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BETWEEN δ-CLOSED SETS AND δ -SEMI-CLOSED SETS

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ABSTRACT

*In this paper we introduce a new class of sets, namely semi*δ-closed sets, as the complement of semi*δ-open sets. We find characterizations of semi*δ-closed sets. We also define the semi*δ-closure of a subset. Further we investigate fundamental properties of the semi*δ-closure.*

Keywords: *δ-semi-open set, δ-semi-closed set, semi*δ-open set, semi*δ-interior, semi*δ-closed set, semi*δ-closure.*

I. INTRODUCTION

In 1963 Levine [3] introduced the concepts of semi-open sets and semi-continuity in topological spaces. Levine [4] also defined and studied generalized closed sets as a generalization of closed sets. Dunham [2] introduced the concept of generalized closure using Levine's generalized closed sets .N.V.Velico [15] introduced the concept of δ-open sets in 1968. In 1997, Park, Lee and Son [17] introduced the concept of δ-semi-open sets in topological spaces. Pasunkili Pandian [9] defined and studied semi*-pre closed sets and investigated its properties.A.Robert [13] defined and studied semi*α-closed sets. The authors [18] have recently introduced the concept of semi*δ-open sets and investigated its properties. The semi*δ-interior of a subset has also been defined and its properties studied.

In this paper, we define a new class of sets, namely semi*δ-closed sets, as the complement of semi*δ-open sets. We further show that the class of semi*δ-closed sets is placed between the class of δ-closed sets and the class of δ-semi-closed sets. We find characterizations of semi*δ-closed sets. We investigate fundamental properties of semi*δ-closed sets. We also define the semi*δ-closure of a subset. We also study some basic properties of semi*δ-closure.

II. PRELIMINARIES

Throughout this paper (X, τ) will always denote a topological space on which no separation axioms are assumed, unless explicitly stated. If A is a subset of the space (X, τ), $Cl(A)$ and $Int(A)$ denote the closure and the interior of A respectively. Also $\mathcal{F}$ denotes the class of all closed sets in the space (X, τ).

Definition 2.1: A subset A of a space X is
- (i) ***generalized closed*** (briefly ***g-closed***) [2] if $Cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in X.
- (ii) ***generalized open*** (briefly ***g-open***) [2] if $X\backslash A$ is g-closed in X.

Definition 2.2: If A is a subset of X,
- (i) the ***generalized closure*** [3] of A is defined as the intersection of all g-closed sets in X containing A and is denoted by $Cl^*(A)$.
- (ii) the ***generalized interior*** of A is defined as the union of all g-open subsets of A and is denoted by $Int^*(A)$.

Definition 2.3: A subset A of a topological space (X, τ) is **semi-open** [3](respectively **semi*-open** [12]) if there is an open set U in X such that $U\subseteq A\subseteq Cl(U)$ (respectively $U\subseteq A\subseteq Cl^*(U)$) or equivalently if $A\subseteq Cl(Int(A))$ (respectively $A\subseteq Cl^*(Int(A))$).

Definition 2.4: Definition 2.4. A subset A of a topological space (X, τ) is **pre-open** [5](respectively **pre*-open** [14]) if $A\subseteq Int(Cl(A))$ (respectively $A\subseteq Int^*(Cl(A))$).

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Definition 2.5: A subset A of a topological space (X, τ) is **α-open** [7] (respectively **α*-open** [10]) if $A\subseteq Int(Cl(Int(A)))$, (respectively $A\subseteq Int^*(Cl(Int^*(A)))$.

Definition 2.6: A subset A of a topological space (X, τ) is **semi-preopen** [1]= β - open (respectively **semi*-preopen** [9]) if $A\subseteq Cl(Int(Cl(A)))$ (respectively $A\subseteq Cl^*(pInt(A))$).

Definition 2.7: A subset A of a topological space (X, τ) is **regular-open** [6] if $A=Int(Cl(A))$.

Definition 2.8: The ***δ-interior*** [15] of A is defined as the union of all regular-open sets of X contained in A. It is denoted by **$\delta Int(A)$.**

Definition 2.9: A subset A of a topological space (X, τ) is **δ-open** [11] if $A=\delta Int(A)$.

