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APPLICATION OF QUEUING THEORY TO WAITING TIME OF OUT-PATIENTS IN HOSPITALS USING M/M/c MODEL

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ABSTRACT

Waiting lines are experienced in our daily life. Waiting line or queue causes many inconveniences among the patients. Queuing theory is a mathematical approach to reduce the waiting time. The present study is about the waiting time, service times of the patients were taken and an effective model is used to reduce the waiting time and improve the service. The data collected from out- patients ward in Government Hospital and the highly suitable modeling tool, M/M/c queueing model, the stochastic Birth-death Markov process is used. The Model M/M/c is a multi channel queueing system with Poisson arrival and exponential distribution. Queue is a discipline which is served as First Come First Served (FCFS) basis.The average number of patients, the average time spent by each patient as well as the probability of arrival of patients into the queuing system will be obtained. The study suggests that more number of doctors may employed in the ward to deliver the effective service among the patients to save the valuable lives.

Keywords: *M/M/c model, Birth-death Markov process, Poisson arrival, FCFS.*

1. INTRODUCTION

Queues are ubiquitous, particularly in health care delivery systems. At the sametime, queues are undesirable because delay in receiving needed services can cause prolonged discomfort and economic loss when patients are unable to work and possible worsening of their medical conditions that can increase subsequent treatment costs and poor health outcomes. In extreme cases, long queues can delay diagnosis and/or treatment to the extent that death occurs while a patient waits. For example, there is a severe shortage of organs in the USA and many patients diewhile waiting for suitable organs for transplant [1] Diwakar Gupta, 2013.

Queuing models using for estimating waiting time of a patient, utilization of service, models system design, and models for evaluating appointment systems [2] Samuel Fomundam, Jeffrey Herrmann 2007. A queuing system helps minimizing the waiting time of patients and maximizing the utilization of the servers i.e. doctors, nurses, hospital beds etc. Queuing is not new but recently hospitals has begun to use it effectively [3] Shanmugasundaram and Umarani 2015. Despite all these efforts, it should be noted that, there are still some avoidable problems which undermine their success of this sector. One of the most frequent of them is the problem of waiting lines (queues) found in hospitals.

Queuing is a very volatile situation which cause unnecessary delay and reduce the service effectiveness of establishments. Apart from the time wasted, it also leads to breakdown of law and order. Many lives and property had been lost in queues at filling stations in the past.

Queues in hospitals often have severe consequences. For instance, delay in treatment of asthma, diabetes, and cardiac disease patients often lead to complications and eventual death. (*The World Bank Illustrated Home Medical Encyclopedia*, 1998). In light of this, there is need for a critical evaluation of patient waiting time as well as reducing or eliminating to avail better health service and save the valuable lives to a considerable extent.

2. METHODOLOGY

A survey was carried out and recorded during the study period for about ten days between 8.00 to 1.00 PM and the data were classified as arrival time of patients, Time of service begins and time of service ends and departure time of 7562 patients. These data were enabled us to obtain the arrival rate λ, the service rate μ, and the traffic intensity ρ of the patients using results from the birth and death model (which is synonymous to arrival and departure).

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3. MODEL SPECIFICATION ((M/M/c): (∞/FIFO) Queuing Model)

The M/M/c Queue (Multi-Channel Queuing System). In this queuing system, the patients arrive according to a Poisson process with rate λ.

The time it takes to serve every patients is an exponential random variable with parameter μ.

The service times are mutually independent and further independent of the inter arrival times.

When a patient enters an empty system, his service starts at once and if the system is non-empty, the incoming patient joins the queue.

Parameters in Multi Server Queuing models:

n = Number of total patients in the system (in queue plus in service)
c = Number of parallel channels (service channels in Hospital)
λ = Arrival rate the patients arrive according to a Poisson process
μ = Serving rate
cμ = Serving rate when c > 1 in a system
ρ = Traffic intensity or load, utilization factor (= λ/(cμ)) (the expected factor of time the server is busy that is, service capability being utilized on the average arriving patients) Departure and arrival rate are state dependent and are in steady-state (equilibrium between events) condition.

Notations and their descriptions

The arrival rate $\lambda = \dfrac{The \quad total \quad no. \;\; of \quad patients}{total \quad waiting \quad time}$

The service rate $\mu = \dfrac{The \;\; total \quad no. \;\; of \quad patients}{Total \quad Service \quad time}$

Traffic intensity, $\rho = \dfrac{\lambda}{c\mu}$

The probability of the system to be ideal $P_0 = \left(\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu} \right)^n \frac{1}{\underline{|n}} + \left(\frac{\lambda}{\mu} \right)^c \frac{1}{\underline{|c}} \left(\frac{c\mu}{c\mu - \lambda} \right) \right)^{-1}$

The average number of patients queue $L_q = \dfrac{\lambda\mu \left(\frac{\lambda}{\mu} \right)^c}{\underline{|(c-1)}(c\mu - \lambda)^2} P_0$

The average number of patients in queue when queue exist, $L_w = \dfrac{\lambda}{c\mu - \lambda}$

The average number of patients in the systems $L_s = \dfrac{\lambda\mu \left(\frac{\lambda}{\mu} \right)^c}{\underline{|(c-1)}(c\mu - \lambda)^2} P_0 + \dfrac{\lambda}{\mu}$

The average time (waiting time) in queue (before service is rendered) $W_q = \dfrac{\mu \left(\frac{\lambda}{\mu} \right)^c}{\underline{|(c-1)}(c\mu - \lambda)^2} P_0$

The average time (waiting time) in system $W_s = \dfrac{\mu\left(\frac{\lambda}{\mu}\right)^c}{\lfloor(c-1)(c\mu-\lambda)^2} P_0 + \dfrac{\lambda}{\mu}$

4. RESULTS

A total of 10 days were used for the data collection. The waiting time was obtained by subtracting arrival time, from the time of service began for each day. Similarly, service time was found by subtracting time of service began from when it ended.

