($V_1$, $E_1$) be a duplicate graph of the kite graph G(V,E). Extended duplicate graph of kite graph is obtained by adding the edge $v_2 v_2'$ to the duplicate graph. It is denoted by EDG ($K_{3,m}$, m ≥ 1). Clearly it has 2m+6 vertices and 2m+7 edges.

**Illustration: 2 EXTENDED DUPLICATE GRAPH OF KITE GRAPH**

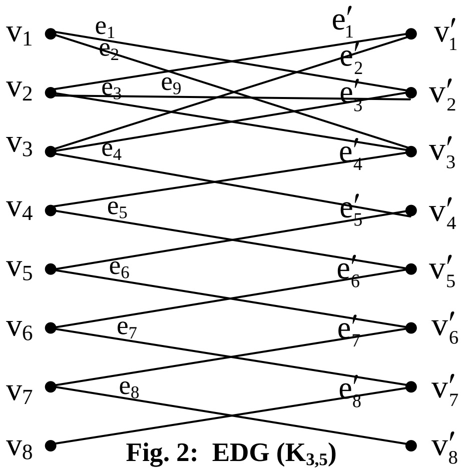


**Fig. 2: EDG ($K_{3,5}$)**

**Definition 2.4 Prime Cordial labeling:** A function $f$: V → {1, 2,.... | V|} such that each edge $v_iv_j$ is assigned the label '1'if g.c.d. (f($v_i$), f($v_j$)) = 1 and '0' if g.c.d. (f($v_i$), f($v_j$)) >1 is said to be Prime cordial labeling, if the number of edges labeled '0' and the number of edges labeled '1' differ by at most one.

**Definition 2.5 Signed product cordial labeling:** A vertex labeling of graph G $f: V(G) \to \{-1, 1\}$ with induced edge labeling $f^*: E(G) \to \{-1, 1\}$ defined by $f^*(uv) = f(u)f(v)$ is called a signed product cordial labeling if $|v_f(-1) - v_f(1)| \leq 1$ and $|e_f(-1) - e_f(1)| \leq 1$, where $v_f(-1)$ is the number of vertices labeled with -1, $v_f(1)$ is the number of vertices labeled with 1, $e_f(-1)$ is the number of edges labeled with $-1$ and $e_f(1)$ is the number of edges labeled with 1.

## 3. MAIN RESULTS

### 3.1 Prime Cordial labeling

In this section, we present an algorithm and prove the existence of Prime cordial labeling for the extended duplicate graph of kite graph $K_{3,m}$, m ≥ 1.

**Algorithm: 3.1: procedure [Prime Cordial labeling for EDG ($K_{3,m}$, m ≥ 1)]**

V ← {$v_1, v_2, \ldots, v_{m+3}, v_1', v_2', \ldots, v_{m+3}'$}
E ← {$e_1, e_2, \ldots, e_{m+4}, e_1', e_2', \ldots, e_{m+3}'$}
  $v_1$ ← 1; $v_2$ ← 6; $v_3$ ← 5
  $v_1'$ ← 3; $v_2'$ ← 4; $v_3'$ ← 2

if 0 < m < 3
  for i = 1 to m **do**
    $v_{3+i}$ ← 7+i
    $v_{3+i}'$ ← 4+3i
  end for
end if
if m = 3
  $v_m$ ← 11; $v_m'$ ← 12
  for i = 1 to 2 **do**
    $v_{3+i}$ ← 7+i
    $v_{3+i}'$ ← 4+3i

end for
end if
if m > 3
$for\ i = 1\ to\ \left\lfloor\frac{m+2}{3}\right\rfloor$ **do**
$v_{3i+1} \leftarrow 2+6i$
$v'_{3i+1} \leftarrow 1+6i$
end for
$for\ i = 1\ to\ \left\lfloor\frac{m+1}{3}\right\rfloor$ **do**
$v_{3i+2} \leftarrow 3+6i$
$v'_{3i+2} \leftarrow 4+6i$
end for
$for\ i = 1\ to\ \left\lfloor\frac{m}{3}\right\rfloor$ do
$v_{3i+3} \leftarrow 5+6i$
$v'_{3i+3} \leftarrow 6+6i$
end for
**end if**
**end procedure.**

**Theorem 3.1:** The extended duplicate graph of kite graph $K_{3,m}$, $m \geq 1$ is prime cordial.

**Proof:** Let $K_{3,m}$, $m \geq 1$ be a kite graph . In order to label the vertices, define a function *f:* $V \rightarrow \{1, 2 \ldots |V|\}$ as given in algorithm 3.1.