Definition 2.10: A subset A of a topological space (X, τ) is **semi α-open** [6] (respectively **semi* α-open** [13]) if there is a α-open set U in X such that $U\subseteq A\subseteq Cl(U)$ (respectively $U\subseteq A\subseteq Cl^*(U)$) or equivalently if $A\subseteq Cl(\alpha Int(A))$. (respectively $A\subseteq Cl^*(\alpha Int(A))$).

Definition 2.11: A subset A is **δ-semi-open** [17] if $A\subseteq Cl(\delta Int(A))$.

Definition 2.12: A subset A of a topological space (X, τ) is called a ***semi*δ-open*** **[**18] if there exists a δ-open set U in X such that $U\subseteq A\subseteq Cl^*(U)$ or equivalently if $A\subseteq Cl^*(\delta Int(A))$.

Definition 2.13:A subset A of a topological space (X, τ) is **semi-closed (**respectively **semi*-closed, pre-closed**[5], **pre*-closed** [14], **α-closed** [7], **α*-closed** [10], **semi-pre closed** [1], **semi*-preclosed** [9], **regular-closed**[6], **δ-closed** [11], **semi α-closed**[6], **semi* α-closed**[13] and **δ-semi-closed** [17]) if $Int(Cl(A))\subseteq A$(respectively $Int^*(Cl(A))\subseteq A$, $Cl(Int(A)\subseteq A$, $Cl^*(Int(A)\subseteq A$,$Cl(Int(Cl(A)))\subseteq A$,$Cl^*(Int(Cl^*(A)))\subseteq A$, $Int(Cl(Int(A)))\subseteq A$,$Int^*(pCl(A))\subseteq A$, $Cl(Int(A)=A$,$\delta Cl(A)=A$, $Int(Cl(Int(Cl(A))))\subseteq A$, $Int^*(\alpha Cl(A))\subseteq A$ and $Int(\delta Cl(A))\subseteq A$).

The class of all semi*δ-open sets in (X, τ) is denoted by $S^*\delta O(X, \tau)$ or simply $S^*\delta O(X)$.

The class of all semi-closed (respectively semi*-closed, pre-closed, pre*-closed, α-closed, α*-closed, semi-preclosed, semi*-preclosed, semi α-closed, semi* α-closed, regular-closed, δ-closed and δ-semi-closed) sets in (X, τ) is denoted by $SC(X)$ (respectively $S^*C(X)$, $PC(X)$. $P^*C(X)$,$\alpha C(X)$, $\alpha^*C(X)$ $SPC(X)$, $S^*PC(X)$,$S\alpha C(X)$, $S^*\alpha C(X)$, $RC(X)$, $\delta C(X)$ and $\delta SC(X)$.

Definition 2.14: A topological space X is $T_{1/2}$[4] if every g-closed set in X is closed.

Theorem 2.15: [2] Cl^* is a Kuratowski closure operator in X.

Definition 2.16: [2] If τ^* is the topology on X defined by the Kuratowski closure operator Cl^*, then (X, τ^*) is $T_{1/2}$.

Definition 2.17: [16] A space X is locally indiscrete if every open set in X is closed.

Theorem 2.18: [18] For a subset A of a topological space (X, τ) the following statements are equivalent:

(i) A is semi*δ-open.
(ii) $A\subseteq Cl^*(\delta Int(A))$.
(iii) $Cl^*(\delta Int(A))=Cl^*(A)$.

Theorem 2.19: [18] Every δ-open set is semi*δ-open.

Theorem 2.20: [18**]** In any topological space,

(i) Every semi*δ-open set is δ-semi-open.
(ii) Every semi*δ-open set is semi - open.
(iii) Every semi*δ-open set issemi* - open.
(iv) Every semi*δ-open set is semi*-preopen.
(v) Every semi*δ-open set is semi-preopen.
(vi) Every semi*δ-open set is semi*α-open
(i) Every semi*δ-open set is semiα-open.

