REMARKS ON STRONGLY $(1,2)^*\pi\alpha$ - CLOSED MAPPINGS

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ABSTRACT

In this paper we investigate the properties of strongly $(1,2)^*\pi\alpha$-closed maps in bitopological spaces.

Keywords: $(1,2)^*\pi\alpha$-closed, $(1,2)^*\pi\alpha$-continuous, $(1,2)^*\pi\alpha$-irresolute.

1. INTRODUCTION

The concept of generalized closed sets was first initiated by Levine [5] in 1970. Malghan [6] introduced generalized closed maps. Lellis Thivagar et al [7] studied stronger form of $\pi\alpha g$ closed mappings in topological spaces. I. Arockiarani and K. Mohana [2, 3] introduced $(1,2)^*\pi\alpha$-continuous functions and $(1,2)^*\pi\alpha$-closed maps in bitopological spaces. In this paper we study the properties of strongly $(1,2)^*\pi\alpha$-closed maps in bitopological spaces.

2. PRELIMINARIES

Throughout this paper by a space $X$ we mean it is a bitopological space. We recall the following definitions which are useful in the sequel.

Definition: 2.1 [4] A subset $S$ of a bitopological space $X$ is said to be $\tau_{1,2}$-open if $S=A \cup B$ where $A \in \tau_1$ and $B \in \tau_2$. A subset $S$ of $X$ is said to be (i) $\tau_{1,2}$-closed if the complement of $S$ is $\tau_{1,2}$-open. (ii) $\tau_{1,2}$-closed if $S$ is both $\tau_{1,2}$-open and $\tau_{1,2}$-closed. Let $S$ be a subset of the bitopological space $X$. Then the $\tau_{1,2}$-interior of $S$ denoted by $\tau_{1,2}$-int($S$) is defined by $\cup \{G: G \subseteq S$ and $G$ is $\tau_{1,2}$-open}$ and the $\tau_{1,2}$-closure of $S$ denoted by $\tau_{1,2}$-cl($S$) is defined by $\cap \{F: S \subseteq F$ and $F$ is $\tau_{1,2}$-closed$$. $\tau_{1,2}$-open sets need not form a topology.

Definition: 2.2 A subset $A$ of a bitopological space $X$ is called

(i) $(1,2)^*\alpha$-regular open [4] if $A = \tau_{1,2} = \text{int}(\tau_{1,2} - c\ell(A))$.

(ii) $(1,2)^*\alpha$-open [4] if $A \subseteq \tau_{1,2} = \text{int}(\tau_{1,2} - c\ell(\tau_{1,2} - \text{int}(A)))$.

(iii) $(1,2)^*\pi\alpha$-closed [1] if $(1,2)^*\alpha - c\ell(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\tau_{1,2} - \pi$-open

The complement of the sets mentioned from (i) and (ii) are called their respective closed sets and the complement of the set mentioned in (iii) is called the respective open set.

The class of all $(1,2)^*\alpha$-open (resp. $(1,2)^*\pi\alpha$-open) subsets of a space $X$ is denoted by $(1,2)^*\alpha O(X)$ ($(1,2)^*\pi\alpha$ $O(X)$).

Definition: 2.5 Let $S$ be a subset of the bitopological space $X$. Then

(i) The $(1,2)^*\alpha$-interior of $S$ denoted by $(1,2)^*\alpha$-int($S$) is defined by $\cup \{G: G \subseteq S$ and $G$ is $(1,2)^*\alpha$-open$}.

(ii) The $(1,2)^*\alpha$-closure of $S$ denoted by $(1,2)^*\alpha$-cl($S$) is defined by $\cap \{F: S \subseteq F$ and $F$ is $(1,2)^*\alpha$-closed$$. 

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Definition: 2.6 [2] A function \( f: X \rightarrow Y \) is called

(i) \((1,2)^* \)-\( ag \)- irresolute if \( f^{-1}(V) \) is \((1,2)^* \)-\( ag \)-closed in \( X \), for every \((1,2)^* \)-\( ag \)-closed set \( V \) of \( Y \).

(ii) \((1,2)^* \)-continuous if \( f^{-1}(V) \) is \((1,2)^* \)-\( ag \)-closed in \( X \), for every \( \sigma_{1,2} \)-closed set \( V \) of \( Y \).

Definition 2.7 [3] A map \( f: X \rightarrow Y \) is called

(i) \((1,2)^* \)-closed if \( f(V) \) is \((1,2)^* \)-\( ag \)-closed in \( Y \), for every \( \tau_{1,2} \)-closed set \( V \) of \( X \).