If **m ≥ 1**, the vertices $v_1, v_2, v_3, v'_1, v'_2$ and $v'_3$ receive labels 1, 6, 5, 3, 4 and 2 respectively.

**Case-(i): If 0 < m < 3;** for $1 \leq i \leq m$, the vertices $v_{3+i}$ receive labels '7+i' and $v'_{3+i}$ receive labels '3i+4'.

**Case-(ii): If m =3;** for $1 \leq i \leq 2$ , the vertices $v_{3+i}$ receive labels '7+i' and the vertices $v'_{3+i}$ receive labels '3i+4'; the vertex $v_m$ receive label 11 and the vertex $v'_m$ receive label 12.

**Case-(iii): If m >3;** $for\ 1 \leq i \leq \left\lfloor\frac{m+2}{3}\right\rfloor$, the vertices $v_{3i+1}$ receive labels 2+6i and the vertices $v'_{3i+1}$ receive labels 1+6i; $for\ 1 \leq i \leq \left\lfloor\frac{m+1}{3}\right\rfloor$, the vertices $v_{3i+2}$ receive labels 3+6i and the vertices $v'_{3i+2}$ receive labels 4+6 i; $for\ 1 \leq i \leq \left\lfloor\frac{m}{3}\right\rfloor$, the vertices $v_{3i+3}$ receive labels 5+6i and the vertices $v'_{3i+3}$ receive labels 6+6i.

Thus in all the cases, the entire 2m+6 vertices are labeled by 1, 2 ...2m+6.

The induced function $f^*: E \rightarrow \{0, 1\}$ is defined as

$$f^*(v_i v_j) = \begin{cases} 1 & if\ g.c.d\ \left(f(vi), f(vj)\right) = 1 \\ 0 & if\ g.c.d\ \left(f(vi), f(vj)\right) > 1 \end{cases}$$

**Case-(i):** If m = 3n–2; n ∈ N, the induced function yields the label '1' for the edge $e_2$**;** label '0' for the edges $e'_1$ and $e_{m+4}$**;** for $1 \leq i \leq (m+2)/3$, the edges $e_{3i}$ and $e'_{3i+1}$ receive label '0'; for $1 \leq i \leq (m+2)/3$ and $1 \leq j \leq 2$ , the edges $e'_{3i+j-2}$, receive label '1'; for $1 \leq i \leq (m+5)/3$, the edges $e_{3i-2}$ receive label '1'; for $1 \leq i \leq (m-1)/3$ and m >1 the edges $e_{3i+2}$ receive label '0'.

Thus all the 2m +7 edges namely the edges $e_3, e_5, e_6, e_8, e_9, \ldots, e_{m+2}$ and $e_{m+4}$ receive label '0' ; the edges $e'_1, e'_4, e'_7, e'_{10}, \ldots, e'_{m+3}$ receive label '0'; the edges $e_1, e_2, e_4, e_7, e_{10}, \ldots, e_{m+3}$ receive label '1'; the edges $e'_2, e'_3, e'_5, e'_6, e'_8, e'_9, \ldots, e'_{m+2}$ receive label '1' which differ by atmost one and satisfies the required condition.

**Case-(ii):** If m = 3n-1; n ∈ N, the induced function yields the label '1' for the edges $e_2$ and $e'_{m+1}$**;** label '0' for the edges $e_{m+4}$ and $e'_1$**;** for $1 \leq i \leq (m+1)/3$, the edges $e_{3i}$, $e_{3i+2}$ and $e'_{3i+1}$ receive label '0'**;** for $1 \leq i \leq (m+1)/3$ and $1 \leq j \leq 2$ ,the edges $e'_{3i+j-2}$, receive label '1'; for $1 \leq i \leq (m+4)/3$, the edges $e_{3i-2}$ receive label '1'.

Thus all the 2m +7 edges namely the edges $e_3, e_5, e_6, e_8, e_9 \ldots\ldots\ldots e_{m+1}$ $e_{m+3}$ and $e_{m+4}$ receive label '0'; the edges $e'_1, e'_4, e'_7, e'_{10}, \ldots, e'_{m+2}$ receive label '0'; the edges $e_1, e_2, e_4, e_7, e_{10} \ldots\ldots\ldots e_{m+2}$ receive label '1'; the edges $e'_2, e'_3, e'_5, e'_6, e'_8, e'_9, \ldots, e'_{m+1}$ $e'_{m+3}$ receive label '1' which differ by atmost one and satisfies the required condition.