Theorem 2.21: [18] In any topological space, arbitrary union semi*δ-open sets is semi*δ-open.

Theorem 2.22: [18] If A is semi*δ-open in X and B is open in X, then $A\cap B$ is semi*δ-open in X.

Theorem 2.23: [18] If A is semi*δ-open in X and $B\subseteq X$ is such that $\delta Int(A)\subseteq B\subseteq Cl^*(A)$. Then B is semi*δ-open.

III. SEMI*δ-CLOSED SETS

Definition 3.1: The complement of a semi*δ-open set is called ***semi*δ-closed.*** The class of all semi*δ-closed sets in (X, τ) is denoted by $S^*\delta C(X, \tau)$ or simply $S^*\delta C(X)$

Definition 3.2: A subset A of X is called ***semi*δ-regular*** if it is both semi*δ-open and semi*δ-closed.

Theorem 3.3: For a subset A of a topological space (X, τ), the following statements are equivalent:

(i) A is semi*δ-closed.
(ii) $Int^*(\delta Cl(A))\subseteq A$.
(iii) $Int^*(\delta Cl(A))=Int^*(A)$.

Proof: (i)⟹(ii): Suppose A is semi*δ-closed.Then $X\backslash A$ is semi*δ-open.Then by Theorem 2.18 $X\backslash A \subseteq Cl^*(\delta Int(X\backslash A))$. Taking the complements we get, $A\supseteq X\backslash Cl^*(\delta Int(X\backslash A) \Rightarrow A \supseteq Int^*(\delta Cl(A))$.

(ii)⟹(iii): By assumption, $Int^*(\delta Cl(A))\subseteq A$. This implies that $Int^*(\delta Cl(A))\subseteq Int^*(A)$. Since it is true that $A\subseteq\delta Cl(A)$, we have $Int^*(A)\subseteq Int^*(\delta Cl(A))$. Therefore $Int^*(\delta Cl(A))=Int^*(A)$.

(iii)⟹(i): If $Int^*(\delta Cl(A))=Int^*(A)$, then taking the complements, we get $X\backslash Int^*(\delta Cl(A))=X\backslash Int^*(A)$. Hence $Cl^*(\delta Int(X\backslash A))=Cl^*(X\backslash A)$. Therefore by Theorem 2.18, $X\backslash A$ is semi*δ-open and hence A is semi*δ-closed.

Theorem 3.4: A subset A of a space (X, τ) is semi*δ-closed iff there is a δ-closed set F in (X, τ) such that $Int^*(F)\subseteq A\subseteq F$.

Proof: ***Necessity*:** Suppose A is semi*δ-closed. Then X\A is semi*δ-open. Then by definition 2.12 there exists a δ- open set U in X such that $U\subseteq X\backslash A\subseteq Cl^*(U)$ which implies $X\backslash U\supseteq A\supseteq X\backslash Cl^*(U)$. Note that in any space, $X\backslash Cl^*(U)=Int^*(X\backslash U)$. Therefore $F\supseteq A\supseteq Int^*(F)$ where $F=X\backslash U$ is δ-closed in X.

***Sufficiency*:** Suppose there is a δ-closed set F in (X, τ) such that $Int^*(F)\subseteq A\subseteq F$ which implies $X\backslash Int^*(F)\supseteq X\backslash A\supseteq X\backslash F$. Since $X\backslash Int^*(F)=Cl^*(X\backslash F)$, we have $Cl^*(X\backslash F)\supseteq X\backslash A\supseteq X\backslash F$ where X\F is a δ-open set. Hence by Definition 2.12, X\A is semi*δ-open. Therefore A is semi*δ-closed.

Remark 3.5:

(i) In any topological space (X, τ), ϕ and X are semi*δ-closed sets.
(ii) In a $T_{1/2}$ space, the semi*δ-closed sets and the δ-semi-closed sets coincide. In particular, in the real line with usual topology the semi*δ-closed sets and the δ-semi-closed sets coincide.

Theorem 3.6: Arbitrary intersection of semi*δ-closed sets is also semi*δ-closed.

