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FINITE ELEMENT SOLUTIONS OF MAGNETOHYDRODYNAMIC CHEMICALLY REACTING FREE CONVECTIVE FLUID FLOW PAST VERTICAL POROUS PLATE WITH THERMAL RADIATION AND VISCOUS DISSIPATION

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ABSTRACT

This is a theoretical investigation on the chemical reaction and thermal radiation effects on unsteady magnetohydrodynamic free convective on a viscous, incompressible, electrically conducting fluid flow past a vertical plate embedded in porous medium in presence of viscous dissipation, heat and mass transfer. The chemical reaction has been assumed to be homogeneous of first-order. The basic non-linear coupled partial differential equations governing the flow have been solved numerically using an efficient, flexible finite element method. Graphical results for velocity, temperature and concentration profiles have been obtained, to show the effects of different parameters entering in the problem. Such flow problems are important in many processes, in which there is combined heat and mass transfer with chemical reaction, such as drying, evaporation at the surface of water body etc.

Key words: *Thermal radiation; MHD; Chemical Reaction; Viscous dissipation; Finite element method;*

NOMENCLATURE

List of Variables:

A	A constant
B_o	Magnetic field component along $y'-$ axis, *Tesla*
C_p	Specific heat at constant pressure, *J/kg-K*
Gr	Grashof number
Gc	Modified Grashof number
g	Acceleration of gravity, 9.81 m/s^2
K'	The permeability of medium
K	The permeability parameter
M	Hartmann number
Pr	Prandtl number
Sc	Schmidt number
N	Thermal radiation parameter
Ec	Eckert number
D	Chemical molecular diffusivity, m^2/s
T'	Temperature of fluid near the plate, *K*
T'_w	Temperature of the fluid far away of the fluid from the plate, *K*
T'_∞	Temperature of the fluid at infinity, *K*
C'	Concentration of fluid near the plate, Kg/m^3
C'_w	Concentration of the fluid far away from the plate, Kg/m^3
C'_∞	Concentration of the fluid at infinity, Kg/m^3
t'	Time in x', y' coordinate system, *Seconds*
t	Time in dimensionless co-ordinates, *Sec*
u'	Velocity component in $x'-$ direction, *m/s*
u	Dimensionless velocity component in $x'-$ direction, *m/s*
Nu	Nusselt number
Sh	Sherwood number
q_r	Radiative heat flux
R_{e_x}	Reynold's number
k_1	Mean absorption coefficient
$\overline{K}$	Chemical reaction of first order with rate constant
x', y'	Co-ordinate system, *m*
x, y	Dimensionless coordinates, *m*
U_o	Reference velocity, *m/s*

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Greek symbols:

β	Coefficient of volume expansion for heat transfer, K^{-1}
β^*	Coefficient of volume expansion for mass transfer, m^3/Kg
κ	Thermal conductivity of the fluid
σ	Electrical conductivity of the fluid, *mho/m*
ν	Kinematic viscosity, m^2/s

θ	Non-dimensional temperature, *K*
ϕ	Concentration of the fluid, *Kg/m^3*
ρ	Density of the fluid, *kg/m^3*
τ	Skin-friction
μ	Viscosity, *Ns/m^2*
λ	Chemical reaction parameter

Superscripts:

$/$	Dimensionless Properties

Subscripts:

p	Plate
w	Wall condition
∞	Free stream condition

1. INTRODUCTION

In several problems related to demanding of efficient transfer of mass over inclined beds related to geophysical, petroleum, chemical, bio-mechanical, chemical technology and in situations the viscous drainage over an inclined porous plane is a subject of considerable interest to both theoretical and experimental investigators. Especially, in the flow of oil through porous rock, the extraction of geo-thermal energy from the deep interior of the earth to the shallow layers, the evaluation of the capability of heat removal from particulate nuclear fuel debris that may result from hypothetical accident in a nuclear reactor, the filtration of solids from liquids, flow of liquids through ion exchange beds, drug permeation through human glands, chemical reactor for economical separation or purification of mixtures flow through porous medium has been the subject of considerable research activity in recent years due to its notable applications. An important application in the petroleum industry where crude oil is trapped from natural underground reservoirs in which oil is entrapped since the flow behavior of fluids in petroleum reservoir rock depends to a large extent on the properties of the rock, techniques that yield new or additional information on the characteristics of the rock would enhance the performance of petroleum reservoirs. An important bio-medical application is the flow of fluids in lungs, blood vessels, arteries and so on, where the fluid is bounded by two layers which are held together by a set of fairly regularly spaced tissues. Slurry adheres to the reactor vessel and gets consolidated in many chemical processing industries, as a result of which chemical compounds within the reactor vessels percolates through the boundaries. Thus adhered substance within the reactor vessel acts as a porous boundary. The problem assumes greater importance in all such situations. The thin film adhering to the surfaces of the container must be taken into account for the purpose of precise chemical calculations in all such situations wherein hear and mass transfer occurs. Failure to do so leads to severe experimental errors. Hence, there is a need for such an analysis in detail. A mathematical model related to such a situation has been studied in detail. Anand Rao *et al.* ([1]-[11]) discussed Soret, Dufour, Hall current on an unsteady magnetohydrodynamic free convective fluid flow past a vertical plate filled in porous medium in presence of heat and mass transfer using finite element method. Dharmendar Reddy *et al.* ([12] and [13]) studied the chemical reaction, heat and mass transfer effects on an unsteady MHD natural convective fluid flow past an infinite vertical porous plate in presence of Hall current. Srinivasa Raju *et al.* ([14]-[31]) and his co-workers found the numerical solutions of unsteady magnetohydrodynamic free convective flow past an infinite vertical porous plate in presence of Soret, Dufour, thermal radiation, Chemical reaction, Hall current, heat and mass transfer using finite element technique. Jithender Reddy *et al.* ([32]-[39]) studied the simultaneous effects of viscous dissipation, thermal diffusion, diffusion thermo, chemical reaction on an unsteady magnetohydrodynamic free convective fluid flow past an infinite vertical plate filled in porous medium in presence of heat and mass transfer using numerical solutions by an efficient finite element method. Sudhakar *et al.* ([40]-[42]) studied the joint effects of thermal diffusion and diffusion thermo on an unsteady MHD natural convective fluid flow along a porous plate in presence of viscous dissipation, heat and mass transfer using finite element technique. Ramya *et al.* ([43]-[45]) found the numerical solutions of magnetohydrodynamic free convective nanofluid flow over a nonlinearly-stretching sheet in presence of thermal radiation, heat absorption/generation and thermal wall slips using Keller-box method. Aruna *et al.* [46] studied the combined influence of Soret and Dufour effects on unsteady hydromagnetic mixed convective flow in an accelerated vertical wavy plate through a porous medium. Unsteady MHD free convection flow near on an infinite vertical plate embedded in a porous medium with chemical reaction, Hall current and thermal radiation studied by Sarada *et al.* [47].

Hence, motivate by above reference work, the aim of the present work is to investigate the effect of thermal radiation on MHD free convection flow past an impulsively started semi-infinite porous plate with variable temperature in the presence of chemical reaction and viscous dissipation by finite element method which is more economical from computational view point. It is also assumed that temperature of the plate and concentration near the plate varies linearly with time.

2. MATHEMATICAL FORMULATION

Consider a two-dimensional unsteady MHD free convection flow of a viscous, incompressible, electrically conducting fluid past a semi-infinite tilted porous plate with chemical reaction and thermal radiation. In Cartesian coordinate system, let $x'-$axis is taken to be along the plate and the $y'-$axis normal to the plate. Since the plate is considered infinite in $x'-$direction, hence all physical quantities will be independent of $x'-$direction. The wall is maintained at constant temperature (T'_w) and concentration (C'_w) higher than the ambient temperature (T'_∞) and concentration

(C'_∞) respectively. A uniform magnetic field of magnitude B_o is applied normal to the plate. The transverse applied magnetic field and magnetic Reynold's number are assumed to be very small, so that the induced magnetic field is negligible. The homogeneous chemical reaction of first order with rate constant $\overline{K}$ between the diffusing species and the fluid is assumed. It is assumed that there is no applied voltage which implies the absence of an electric field. The fluid has constant kinematic viscosity and constant thermal conductivity, and the Boussinesq's approximation have been adopted for the flow. The fluid is considered to be gray absorbing-emitting radiation but non-scattering medium and the Roseland's approximation is used to describe the radiative heat flux. It is considered to be negligible in $x'-$ direction as compared in $y'-$ direction. At time $t' > 0$ the plate is given an impulsive motion in the direction of flow i.e. along $x'-$axis against the gravity with constant velocity U_o, it is assumed that the plate temperature and concentration at the plate are varying linearly with time. The concentration of the diffusing species in the binary mixture is assumed to be very small in comparison with the other chemical species, which are present and hence Soret and Dufour effects are negligible. Under these assumptions the equations governing the flow are:

Momentum Equation:

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} - \left(\frac{\sigma B_o^2}{\rho} + \frac{\nu}{K'}\right)u' \tag{1}$$

Energy Equation:

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} + \nu\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{2}$$

Species Diffusion Equation:

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K_r(C' - C'_\infty) \tag{3}$$

With the following initial and boundary conditions:

$$\left.\begin{array}{l} t' \le 0: u' = 0,\ T' = T'_\infty,\ C' = C'_\infty \quad for \quad all \quad y' \\ t' > 0: \begin{cases} u' = U_o,\ T' = T'_\infty + (T'_w - T'_\infty)At',\ C' = C'_\infty + (C'_w - C'_\infty)At',\ at\ \ y' = 0 \\ u' \to 0,\ T' \to 0,\ C' \to 0 \quad as \quad y' \to \infty \end{cases} \end{array}\right\} \tag{4}$$

Where $A = \dfrac{U_o^2}{\nu}$. The radiative heat flux q_r, under Rosseland approximation has the form

$$q_r = -\frac{4\sigma_1}{3\kappa}\frac{\partial^2 T'^4}{\partial y'} \tag{5}$$

It is assumed that, the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature T'_∞. This is obtained by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher order terms. Thus, we get