**Case-(iii):** If m = 3n; n $\in$ N, the induced function yields the label '1' for the edge $e_2$**;** label '0' for the edges $e'_1$ and $e_{m+4}$**;** for $1 \le i \le (m)/3$ ,the edges $e_{3i+2}$ and $e'_{3i+1}$ receive label '0'; for $1 \le i \le (m+3)/3$, the edges $e_{3i}$ receive label '0'; the edges $e'_{3i-2}$ and $e'_{3i+j-2}$, $1 \le j \le 2$ receive label '1'.

Thus all the 2m +7 edges namely the edges $e_3, e_5, e_6, e_8, e_9$ ………. $e_{m+2}$ $e_{m+3}$ and $e_{m+4}$ receive label '0' ; the edges $e'_1, e'_4, e'_7, e'_{10}, \ldots, e'_{m+1}$ receive label '0'; the edges $e_1, e_2, e_4, e_7, e_{10}$ ………. $e_{m+1}$ receive label '1'; the edges $e'_2, e'_3, e'_5, e'_6, e'_8, e'_9, \ldots, e'_{m+2}$ $e'_{m+3}$ receive label '1' which differ by atmost one and satisfies the required condition.

Hence the extended duplicate graph of kite graph $K_{3,m}$, $m \ge 1$ is prime cordial labeling.

**Illustration: 3** Prime Cordial labeling for the graph **EDG ($K_{3,5}$)**

**PRIME CORDIAL LABELING FOR THE EXTENDED DUPLICATE GRAPH OF KITE GRAPH**

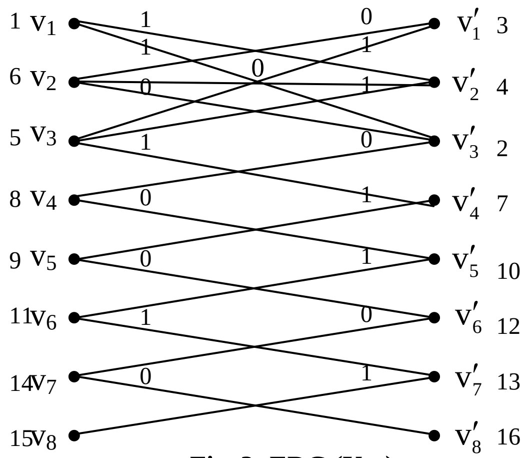


**Fig. 3: EDG ($K_{3,5}$)**

### 3.2 Signed product Cordial labeling

In this section, we present an algorithm and prove the existence of signed product cordial labeling for the extended duplicate graph of kite graph $K_{3,m}$, $m \ge 1$.

**Algorithm: 3.2: procedure (signed product cordial labeling for EDG ($K_{3,m}$, $m \ge 1$)**

$V \leftarrow \{v_1, v_2, \ldots, v_{m+3}, v'_1, v'_2, \ldots, v'_{m+3}\}$

$E \leftarrow \{e_1, e_2, \ldots, e_{m+4}, e'_1, e'_2, \ldots, e'_{m+3}\}$

```
if m = 4n-3
        for i = 0 to (m-1)/4 do
                j = 0 to 1 do
                     v_{1+4i+j}   ← -1
                     v_{3+4i+j}   ← 1
                     v'_{1+4i+j}  ← 1
                     v'_{3+4i+j}  ← -1
        end for
else
if m = 4n-2
        for i=0 to (m+2)/4 do
                v_{1+4i}   ← -1
                v'_{1+4i}  ← 1
        end for
        for i = 0 to (m-2)/4 do
                j = 0 to 1 do
                     v_{2+4i}     ← -1
                     v_{3+4i+j}   ← 1
                     v'_{2+4i}    ← 1
                        v'_{3+4i+j}  ← -1
        end for
else
```

if m = 4n-1
  for i = 0 to (m+1)/4 **do**
    j = 0 to 1 **do**
      $v_{1+4i+j} \leftarrow -1$
      $v'_{1+4i+j} \leftarrow 1$
  end for

  for i = 0 to (m-3)/4 **do**
    j = 0 to 1 **do**
      $v_{3+4i+j} \leftarrow 1$
      $v'_{3+4i+j} \leftarrow -1$
  end for
else
if m = 4n
  for i = 0 to m/4 **do**
    j = 0 to 1 **do**
      $v_{1+4i+j} \leftarrow -1$
      $v_{3+4i} \leftarrow 1$
      $v'_{1+4i+j} \leftarrow 1$
      $v'_{3+4i} \leftarrow -1$
  end for
  for i=0 to (m-4)/4 **do**
    $v_{4+4i} \leftarrow 1$
    $v'_{4+4i} \leftarrow -1$
  end for
**end if**
**end procedure**

**Theorem 3.2:** The extended duplicate graph of kite graph $K_{3,m}$, $m \geq 1$ is Signed product cordial labeling.

**Proof:** Let $K_{3,m}$, $m \geq 1$ be a kite graph. In order to label the vertices, define a function $f$: $V \rightarrow \{-1, 1\}$ as given in algorithm 3.2.