(ii) almost \((1,2)^* \)-\( ag \)-closed if \( f(V) \) is \((1,2)^* \)-\( ag \)-closed in \( Y \), for every \((1,2)^* \)-regular closed set \( V \) of \( X \).

(iii) \((1,2)^* \)-\( ag \)-closed in \( Y \), for every \((1,2)^* \)-\( ag \)-closed set \( V \) of \( X \).

(iv) \((1,2)^* \)-\( ag \)-irresolute if \( f^{-1}(V) \) is \( \tau_{1,2} \)-\( ag \)-closed in \( X \), for every \( \sigma_{1,2} \)-\( ag \)-closed set \( V \) of \( Y \).

3. STRONGLY \((1,2)^* \)-\( ag \)-CLOSED MAPS

Definition: 3.1 A map \( f: X \rightarrow Y \) is called strongly \((1,2)^* \)-\( ag \)-closed or \((1,2)^* \)-\( M-ag \)-closed [3] (resp. strongly \((1,2)^* \)-\( M-ag \)-open or \((1,2)^* \)-\( M-ag \)-open) if the image of every \((1,2)^* \)-\( ag \)-closed (resp. \((1,2)^* \)-\( ag \)-open) set in \( X \) is \((1,2)^* \)-\( ag \)-closed (resp. \((1,2)^* \)-\( ag \)-open) set in \( Y \).

Theorem: 3.2 If \( f: X \rightarrow Y \) is \((1,2)^* \)-\( ag \)-closed map and \( g: Y \rightarrow Z \) is \((1,2)^* \)-\( ag \)-closed map then \( g \circ f: X \rightarrow Z \) is \((1,2)^* \)-\( ag \)-closed map.

Proof: The proof is obvious.

Theorem: 3.3 If \( f: X \rightarrow Y \) is \((1,2)^* \)-closed map and \( g: Y \rightarrow Z \) is \((1,2)^* \)-\( ag \)-closed map then \( g \circ f: X \rightarrow Z \) is \((1,2)^* \)-\( ag \)-closed map.

Proof: Straight forward.

Remark: 3.4 If \( f: X \rightarrow Y \) is \((1,2)^* \)-\( ag \)-closed map and \( g: Y \rightarrow Z \) is \((1,2)^* \)-closed map then also the composite map \( g \circ f \) may not be \((1,2)^* \)-\( ag \)-closed map. The following example shows this result.

Example: 3.5 Let \( X = \{a, b, c\} = Y, Z = \{a, b, c, d\}, \tau_1 = \{\phi, X, \{a\}\}, \tau_2 = \{\phi, X, \{b\}\}, \sigma_1 = \{\phi, Y, \{b\}\}, \sigma_2 = \{\phi, Y, \{b, c\}\}, \eta_1 = \{\phi, Z, \{a\}, \{b, c\}\}, \eta_2 = \{\phi, Z, \{a, c\}\} \) and \( f: X \rightarrow Y \) be an identity map. \( g: Y \rightarrow Z \) be a map defined by \( f[a] = \{a\}, f[b] = \{b\}, f[c] = \{d\} = \{c\} \). Then \( f \) is \((1,2)^* \)-\( ag \)-closed map and \( g \) is \((1,2)^* \)-\( ag \)-closed map, but \( g \circ f \) is not \((1,2)^* \)-\( ag \)-closed map.

Proposition: 3.6 Every strongly \((1,2)^* \)-\( ag \)-closed map is almost \((1,2)^* \)-\( ag \)-closed map but not conversely.

Example: 3.7 Let \( X = \{a, b, c\} = Y, \tau_1 = \{\phi, X, \{a, b\}\}, \tau_2 = \{\phi, X, \{c\}, \{b, d\}, \{b, c, d\}\}, \sigma_1 = \{\phi, Y, \{d\}, \{a, d\}\}, \sigma_2 = \{\phi, Y, \{a\}, \{c\}, \{a, c, d\}\} \) and \( f: X \rightarrow Y \) be an identity map. Then \( f \) is not \((1,2)^* \)-\( ag \)-closed map but not strongly \((1,2)^* \)-\( ag \)-closed map.

Theorem: 3.8 The composite mapping of two strongly \((1,2)^* \)-\( ag \)-closed maps is strongly \((1,2)^* \)-\( ag \)-closed map.

Proof: The proof follows from definitions.

Remark: 3.9 The concept of strongly \((1,2)^* \)-\( ag \)-closed map is independent from the concept of \((1,2)^* \)-\( ag \)-irresolute map as shown in the following example.