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \tag{6}$$

By using equations (5) and (6), equation (2) reduces to

$$\frac{\partial T'}{\partial t'} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{16\sigma_1 T'^3_\infty}{3k_1\rho C_p}\frac{\partial^2 T'}{\partial y'^2} \tag{7}$$

Introducing the following non-dimensional parameters in equations (1), (2), (3) and (7) quantities:

$$\left.\begin{array}{l} y = \dfrac{y'U_o}{\nu},\ t = \dfrac{t'U_o}{\nu},\ u = \dfrac{u'}{U_o},\ \theta = \dfrac{T' - T'_\infty}{T'_w - T'_\infty},\ \varphi = \dfrac{C' - C'_\infty}{C'_w - C'_\infty},\ Gr = \dfrac{g\beta\nu(T'_w - T'_\infty)}{U_o^3},\ Gc = \dfrac{g\beta^*\nu(C'_w - C'_\infty)}{U_o^3}, \\ \Pr = \dfrac{\mu C_p}{\kappa},\ M = \left(\dfrac{\sigma B_o^2}{\rho}\right)\dfrac{\nu}{U_o^2}, \\ Ec = \dfrac{U_o^2}{C_p(T'_w - T'_\infty)},\ Sc = \dfrac{\nu}{D},\ K = \dfrac{K'U_o^2}{\nu^2},\ N = \dfrac{\kappa U_o^2}{4\nu^2\sigma_1 T'^3_\infty},\ \lambda = \dfrac{K_r\nu}{U_o^2} \end{array}\right\} \tag{8}$$

In equations (1), (2), (3) and (7) reduces to

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}-(M+\frac{1}{K})u+(Gr)\theta+(Gc)\phi \tag{9}$$

$$(\Pr)\frac{\partial \theta}{\partial t}=\left(1+\frac{4}{3N}\right)\frac{\partial^2 \theta}{\partial y^2}+(\Pr)(Ec)\left(\frac{\partial u}{\partial y}\right)^2 \tag{10}$$

$$\frac{\partial \phi}{\partial t}=\frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2}-\lambda\phi \tag{11}$$

The corresponding initial and boundary conditions in dimensionless form are:

$$\left.\begin{array}{l} t\le 0:\ u=0,\ \theta=0,\ \phi=0\ \ for\ \ all\ \ y \\ t>0:\begin{cases} u=1,\ \theta=t,\ \phi=t\ \ at\ \ y=0 \\ u\to 0,\ \theta\to 0,\ \phi\to 0\ \ as\ \ y\to\infty \end{cases} \end{array}\right\} \tag{12}$$

All the physical parameters are defined in the nomenclature. It is now important to calculate the physical quantities of primary interest, which are the local wall shear stress, the local surface heat and mass flux. Given the velocity field in the boundary layer, we can now calculate the local wall shear stress (i.e., skin-friction) is given by and in dimensionless form, we obtain Knowing the temperature field, it is interesting to study the effect of the free convection and radiation on the rate of heat transfer.

This is given by which is written in dimensionless form as

$$\tau=\frac{\tau_w}{\rho u_w^2},\ \ \tau_w=\left[\mu\frac{\partial u}{\partial y}\right]_{y'=0}=\rho U_o^2 u'(0)=\left[\frac{\partial u}{\partial y}\right]_{y=0} \tag{13}$$

The dimensionless local surface heat flux (i.e., Nusselt number) is obtained as

$$N_u(x')=-\left[\frac{x'}{(T_w'-T_\infty')}\frac{\partial T'}{\partial y'}\right]_{y'=0} \quad \text{then}$$

$$Nu=\frac{N_u(x')}{R_{e_x}}=-\left[\frac{\partial \theta}{\partial y}\right]_{y=0} \tag{14}$$

The definition of the local mass flux and the local Sherwood number are respectively given by with the help of these equations, one can write

$$S_h(x')=-\left[\frac{x'}{(C_w'-C_\infty')}\frac{\partial C'}{\partial y'}\right]_{y'=0} \quad \text{then} \quad Sh=\frac{S_h(x')}{R_{e_x}}=-\left[\frac{\partial \phi}{\partial y}\right]_{y=0} \tag{15}$$

Where $R_{e_x}=-\frac{U_o x'}{\nu}$ is the Reynold's number.

3. NUMERICAL SOLUTIONS BY FINITE ELEMENT TECHNIQUE

Finite Element Technique: The finite element procedure (FEM) is a numerical and computer based method of solving a collection of practical engineering problems that happen in different fields such as, in heat transfer, fluid mechanics ([48]-[54]) and many other fields. It is recognized by developers and consumers as one of the most influential numerical analysis tools ever devised to analyze complex problems of engineering. The superiority of the method, its accuracy, simplicity, and computability all make it a widely used apparatus in the engineering modeling and design process. It has been applied to a number of substantial mathematical models, whose differential equations are solved by converting them into a matrix equation. The primary feature of FEM ([55] and [56]) is its ability to describe the geometry or the media of the problem being analyzed with huge flexibility. This is because the discretization of the region of the problem is performed using highly flexible uniform or non uniform pieces or elements that can easily describe complex shapes. The method essentially consists in assuming the piecewise continuous function for the results and getting the parameters of the functions in a manner that reduces the fault in the solution. The steps occupied in the finite element analysis areas follows.