Proof: Let $\{A_i\}$ be a collection of semi*δ-closed sets in X. Since each A_i is semi*δ-closed, $X\backslash A_i$ is semi*δ-open. By Theorem 2.21, $X\backslash(\cap A_i)= \cup(X\backslash A_i)$ is semi*δ-open. Hence $\cap A_i$ is semi*δ-closed.

Corollary 3.7: If A is semi*δ-closed and U is semi*δ-open in X, then $A\backslash U$ is semi*δ-closed.

Proof: Since U is semi*δ-open, $X\backslash U$ is semi*δ-closed. Also since $A\backslash U=A\cap (X\backslash U)$, and hence by Theorem 3.6, $A\backslash U$ is semi*δ-closed.

Remark 3.8: Union of two semi*δ-closed sets need not be semi*δ-closed as seen from the following examples.

Example 3.9: Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. In the space (X, τ), the subsets $\{a\}$ and $\{b\}$ are semi*δ-closed but their union $\{a, b\}$ is not semi*δ-closed.

Example 3.10: Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, \{a\},\{b\},\{c\},\{a, b\},\{a, c\}, \{b, c\}, \{a, b, c\}, X\}$. In the space (X, τ), the subsets $\{a, c\}$ and $\{b, c\}$ are semi*δ-closed but their union $\{a, b, c\}$ is not semi*δ-closed.

Theorem 3.11: If A is semi*δ-closed in X and B is closed in X, then $A\cup B$ is semi*δ-closed.

Proof: Since A is semi*δ-closed, $X\backslash A$ is semi*δ-open in X. Also $X\backslash B$ is open. By Theorem 2.22, $(X\backslash A) \cap (X\backslash B)=X\backslash(A\cup B)$ is semi*δ-open in X. Hence $A\cup B$ is semi*δ-closed in X.

Theorem 3.12: Every δ-closed set is semi*δ-closed.

Let U be δ-closed in X. Then X\U is δ-open. By theorem 2.19, X\U is semi*δ-open, Hence U is semi*δ-closed.

Remark 3.13: The converse of the above theorem is not true as shown in the following examples.

Example 3.14: In the space (X, τ) where $X=\{a, b, c\}$ and $\tau=\{\phi, \{a\}, \{b\}, \{a, b\}, X\}$, the subsets $\{a\}$ and $\{b\}$ are semi*δ-closed but not δ-closed.

Example 3.15: In the space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\}, \{b\}, \{a, b\},\{a, b, c\}, X\}$, the subsets $\{c\}$, $\{a, c\}$and $\{b, c\}$ are semi*δ-closed but not δ-closed.

Theorem 3.16: In any topological space,

(i) Every semi*δ-closed set is δ-semi-closed.
(ii) Every semi*δ-closed set is semi-closed.
(iii) Every semi*δ-closed set is semi*-closed.
(iv) Every semi*δ-closed set is semi*-preclosed.
(v) Every semi*δ-closed set is semi-preclosed.
(vi) Every semi*δ-closed set is semi*α-closed.
(vii) Every semi*δ-closed set is semi-α-closed.

Proof:

(i) Let A be a semi*δ-closed set. Then $X\backslash A$ is semi*δ-open. By Theorem 2.20(i), $X\backslash A$ is δ-semi-open. Hence A is δ-semi-closed.
(ii) Suppose A is a semi*δ-closed set. Then $X\backslash A$ is semi*δ-open. By Theorem2.20 (ii), $X\backslash A$ is semi-open. Hence, A is semi-closed.
(iii) Suppose A is a semi*δ-closed set. Then $X\backslash A$ is semi*δ-open. By Theorem 2.20(iii), $X\backslash A$ is semi*-open. Hence, A is semi*-closed.
(iv) Let A be a semi*δ-closed set. Then $X\backslash A$ is semi*δ-open in X. By Theorem 2.20(iv), $X\backslash A$ is semi*-preopen Hence A is semi*-preclosed in X.
(v) his statement follows from (iv) and the fact that every semi*-preclosed set is semi-pre closed.
(vi) Let A be a semi*δ-closed set. Then $X\backslash A$ is semi*δ-open By Theorem 2.20(vi), $X\backslash A$ is semi*α-open. Hence, A is semi*α-closed.
(vii) This statement follows from (vi) and the fact that every semi*α-closed set is semiα-closed.