Example: 3.10 Let \( X = \{a, b, c\} = Y, \tau_1 = \{\phi, X, \{b\}\}, \tau_2 = \{\phi, X, \{a, b\}\}, \sigma_1 = \{\phi, Y, \{a, b\}\}, \sigma_2 = \{\phi, Y, \{a\}\} \) respectively. Let \( f: X \rightarrow Y \) be an identity map. Then \( a, b \) are \((1,2)^* \)-\( ag \)-closed sets of \( Y \), but \( f^{-1} \( \{a, b\}\) \) is not \((1,2)^* \)-\( ag \)-closed set of \( X \). This implies that \( f \) is not \((1,2)^* \)-\( ag \)-irresolute. However, \( f \) is a strongly \((1,2)^* \)-\( ag \)-closed map.
Example: 3.11 Let $X = \{a, b, c\} \rightarrow Y$, $\tau_1 = \{\emptyset, X, \{a\}\}$, $\tau_2 = \{\emptyset, Y, \{b\}\}$, $\sigma_2 = \{\emptyset, Y, \{a\}\}$ and $f : X \rightarrow Y$ be an identity maps. Then $f$ is $(1, 2)^*\pi g a - \text{irresolute}, \text{but not strongly } (1, 2)^*\pi g a - \text{closed map.}$

Theorem: 3.12 A map $f : X \rightarrow Y$ is strongly $(1, 2)^*\pi g a - \text{closed if and only if for each subset } B \text{ of } Y$ and for each $(1, 2)^*\pi g a - \text{open set } U \text{ of } X \text{ containing } f^{-1}(B)$, there exists an $(1, 2)^*\pi g a - \text{open set } V \text{ of } Y, \text{ such that } B \subseteq V \text{ and } f^{-1}(V) \subseteq U$.

Proof: Let $B$ be any subset of $Y$ and $U$ be an $(1, 2)^*\pi g a - \text{open set of } X \text{ containing } f^{-1}(B)$. Put $V = Y - f(X - U)$. Then $V$ is $(1, 2)^*\pi g a - \text{open set in } Y \text{ containing } B$ such that $f^{-1}(V) \subseteq U$.

Sufficiency: Let $F$ be any $(1, 2)^*\pi g a - \text{closed subset of } X$. Then $f^{-1}(Y - f(F)) \subseteq X - F$. Put $B = Y - f(F)$. Also, $X-F$ is $(1, 2)^*\pi g a - \text{open set in } X$. There exists an $(1, 2)^*\pi g a - \text{open set } V$ of $Y$ such that $B \subseteq V$ and $f^{-1}(V) \subseteq X - F$. Hence $f(F)$ is $(1, 2)^*\pi g a - \text{closed set in } Y$.

Proposition: 3.13 If $f : X \rightarrow Y$ is $(1, 2)^*\pi g a - \text{irresolute and } (1, 2)^*\text{-pre-} \alpha - \text{closed then } f$ is a strongly $(1, 2)^*\pi g a - \text{closed map.}$

Proof: Let $A$ be an $(1, 2)^*\pi g a - \text{closed set in } X$. Let $V$ be any $\tau_{1,2} - \pi - \text{open set in } Y$ containing $f(A)$. Then $A \subseteq f^{-1}(V)$. Since $f$ is $\tau_{1,2} - \pi - \text{irresolute, } f^{-1}(V)$ is $\tau_{1,2} - \pi - \text{open set in } X$. Since $A$ is $(1, 2)^*\pi g a - \text{closed in } X, (1, 2)^* - \alpha \text{cl}(A) \subseteq f^{-1}(V)$ and hence $f(A) \subseteq f((1, 2)^* - \alpha \text{cl}(A)) \subseteq V$. Since $f$ is $(1, 2)^*\pi g a - \text{closed and } (1, 2)^* - \alpha \text{cl}(A)$ is $(1, 2)^*\pi g a - \alpha - \text{closed in } X$, $f((1, 2)^* - \alpha \text{cl}(A))$ is $(1, 2)^*\pi g a - \alpha - \text{closed in } Y$ and hence $(1, 2)^* - \alpha \text{cl}(f(A)) \subseteq (1, 2)^* - \alpha \text{cl}(f((1, 2)^* - \alpha \text{cl}(A))) \subseteq V$. This shows that $f(A)$ is $(1, 2)^*\pi g a - \text{closed set in } Y$. Hence $f$ is strongly $(1, 2)^*\pi g a - \text{closed map.}$

Theorem: 3.14 Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be two maps.