Step 1: Discretization of the Domain: The fundamental concept of the FEM is to divide the region of the problem into small connected pieces, called finite elements. The group of elements is called the finite element mesh. These finite elements are associated in a non overlapping manner, such that they completely cover the entire space of the problem.

Step 2: Invention of the Element Equations:

i) A representative element is secluded from the mesh and the variational formulation of the given problem is created over the typical element.

ii) Over an element, an approximate solution of the variational problem is invented, and by surrogating this in the system, the element equations are generated.

iii) The element matrix, which is also known as stiffness matrix, is erected by using the element interpolation functions.

Step 3: Assembly of the Element Equations: The algebraic equations so achieved are assembled by imposing the inter element continuity conditions. This yields a large number of mathematical equations known as the global finite element model, which governs the whole domain.

Step 4: Imposition of the Boundary Conditions: On the accumulated equations, the Dirichlet's and Neumann boundary conditions (12) are imposed.

Step 5: Solution of Assembled Equations: The assembled equations so obtained can be solved by any of the numerical methods, namely, Gauss elimination technique, LU decomposition technique, and the final matrix equation can be solved by iterative technique. For computational purposes, the coordinate y varies from 0 to 10, where y_{max} represents infinity external to the momentum, energy and concentration edge layers.

In one-dimensional space, linear and quadratic elements, or element of higher order can be taken. The entire flow province is divided into 11000 quadratic elements of equal size. Each element is three-noded, and therefore the whole domain contains 21001 nodes. At each node, three functions are to be evaluated; hence, after assembly of the element equations, we acquire a system of 81004 equations which are non-linear. Therefore, an iterative scheme must be developed in the solution. After striking the boundary conditions, a system of equations has been obtained which is solved mathematically by the Gauss elimination method while maintaining a correctness of 0.00001. A convergence criterion based on the relative difference between the present and preceding iterations is employed. When these differences satisfy the desired correctness, the solution is assumed to have been congregated and iterative process is terminated. The Gaussian quadrature is applied for solving the integrations. The computer cryptogram of the algorithm has been performed in MATLAB running on a PC. Excellent convergence was completed for all the results.

4, RESULTS AND DISCUSSION

In order to get a physical insight in to the problem the effects of various governing parameters on the physical quantities are computed and represented in figures (1)-(16) and discussed in detail. Figures (1) and (2) exhibit the effect of Grashof number and Modified Grashof numbers on the velocity profile with other parameters are fixed. The Grashof number signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Also, as *Gr* increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity. The Modified Grashof number defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value. It is noticed that the velocity increases with increasing values of the Modified Grashof number.