**Case-(i):** If $m = 4n-3$; $n \in N$, for $0 \leq i \leq (m-1)/4$ and $0 \leq j \leq 1$, the vertices $v_{1+4i+j}$ and $v'_{3+4i+j}$ receive label '-1'; the vertices $v_{3+4i+j}$ and $v'_{1+4i+j}$ receive label '1'. Hence all the 2m+6 vertices namely the vertices $v_1, v_2, v_5, v_6, \ldots, v_m, v_{m+1}$ receive label '-1'; the vertices $v_3, v_4, v_7, v_8, \ldots, v_{m+2}, v_{m+3}$ receive label '1'; the vertices $v'_1, v'_2, v'_5, v'_6, \ldots, v'_m, v'_{m+1}$ receive label 1; the vertices $v'_3, v'_4, v'_7, v'_8, \ldots, v'_{m+2}, v'_{m+3}$ receive label '-1'.

**Case-(ii):** If $m = 4n-2$ ; $n \in N$, for $0 \leq i \leq (m+2)/4$, the vertices $v_{1+4i}$ receive label '-1' and the vertices $v'_{1+4i}$ receive label '1'**;** for $0 \leq i \leq (m-2)/4$ and $0 \leq j \leq 1$, the vertices $v_{2+4i}$ and $v'_{3+4i+j}$ receive label '-1' ; the vertices $v_{3+4i+j}$ and $v'_{2+4i}$ receive label '1'. Hence all the 2m+6 vertices namely the vertices $v_1, v_2, v_5, v_6, v_9, \ldots, v_{m-1}, v_m, v_{m+3}$ receive label '-1'; the vertices $v_3, v_4, v_7, v_8, \ldots, v_{m+1}, v_{m+2}$ receive label '1'; the vertices $v'_1, v'_2, v'_5, v'_6, v'_9, \ldots, v'_{m-1}, v'_m, v'_{m+3}$ receive label 1; the vertices $v'_3, v'_4, v'_7, v'_8, \ldots, v'_{m+1}, v'_{m+2}$ receive label '-1'.

**Case-(iii):** If $m = 4n-1$ ; $n \in N$, for $0 \leq i \leq (m+1)/4$ and $0 \leq j \leq 1$, the vertices $v_{1+4i+j}$ receive label '-1' and the vertices $v'_{1+4i+j}$ receive label '1'**;** for $0 \leq i \leq (m-3)/4$ and $0 \leq j \leq 1$, the vertices $v_{3+4i+j}$ receive label '1' and the vertices $v'_{3+4i+j}$ receive label '-1'. Hence all the 2m+6 vertices namely the vertices $v_1, v_2, v_5, v_6, \ldots, v_{m+2}, v_{m+3}$ receive label '-1'; the vertices $v_3, v_4, v_7, v_8, \ldots, v_m, v_{m+1}$ receive label'1'; the vertices $v'_1, v'_2, v'_5, v'_6, \ldots, v'_{m+2}, v'_{m+3}$ receive label 1 ; the vertices $v'_3, v'_4, v'_7, v'_8, \ldots, v'_m, v'_{m+1}$ receive label '-1'.

**Case-(iv):** If $m = 4n$ ; $n \in N$, for $0 \leq i \leq m/4$ and $0 \leq j \leq 1$ , the vertices $v_{1+4i+j}$ and $v'_{3+4i}$ receive label '-1' ; the vertices $v_{3+4i}$ and $v'_{1+4i+j}$ receive label '1'. Hence all the 2m+6 vertices namely the vertices $v_1, v_2, v_5, v_6, \ldots, v_{m+1}, v_{m+2}$ receive label '-1'; the vertices $v_3, v_4, v_7, \ldots, v_{m-1}, v_m, v_{m+3}$ receive label '1'; the vertices $v'_1, v'_2, v'_5, v'_6, \ldots, v'_{m+1}, v'_{m+2}$ receive label 1; the vertices $v'_3, v'_4, v'_7, \ldots, v'_{m-1}, v'_m, v'_{m+3}$ receive label '-1'.