The following results were calculated for ten days of the study period, Therefore:

- Total waiting times = 42553 minutes
- Total service times = 24000 minutes
- This follows the arrival of 7562 patients.

By applying the above formula we arrive the following,

The arrival rate $\lambda = \dfrac{The\ \ total\ \ no.\ \ of\ \ patients}{total\ \ waiting\ \ times}$ = 7562/42553 = 0.1777078

The service rate $\mu = \dfrac{The\ \ total\ \ no.\ \ of\ \ patients}{Total\ \ Service\ \ time}$ = 7562/24000= 0.315083333

Traffic intensity $\rho = \dfrac{\lambda}{c\mu}$

= 0.1777078 / 8 (0.315083333)

= 0.0705003181

The probability of the system to be ideal = 0.5689

The average number of patients queue L_q = 1.179×10^{-8}

The average number of patients in the system, L_s = 0.564

The average time (waiting time) in queue before service is renderedW_q = 0.0000000664

The average time (waiting time) in system W_s = 0.5640000664

Table showing average no. of patients and their waiting time for the study period (day wise)

Day	Arrival of patients/day	ρ	Ls	Lq	Ws	Wq
1	750	0.0694	0.5552	1.0300769×10^{-8}	0.5552000594	5.94×10^{-8}
2	730	0.06911625	0.55293	$9.948012814 \times 10^{-9}$	0.552930059	$5.914395252 \times 10^{-8}$
3	700	0.07074875	0.565990012	$1.21566495 \times 10^{-8}$	0.565990073	$7.363203816 \times 10^{-8}$
4	770	0.069875	0.55900001	$1.092418628 \times 10^{-8}$	0.55900006	$6.092006625 \times 10^{-8}$
5	830	0.069375	0.55500001	$1.028298071 \times 10^{-8}$	0.555000053	$5.35014605 \times 10^{-8}$
6	840	0.06325	0.506000004	$4.632485299 \times 10^{-9}$	0.506000026	$2.61574551 \times 10^{-8}$
7	698	0.08033872	0.643000036	$3.620946541 \times 10^{-8}$	0.643000193	$1.937371075 \times 10^{-7}$
8	738	0.07025	0.562000011	$1.143943696 \times 10^{-8}$	0.562000066	$6.616215789 \times 10^{-8}$
9	762	0.066875	0.535000007	$7.483008663 \times 10^{-9}$	0.535000044	$4.403323916 \times 10^{-8}$
10	744	0.069375	0.55480001	$1.026090634 \times 10^{-8}$	0.554800059	$5.966336981 \times 10^{-8}$

5. DISCUSSION OF RESULTS

In the present study the data obtained for the study period were calculated and tabulated. The table shows the traffic intensity, average waiting time of patients in the queue, average waiting time in the system, average number of patients in the system for every day in the study period.

Queuing theory was first developed for studying queuing phenomena in various departments viz., commerce, telephonic traffic, transportation and business industrial servicing etc., [4] Gross and Haris, 1985. Applications of queuing theory on reducing the outpatients waiting time in general hospitals was extensively studied by [5] Adeleke *et al.*,; [6] Fatma Poni Mardiah and Mursyid Hasan Basri, 2013; Similarly use of Multi channel models i.e., M/M/c in health care system to reduce the outpatients waiting time was explained [3] Shanmugasundaram and Banumathi, 2016).

(i) The traffic intensity (ρ) value of maximum no. of individual days was lesser when compared with the traffic intensity values for entire ten days.
(ii) The average waiting time of patients in system (Ws) values of maximum no. of individual days were lesser than the Ws values of entire ten days.
(iii) The table also explains that when the no. of patients arrival rate was lesser, the service time is increased. This may leads to a doctor or health care personnel can render his duty effectively. In vice versa when the patients arrival rate was higher then the service time is reduced considerably. This implies that doctors or health care officials can render their duties comparatively less effective or the possibilities for poor diagonisation.

6. CONCLUSION AND RECOMMENDATION

The queuing theory is a useful statistical technique for solving peculiar problems. Its applications in the organization are indispensable. Excessive waste of time in the hospitals or health centres may lead to patients health complications and in some cases eventual death which may be avoided. As a result, it is recommended that more doctors should be deployed to these centres. It is also recommended that more health care centers should be created to take care of all categories of patients in the community. This will reduce the service time spent by the doctors in attending to patients and hence the service efficiency. From the above discussed results the author suggests that more number of doctors may deployed for better health care service while the outburst of many strange diseases to diagnose better and assure the best service to the people.

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