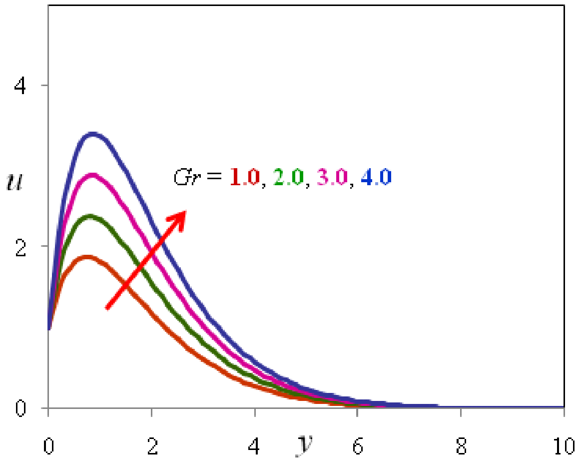

Figure 1. Effect of *Gr* on Velocity profiles

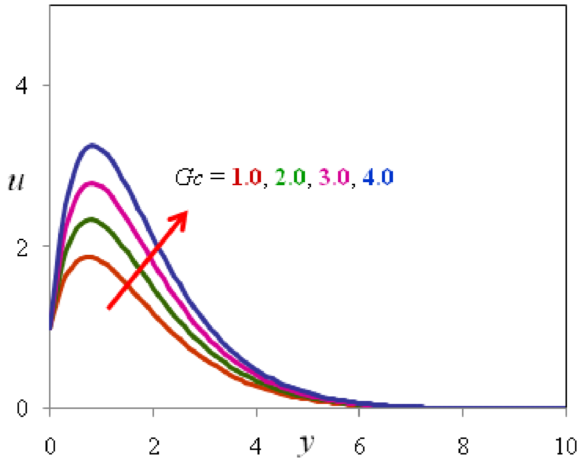

Figure 2. Effect of *Gc* on Velocity profiles

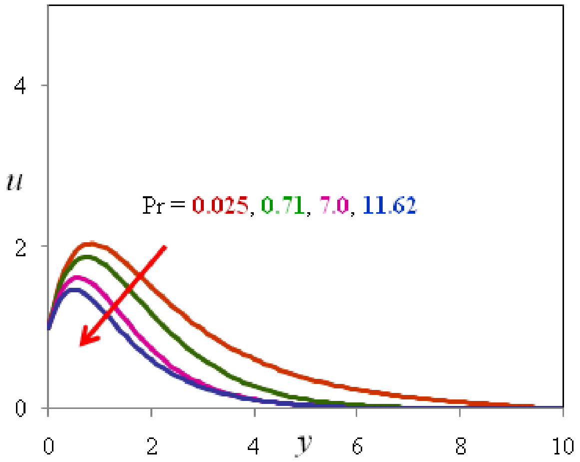

Figure 3. Effect of Pr on Velocity profiles

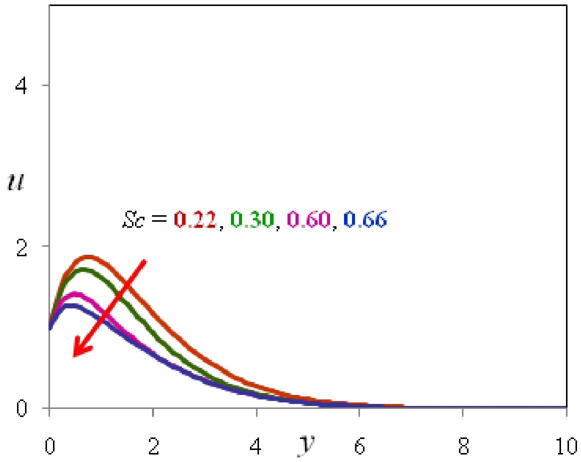

Figure 4. Effect of *Sc* on Velocity profiles

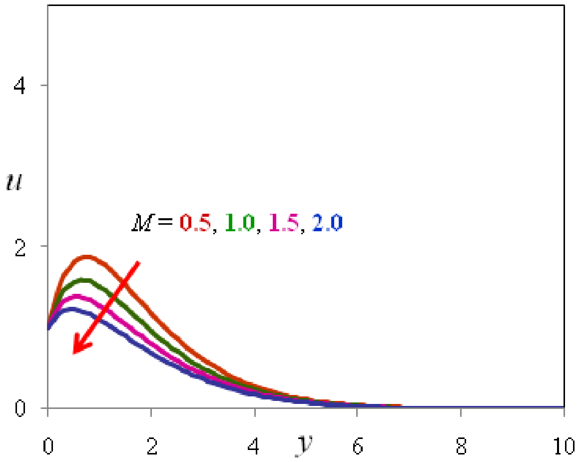

Figure 5. Effect of *M* on Velocity profiles

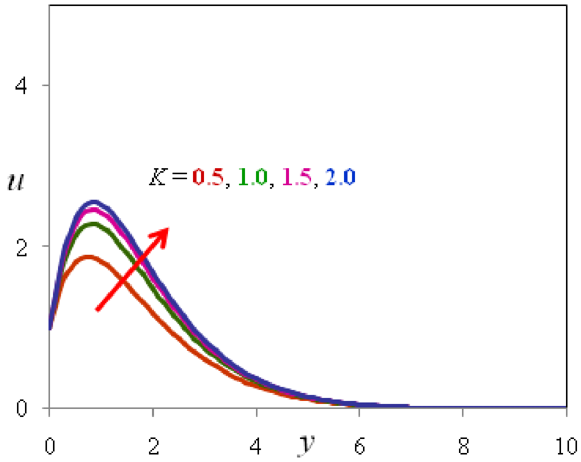

Figure 6. Effect of *K* on Velocity profiles

Figure (3) depicts the effect of Prandtl number on velocity profiles in presence of foreign species such as Mercury ($\Pr$ = 0.025), Air ($\Pr$ = 0.71), Water ($\Pr$ = 7.00) and Water at $4^{o}C$ ($\Pr$ = 11.62) are shown in figure (3). It is observed that from figure (3), the velocity decreases with increasing of Prandtl number $(\Pr)$. The nature of velocity profiles in presence of foreign species such as Hydrogen (Sc = 0.22), Helium (Sc = 0.30), Oxygen (Sc = 0.60) and Water-vapour (Sc = 0.66) are shown in figure (4). The flow field suffers a decrease in velocity at all points in presence of heavier diffusing species. The effect of the Hartmann number is shown in figure (5). It is observed that the velocity of the fluid decreases with the increase of the magnetic field number values. The decrease in the velocity as the Hartmann number increases is because the presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction, as in the present study. This resistive force slows down the fluid velocity component as shown in figure (5). The effect of Permeability parameter is presented in the figure (6). From this figure it observe that, the velocity is increases with increasing values of *K*.