Remark 3.17: The converse of each of the statements in Theorem 3.16 is not true as shown in the following examples.

Example 3.18: In the space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\}, \{b\}, \{a, b\},\{a, b, c\}, X\}$, the subsets $\{a\}$, $\{b\}$,$\{d\}$, $\{a, d\}$ and $\{b, d\}$ are semiδ-closed but not semi*δ-closed.

Example 3.19: In the space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\}, X\}$, the subsets$\{b\}$,$\{c\}$, $\{d\}$,$\{b, c\}$,$\{b, d\}$,$\{c, d\}$ and $\{b, c, d\}$ are semi-closed but not semi*δ-closed.

Example 3.20: In the space (X, τ) where $X= \{a, b, c, d\}$ and $\tau=\{\phi, \{a, b, c\}, X\}$, the subset {d} is semi*-closed but not semi*δ-closed.

Example 3.21: In the space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a, b\},\{a, b, c\}, X\}$, the subsets $\{a\}$, {b}, {c}, {d},$\{a, c\}$,$\{a, d\}$, $\{b, c\}$, $\{b, d\}$,{c, d},$\{a, c, d\}$ and {b, c, d} are semi*-preclosed but not semi*δ-closed.

Example 3.22: In the space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\},\{b, c, d\}, X\}$, the subsets {b}, {c},{d},$\{a, b\}$, {a, c},$\{a, d\}$,$\{b, c\}$, $\{b, d\}$,{c, d},$\{a, b, c\}$,$\{a, b, d\}$ and $\{a, c, d\}$ are semi-preclosed but not semi*δ-closed.

Example 3.23: In the space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\},\{a, b, c\}, X\}$, the subsets{b}, $\{c\}$, {d},{b, c},$\{b, d\}$,{c, d} and {b, c, d} are semi*α-closed but not semi*δ-closed.

Example 3.24: In the space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\},\{a, b\},\{a, b, c\}, X\}$, the subsets{b}, {c}, {d}, {b, c},$\{b, d\}$,$\{c, d\}$ and {b, c, d} are semiα-closed but not semi*δ-closed.

Corollary 3.25: If A is semi*δ-closed and F is δ-closed in X, then $A\cap F$ is semi*δ-closed in X.

Proof: Since F is δ-closed, by Theorem 3.12 F is semi*δ-closed. Then by Theorem 3.6, $A\cap F$ is semi*δ-closed

Theorem 3.26: In any topological space (X,τ), $\delta C(X, \tau)\subseteq S^*\delta C(X, \tau)\subseteq\delta SC(X, \tau)$ .That is the class of semi*δ-closed set is placed between the class of δ-closed sets and the class of δ-semi-closed sets.

Proof: Follows from Theorem 3.12 and Theorem 3.16.

Remark 3.27:
(i) If (X, τ) is a locally indiscrete space,
$\mathcal{F}=\delta C(X,\tau)=S^*\delta C(X,\tau)=\delta SC(X,\tau)=S^*C(X,\tau)=SC(X,\tau)=\alpha C(X,\tau)=S^*CO(X,\tau)=S\alpha C(X,\tau)=RC(X, \tau)$.
(ii) The inclusions in Theorem 3.26 may be strict and equality may also hold. This can be seen from the following examples.

Example 3.28: In the topological space (X, τ) where $X=\{a, b, c, d\}$ and $\tau =\{\phi, \{a\}, \{b, c, d\}, X\}$,$\delta C(X, \tau)=S^*\delta C(X, \tau)=\delta SC(X, \tau)$.

Example 3.29: In the topological space (X, τ) where $X=\{a, b, c, d\}$ and $\tau =\{\phi,\{a\},\{b\},\{c\},\{a, b\},\{a, c\},\{b, c\}, \{a, b, c \}, X\}$, $\delta C(X, \tau) \subsetneq S^*\delta C(X, \tau) = \delta SC(X, \tau)$.