Thus in all the cases, the entire 2m+6 vertices are labeled in such a way that the number of vertices labeled -1 and 1 differ by at most one, which satisfies the required condition.

The induced function $f^*: E \rightarrow \{-1, 1\}$ is defined as

$$f^*(v_i v_j) = f(v_i) \times f(v_j); \quad v_i, v_j \in V$$

The induced function yields the label '-1' for the edges $e_1$, $e'_1$ and $e_{m+4}$; label '1' for the edges $e_2$ and $e'_2$; label '1' for the edges $e_{2i+1}$ and $e'_{2i+1}$ if $1 \le i \le (m+1)/2$, 'm' is odd and if $1 \le i \le (m+2)/2$, 'm' is even; label '-1' for the edges $e_{2i+2}$ and $e'_{2i+2}$ if $1 \le i \le (m+1)/2$, 'm' is odd and if $1 \le i \le m/2$, 'm' is even.

Thus the entire 2m+7 edges are labeled namely when 'm' is odd, m+3 edges $e_2, e_3, e_5, e_7, e_9, \ldots, e_{m+2}, e'_2, e'_3, e'_5, e'_7, e'_9, \ldots, e'_{m+2}$ receive label '1' and m+4 edges $e_1, e_4, e_6, e_8, \ldots, e_{m+3}, e'_1, e'_4, e'_6, e'_8, \ldots, e'_{m+3}$ and the edge $e_{m+4}$ receive label '-1' and when 'm' is even, m+4 edges $e_2, e_3, e_5, e_7, e_9, \ldots, e_{m+3}, e'_2, e'_3, e'_5, e'_7, e'_9, \ldots, e'_{m+3}$ receive label '1' and m+3 edges $e_1, e_4, e_6, e_8, \ldots, e_{m+2}, e'_1, e'_4, e'_6, e'_8, \ldots, e'_{m+2}$ and the edge $e_{m+4}$ receive label '-1' which differ by atmost one and satisfies the required condition.

Hence the extended duplicate graph of kite graph $K_{3,m}$, $m \ge 1$ is signed product cordial labeling.

**Illustration: 4** Signed product cordial labeling for the graphs **EDG ($K_{3,5}$)** and **EDG($K_{3,6}$)**

**SIGNED PRODUCT CORDIAL LABELING FOR THE EXTENDED DUPLICATE GRAPH OF KITE GRAPH**

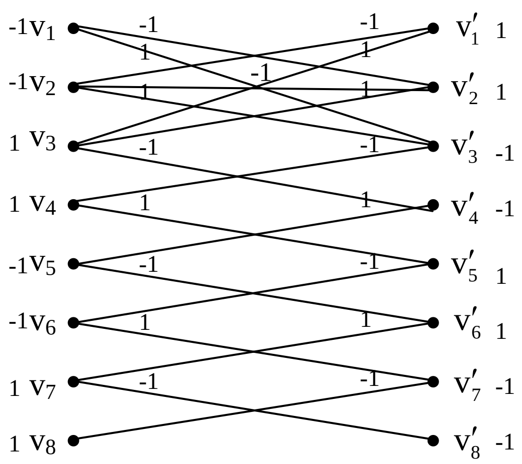


**Fig. 4: EDG ($K_{3,5}$)**

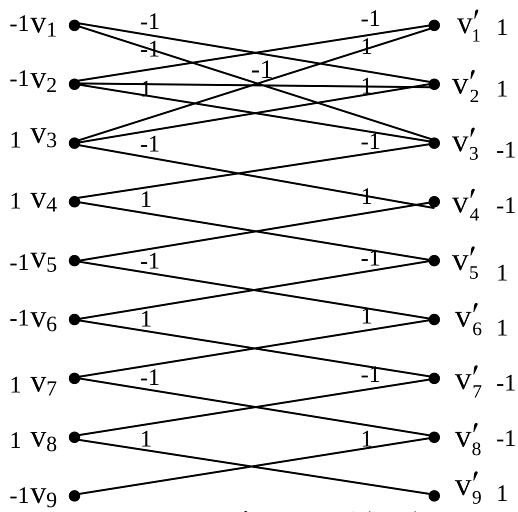


**Fig. 5: EDG($K_{3,6}$)**

## 4. CONCLUSION

In this paper, we presented algorithms and prove that the extended duplicate graph of kite graph $K_{3,m}$, $m \ge 1$ is prime cordial and signed product cordial labeling.

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