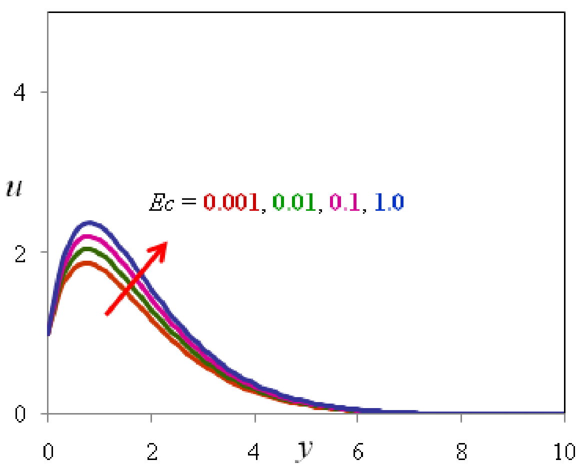

Figure 7. Effect of *Ec* on Velocity profiles

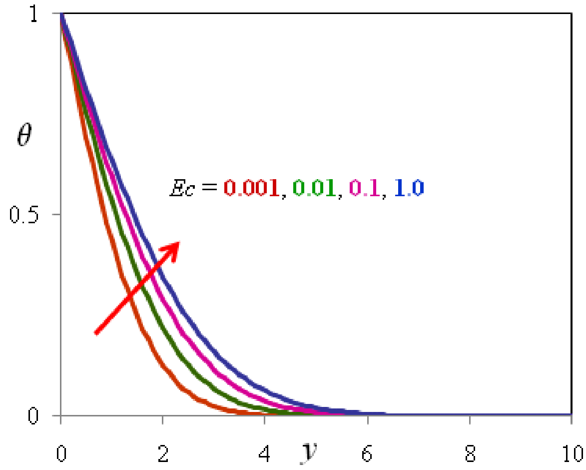

Figure 8. Effect of *Ec* on Temperature profiles

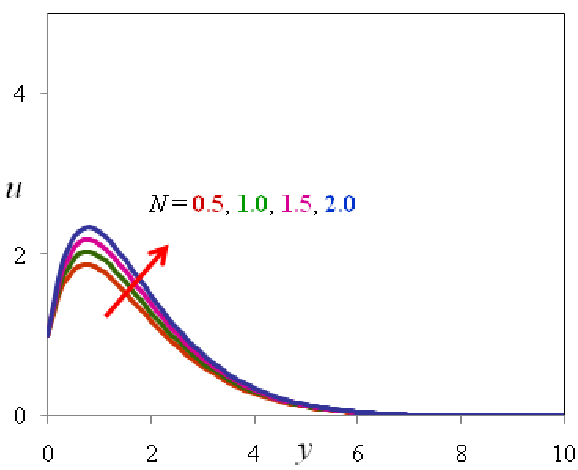

Figure 9. Effect of *N* on Velocity profiles

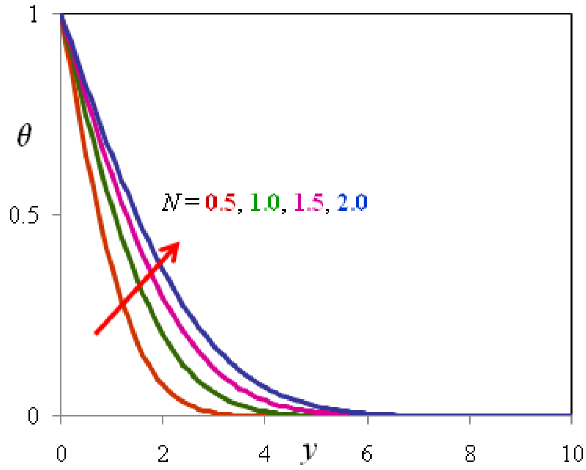

Figure 10. Effect of *N* on Temperature profiles

The influence of the viscous dissipation parameter i.e., the Eckert number on the velocity and temperature are shown in figures (7) and (8) respectively. The Eckert number expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. Greater viscous dissipative heat causes a rise in the temperature as well as the velocity. This behavior is evident from figures (7) and (8). The effects of the thermal radiation parameter on the velocity and temperature profiles in the boundary layer are illustrated in figures (9) and (10) respectively. Increasing the thermal radiation parameter produces significant increase in the thermal condition of the fluid and its thermal boundary layer. This increase in the fluid temperature induces more flow in the boundary layer causing the velocity of the fluid there to increase. Figures (11) and (12) display the effects of the chemical reaction parameter on the velocity and concentration profiles, respectively. As expected, the presence of the chemical reaction significantly affects the concentration profiles as well as the velocity profiles. It should be mentioned that the studied case is for a destructive chemical reaction. In fact, as chemical reaction increases, the considerable reduction in the velocity profiles is predicted, and the presence of the peak indicates that the maximum value of the velocity occurs in the body of the fluid close to the surface but not at the surface. Also, with an increase in the chemical reaction parameter, the concentration decreases. It is evident that the increase in the chemical reaction significantly alters the concentration boundary layer thickness but does not alter the momentum boundary layers.