Example 3.30: In the topological space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\}, \{b\}, \{a, b\},\{a, b, c\}, X\}$, $\delta C(X, \tau) \subsetneq S^*\delta C(X, \tau) \subsetneq \delta SC(X, \tau)$.

Theorem 3.31: If A is semi*δ-closed in X and B be a subset of X such that $Int^*(A)\subseteq B\subseteq\delta Cl(A)$, then B is semi*δ-closed in X.

Proof: Since A is semi*δ-closed, $X\backslash A$ is semi*δ-open. Now $Int^*(A)\subseteq B\subseteq\delta Cl(A)$ which implies $X\backslash Int^*(A)\supseteq X\backslash B\supseteq X\backslash\delta Cl(A)$. That is, $Cl^*(X\backslash A)\supseteq X\backslash B\supseteq\delta Int(X\backslash A)$. Therefore by Theorem 2.23, $X\backslash B$ is semi*δ-open. Hence B is semi*δ-closed.

Remark 3.32: The concept of semi*δ-closed sets and closed sets are independent as seen from the following example:

Example 3.33: In the topological space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$, the subsets $\{c\}$,$\{a, c\}$and $\{b, c\}$ are semi*δ-closed but not closed and $\{d\}$ is closed but not semi*δ-closed.

Remark 3.34: The concept of semi*δ-closed sets and g-closed sets are independent as seen from the following example:

Example 3.35: In the topological space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$, the subsets $\{c\}$,$\{a, c\}$and $\{b, c\}$ are semi*δ-closed but not g-closed and $\{d\}$,$\{a, d\}$,$\{b, d\}$and$\{a, b, d\}$ are g-closed but not semi*δ-closed.

Remark 3.36: The concept of semi*δ-closed sets and α-closed sets are independent as seen from the following examples:

Example 3.37: In the topological space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi,\{a, b\}, \{a, b, c\}, X\}$, the subsets $\{c\}$,$\{d\}$and $\{c, d\}$ are α-closed but not semi*δ-closed.

Example 3.38: In the topological space (X, τ) where $X=\{a, b, c, d\}$ and $\tau =\{\phi, \{a\}, \{b\},\{c\},\{a, b\},\{a, c\}, \{b, c\}, \{a, b, c\}, X\}$,the subsets $\{a\}$,$\{b\}$, $\{c\}$,$\{a, b\}$,$\{a, c\}$ and $\{b, c\}$ are semi*δ-closed but not α-closed.

Remark 3.39: The concept of semi*δ-closed sets and pre-closed sets are independent as seen from the following examples:

Example 3.40: In the topological space (X, τ) where $X= \{a, b, c\}$ and $\tau=\{\phi,\{a\},\{b\},\{a, b\}, X\}$, the subsets $\{a\}$ and$\{b\}$ are semi*δ-closed but not pre-closed.

Example 3.41: In the topological space (X, τ) where $X= \{a, b, c\}$ and $\tau =\{\phi, \{a, b\}, X\}$, the subsets $\{a\}$,$\{b\}$, $\{c\}$,$\{ a, c\}$and$\{b, c\}$ are pre-closed but not semi*δ-closed.