(i) If $f$ is $(1, 2)^*\pi g a - \text{closed map and } g$ is strongly $(1, 2)^*\pi g a - \text{closed then } g \circ f$ is $(1, 2)^*\pi g a - \text{closed map.}$

(ii) If $f$ is $(1, 2)^*\pi g a - \text{closed map and } g$ is $(1, 2)^* - \pi - \text{irresolute and } (1, 2)^*\pi g a - \text{closed then } g \circ f$ is $(1, 2)^*\pi g a - \text{closed map.}$

(iii) If $g \circ f$ is strongly $(1, 2)^*\pi g a - \text{closed and } f$ is a $(1, 2)^*\text{-continuous surjection then } g \circ f$ is $(1, 2)^*\pi g a - \text{closed map.}$

Proof:

(i) By Proposition 2. 13[3], the proof is obvious.

(ii) By Proposition 3.13, $g$ is strongly $(1, 2)^*\pi g a - \text{closed map. Hence by (i), } g \circ f$ is $(1, 2)^*\pi g a - \text{closed map.}$

(iii) Straight forward.

Definition: 3.15

A subset $A$ of a bitopological space $X$ is called $\tau_{1,2} - \pi - \text{closed space if every } \tau_{1,2} - \text{closed set is } \tau_{1,2} - \pi - \text{closed set.}$

Theorem: 3.16 If $f : X \rightarrow Y$ is $(1, 2)^*\pi g a - \text{continuous strongly } (1, 2)^*\pi g a - \text{open bijective map and if } X$ is a $(1, 2)^*\pi g a - \text{normal space and } Y$ is $\sigma_{1,2} - \pi - \text{closed space, then } Y$ is $(1, 2)^*\pi g a - \text{normal.}$

Proof: Let $A$ and $B$ be disjoint $\sigma_{1,2} - \pi - \text{closed sets of } Y$. Since $f$ is $(1, 2)^*\pi g a - \text{continuous Bijective, } f^{-1}(A)$ and $f^{-1}(B)$ are disjoint $\tau_{1,2} - \text{closed sets of } X$. Since $X$ is $(1, 2)^*\pi g a - \text{normal, there exist disjoint } \tau_{1,2} - \text{open sets } G$ and $H$ of $X$, such that $G \supseteq f^{-1}(A)$ and $H \supseteq f^{-1}(B)$. Every $\tau_{1,2} - \text{open set is } (1, 2)^*\pi g a - \text{open and hence } G$ and $H$ are disjoint $(1, 2)^*\pi g a - \text{open sets of } X$.

Since $f$ is $(1, 2)^*\pi g a - \text{continuous bijective, } f(G)$ and $f(H)$ are disjoint $(1, 2)^*\pi g a - \text{open sets of } Y$ containing $A$ and $B$ respectively. Since $Y$ is $\sigma_{1,2} - \pi - \text{closed space, } A$ and $B$ are $\sigma_{1,2} - \pi - \text{closed sets in } Y$.

Then we have $(1, 2)^* - \alpha \text{int}(f(G)) \supseteq A$ and $(1, 2)^* - \alpha \text{int}(f(H)) \supseteq B$
and $(1, 2)^* - \alpha \text{int}(f(G)) \cap (1, 2)^* - \alpha \text{int}(f(H)) \subseteq f(G) \cap f(H) = \emptyset$. 

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Therefore, there exist disjoint $(1, 2)^\ast$-$\alpha$-open sets $(1, 2)^\ast - \alpha \text{int}(f(G))$ say $U$ and $(1, 2)^\ast - \alpha \text{int}(f(H))$ say $V$ of $Y$ containing $A$ and $B$ respectively. $U$ and $V$ are $(1, 2)^\ast$-$\alpha$-open sets imply

$$U \subseteq \sigma_{1,2} - \text{int}(\sigma_{1,2} - \text{cl}(\sigma_{1,2} - \text{int}(U))) \quad \text{and} \quad V \subseteq \sigma_{1,2} - \text{int}(\sigma_{1,2} - \text{cl}(\sigma_{1,2} - \text{int}(V))).$$

Since $\sigma_{1,2} - \text{int}(\sigma_{1,2} - \text{cl}(\sigma_{1,2} - \text{int}(U))) \cap \sigma_{1,2} - \text{int}(\sigma_{1,2} - \text{cl}(\sigma_{1,2} - \text{int}(V))) = \phi,$

$$A \subseteq (1, 2)^\ast - \alpha \text{int}(f(G)) = U \subseteq \sigma_{1,2} - \text{int}(\sigma_{1,2} - \text{cl}(\sigma_{1,2} - \text{int}(U))) \quad \text{and}$$
$$B \subseteq (1, 2)^\ast - \alpha \text{int}(f(H)) = V \subseteq \sigma_{1,2} - \text{int}(\sigma_{1,2} - \text{cl}(\sigma_{1,2} - \text{int}(V))).$$

Hence, $Y$ is $(1, 2)^\ast$-normal.

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