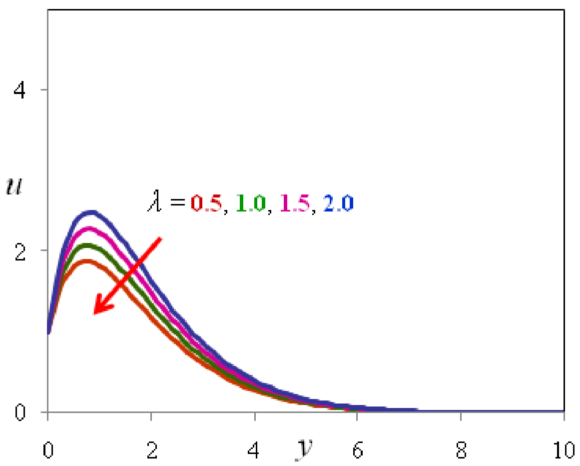

Figure 11. Effect of λ on Velocity profiles

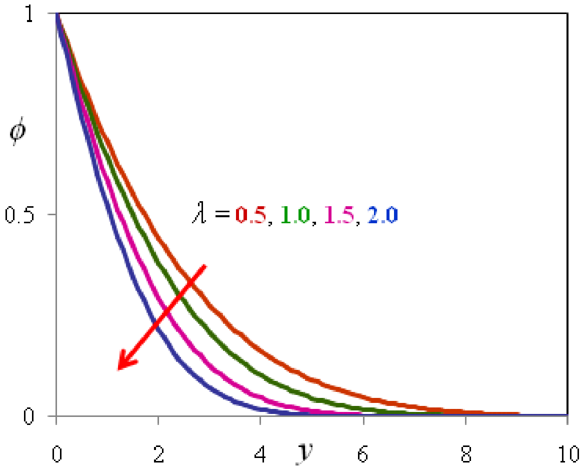

Figure 12. Effect of λ on Concentration profiles

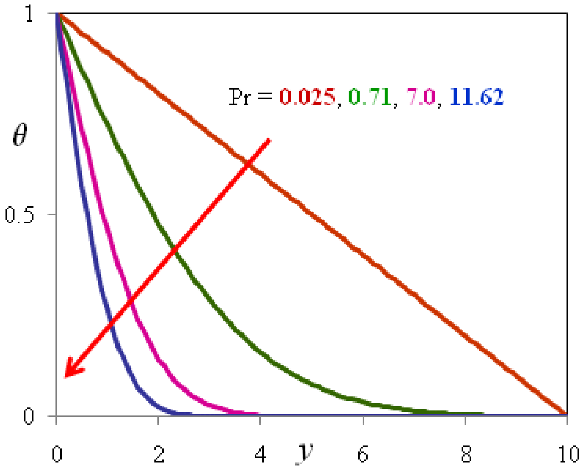

Figure 13. Effect of Pr on Temperature profiles

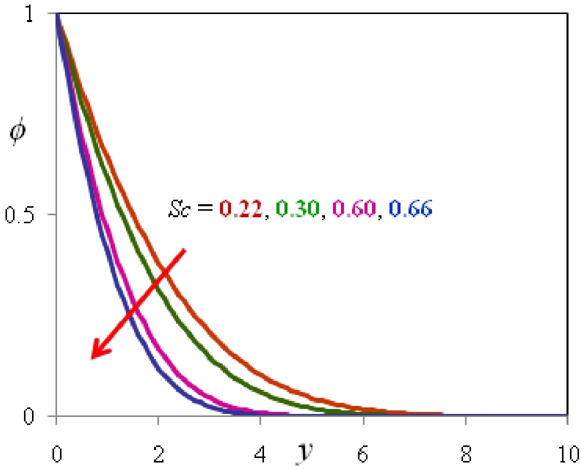

Figure 14. Effect of *Sc* on Concentration profiles

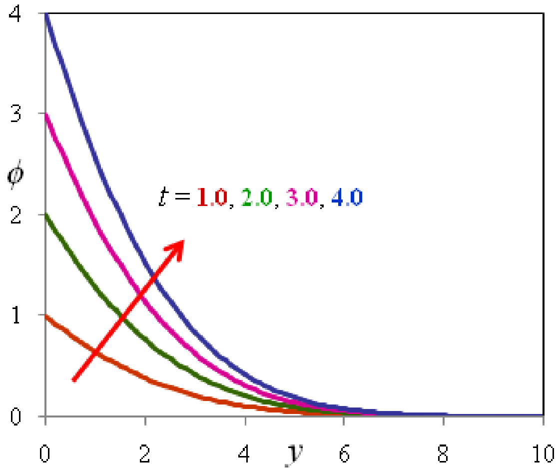

Figure 15. Effect of *t* on Temperature profiles

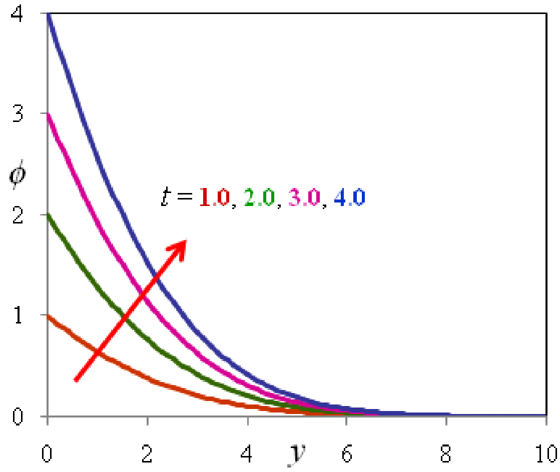

Figure 16. Effect of *t* on Concentration profiles

In figure (13) it depict the effect of Prandtl number on the temperature field. It is observed that an increase in the Prandtl number leads to decrease in the temperature field. Also, temperature field falls more rapidly for water in comparison to air and the temperature curve is exactly linear for mercury, which is more sensible towards change in temperature. From this observation it is conclude that mercury is most effective for maintaining temperature differences and can be used efficiently in the laboratory. Air can replace mercury, the effectiveness of maintaining temperature changes are much less than mercury. However, air can be better and cheap replacement for industrial purpose. This is because, either increase of kinematic viscosity or decrease of thermal conductivity leads to increase in the value of Prandtl number. Hence temperature decreases with increasing of Prandtl number. Figure (14) illustrates the effect of Schmidt number on the concentration field. It is noticed that as the Schmidt number increases, the concentration of the fluid medium decreases significantly in the boundary layer region and thereafter not much of variation is noticed. Figures (15) and (16) display the effect of the time on temperature and concentration profiles respectively. From these two figures it observed that, both temperature and concentration are increasing with increasing values of time.

The profiles for skin-friction due to velocity under the effects of Grashof number, Modified Grashof number, Prandtl number, Schmidt number, Hartmann number, Eckert number, Permeability parameter, Thermal radiation parameter and Chemical reaction parameter are presented in the table-1 respectively. It is observed from this table-1, the skin-friction rises under the effects of Grashof number, Modified Grashof number, Eckert number, Permeability parameter and Thermal radiation parameter. And falls under the effects of Prandtl number, Schmidt number, Hartmann number and Chemical reaction parameter.

Table-1: Skin-friction results (τ) for the values of Gr, Gc, Pr, Sc, M, K, Ec, N and λ

Gr	Gc	Pr	Sc	M	K	Ec	N	λ	τ
1.0	1.0	0.71	0.22	1.0	1.0	0.001	1.0	1.0	1.35831592
2.0	1.0	0.71	0.22	1.0	1.0	0.001	1.0	1.0	2.55841064
1.0	2.0	0.71	0.22	1.0	1.0	0.001	1.0	1.0	3.20690598
1.0	1.0	7.00	0.22	1.0	1.0	0.001	1.0	1.0	1.21241232
1.0	1.0	0.71	0.30	1.0	1.0	0.001	1.0	1.0	1.22361664