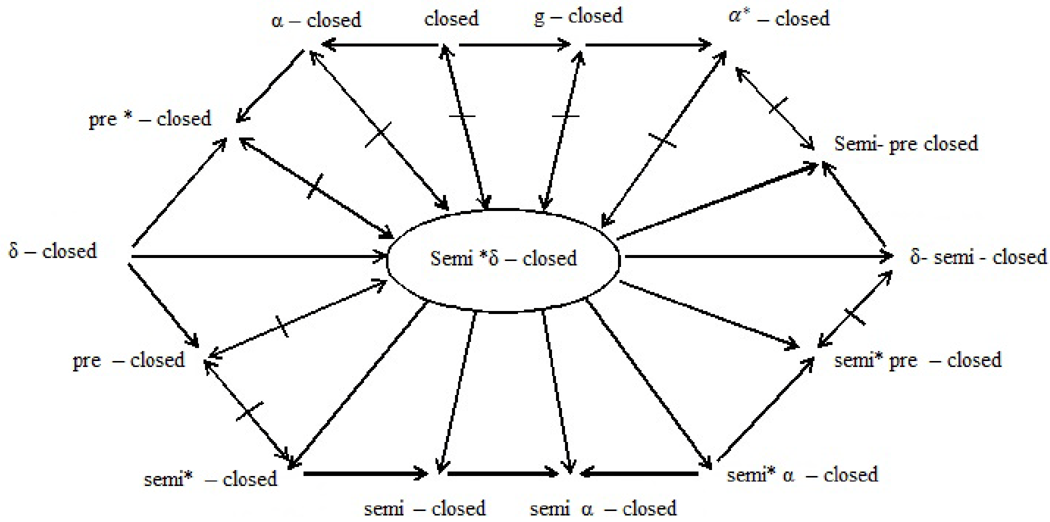
Remark 3.42: The concept of semi*δ-closed sets and α*-closed sets are independent as seen from the following examples:

Example 3.43: In the topological space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$, the subsets $\{a, c\}$ and $\{b, c\}$ are semi*δ-closed but not α*-closed and $\{d\}$,$\{ a, d\}$,$\{b, d\}$and $\{a, b, d\}$ are α*-closed but not semi*δ-closed.

Remark 3.44: The concept of semi*δ-closed sets and pre*-closed sets are independent as seen from the following examples:

Example 3.45: In the topological space (X, τ) where X={a, b, c, d} and τ={ϕ, {a}, {b}, {a, b}, {a, b, c}, X}, the subsets {a, c}and{b, c} are semi*δ-closed but not pre*-closed and {d},{ a, d},{b, d} and {a, b, d} are pre*-closed but not semi*δ-closed.

From the above discussions we have the following diagram:



IV. SEMI*δ-CLOSURE OF A SET

Definition 4.1: If A is a subset of a topological space X, the ***semi*δ-closure*** of A is defined as the intersection of all semi*δ-closed sets in X containing A. It is denoted by $s^*\delta Cl(A)$.

Theorem 4.2: If A is any subset of a topological space (X, τ), then
(i) $s^*\delta Cl(A)$ is the smallest semi*δ-closed set in X containing A.
(ii) A is semi*δ-closed if and only if $s^*\delta Cl(A)=A$.

Proof:
(i) Since $s^*\delta Cl(A)$ is the intersection of all semi*δ-closed supersets of A, by Theorem 3.6, it is semi*δ-closed and is contained in every semi*δ-closed set containing A and hence it is the smallest semi*δ-closed set in X containing A.
(ii) If A is semi*δ-closed, then $s^*\delta Cl(A) = A$ is obvious from definition 4.1. Conversely, let $s^*\delta Cl(A)=A$. By (i) $s^*\delta Cl(A)$ is semi*δ-closed and hence A is semi*δ-closed.

Theorem 4.3: (Properties of Semi*δ-Closure)
In any topological space (X, τ) the following statements hold:
(i) $s^*\delta Cl(\phi)=\phi$.
(ii) $s^*\delta Cl(X)=X$.

If A and B are subsets of X,
(iii) $A\subseteq s^*\delta Cl(A)$.
(iv) $A\subseteq B \Longrightarrow s^*\delta Cl(A)\subseteq s^*\delta Int(B)$.
(v) $s^*\delta Cl(s^*\delta Cl(A))=s^*\delta Cl(A)$.
(vi) $A \subseteq \delta sCl(A)\subseteq s^*\delta Cl(A) \subseteq \delta Cl(A$
(vii) $s^*\delta Cl(A\cup B)\supseteq s^*\delta Cl(A)\cup s^*\delta Cl(B)$.
(viii) $s^*\delta Cl(A\cap B)\subseteq s^*\delta Cl(A) \cap s^*\delta Cl(B)$.

Proof: (i), (ii), (iii) and (iv) follow from Definition 4.1. By Theorem 4.2(i), $s^*\delta Cl(A)$ is semi*δ-closed and by Theorem 4.3(ii), $s^*\delta Cl(s^*\delta Cl(A))=s^*\delta Cl(A)$. Thus (v) is proved. The statements (vi) follows from Theorem 3.12and Theorem 3.16(i). Since $A\subseteq A\cup B$, from statement (iv) we have $s^*\delta Cl(A)\subseteq s^*\delta Cl(A\cup B)$. Similarly, $s^*\delta Cl(B)\subseteq s^*\delta Cl(A\cup B)$. This proves (vii). The proof for (viii) is similar.

Remark 4.4: In (vi) of Theorem 4.3,each of the inclusions may be strict and equality may also hold. This can be seen from the following examples:

Example 4.5: In the space (X, τ) where X={a, b, c, d} and τ={ϕ,{a},{b},{a, b}, {a, b, c}, X}.
$\mathcal{F}$={ϕ,{d},{c, d},{a, c, d}, {b, c, d}, X}.

Let A= {a, c, d}. Then δsCl(A) =$s^*\delta Cl$(A)= δCl(A)={a, c, d}=A.

Let B={a, c}.Then δsCl(B)= $s^*\delta Cl$(B)= {a, c}; δCl(B)={a, c, d};

Here B=δsCl(B)= $s^*\delta Cl$(B) $\subsetneq$ δCl(B)

Let C= { a, d}.Then δsCl(C)={ a, d}; $s^*\delta Cl$(C)= δCl(C)={a, c, d}

Here C= δsCl(C) $\subsetneq$ $s^*\delta Cl$(C) = δCl(C).

Let D= {a, b}.Then δsCl(D)= $s^*\delta Cl$(D)= δCl(D)= X.

Here D$\subsetneq$$\delta sCl$(D)=$s^*\delta Cl$(D)= δCl(D).

Let E={b}.Then δsCl(E)={b}; $s^*\delta Cl$(E)={b, c}; δCl(E)={b, c, d};

Here E = δsCl(E) $\subsetneq$ $s^*\delta Cl$(E) $\subsetneq$ δCl(E).

Remark 4.6: The inclusions in (vii) and (viii) of Theorem 4.3 may be strict and equality may also hold. This can be seen from the following examples.

Example 4.7: Consider the space (X**,** τ) in Example 4.5

Let A= {a, c} and B={c, d} then A∪B={a, c, d}; $s^*\delta Cl$(A) ={a, c}; $s^*\delta Cl$(B)={c, d}; $s^*\delta Cl$(A∪B)={a, c, d};

Here $s^*\delta Cl$(A∪B)= $s^*\delta Cl$(A) ∪$s^*\delta Cl$(B)

Let C= {a, c} and D ={b, c} then C∩D={c}; $s^*\delta Cl$(C)={a, c}; $s^*\delta Cl$(D)={b, c}; $s^*\delta Cl$(C∩D)={c};

Here $s^*\delta Cl$(C∩D)= $s^*\delta Cl$(C) ∩ $s^*\delta Cl$(D)

Let E= {a, b} and F ={c, d} then E∩F = ϕ; $s^*\delta Cl$(E) = X; $s^*\delta Cl$(F)={c, d}; $s^*\delta Cl$(E∩F)=ϕ; $s^*\delta Cl$(E) ∩$s^*\delta Cl$(F)={c, d}

Here $s^*\delta Cl$(E∩F)$\subsetneq$$s^*\delta Cl$(E) ∩$s^*\delta Cl$(F)

Let G={a} and H={ b} then G∪H={a, b};
$s^*\delta Cl$(G)={a, c}; $s^*\delta Cl$(H)= { b, c}; $s^*\delta Cl$(G∪H)= X ; $s^*\delta Cl$(G) ∪$s^*\delta Cl$(H)={a, b, c};

Here s$^*\delta Cl$(G) ∪$s^*\delta Cl$(H)$\subsetneq$$s^*\delta Cl$(G∪H).

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