1.0	1.0	0.71	0.22	2.0	1.0	0.001	1.0	1.0	1.15888106
1.0	1.0	0.71	0.22	1.0	2.0	0.001	1.0	1.0	1.41703941
1.0	1.0	0.71	0.22	1.0	1.0	0.100	1.0	1.0	1.36513887
1.0	1.0	0.71	0.22	1.0	1.0	0.001	2.0	1.0	1.38441964
1.0	1.0	0.71	0.22	1.0	1.0	0.001	1.0	2.0	1.31157298

The profiles for Nusselt number due to temperature profile under the effect of Prandtl number, Eckert number, Thermal radiation parameter and time are presented in the table-2. From this table it observed that, the Nusselt number due to temperature profiles rises under the effect of Eckert number, Thermal radiation parameter and time. And temperature profiles falls under the effects of Prandtl number. The profiles for Sherwood number due to concentration profiles under the effect of Schmidt number, Chemical reaction parameter and time are presented in the table-3. From this table, the Sherwood number due to concentration profiles decreases under the effects of Schmidt number and Chemical reaction parameter and increases with the increasing values of time.

Table-2: Rate of heat transfer (Nu) values for different values of Pr, Ec, N and t

Pr	Ec	N	t	Nu
0.71	0.001	1.0	1.0	0.68749421
7.00	0.001	1.0	1.0	0.55419631
0.71	0.100	1.0	1.0	0.72594852
0.71	0.001	2.0	1.0	0.71434208
0.71	0.001	1.0	2.0	0.70157693

Table-3: Rate of mass transfer (Sh) values for different values of Sc, λ and t

Sc	λ	t	Sh
0.22	1.0	1.0	1.42919663
0.30	1.0	1.0	1.32481477
0.22	2.0	1.0	1.36825118
0.22	1.0	2.0	1.44576324

6. CONCLUSIONS

The author summarize below the following results of physical interest on the velocity, temperature and concentration distributions of the flow field and also on the skin-friction, rate of heat and mass transfer at the wall.

1. A growing Hartmann number or Prandtl number or Schmidt number or Chemical reaction parameter retards the velocity of the flow field at all points.
2. The effect of increasing Grashof number or Modified Grashof number or Permeability parameter or Eckert number or Thermal radiation parameter is to accelerate velocity of the flow field at all points.
3. A growing Prandtl number decreases temperature of the flow field at all points and increases with increasing of Eckert number or Time or Thermal radiation parameter.
4. The Schmidt number and Chemical reaction parameter decreases the concentration of the flow field at all points.

5 A growing Hartmann number or Prandtl number or Schmidt number or Chemical reaction parameter decreases the skin-friction while increasing Grashof number or Modified Grashof number or Permeability parameter or Eckert number or Thermal radiation parameter increases the skin-friction.

6 The rate of heat transfer is decreasing with increasing of Prandtl number and increases with increasing of Eckert number and Thermal radiation parameter.

7 The rate of mass transfer is decreasing with increasing of Schmidt number and Chemical reaction parameter.

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